Pecuniary externalities and wealth inequality*

Konstantinos Angelopoulos
University of Glasgow and CESifo

Spyridon Lazarakis
University of Glasgow

James Malley
University of Glasgow and CESifo

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Abstract

Differences in earnings processes between education groups create a pecuniary externality that works via group-level savings and the interest rate. Using longitudinal data for Great Britain, we find that the university educated have higher average wealth, higher net labour income risk but lower within group wealth inequality. We calibrate an incomplete markets model with *ex ante* heterogeneous households and find that, due to the externality, its predictions regarding wealth inequality within and between the groups cohere with the data. The externality also works to increase aggregate wealth inequality and bring this prediction closer to the data than a model with identical households.

Keywords: incomplete markets, education differences, pecuniary externalities

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1 Introduction

There is an extensive literature which has examined wealth inequality under idiosyncratic earnings shocks when agents cannot fully insure against uncertain income streams. The benchmark incomplete markets model featuring stochastic labour income, one asset and *ex ante* identical agents captures qualitative properties of the wealth distribution (see, Bewley (1986), Imrohoroglu (1989), Huggett (1993) and Aiyagari (1994)). However, quantitatively it under predicts the extent of inequality, both overall (e.g. as captured by measures such as the Gini index) and at the top end of the wealth distribution.

In the benchmark model, wealth differences in the stationary equilibrium are attributed to exogenous conditions that change over time. In this framework, different histories of earnings shocks received by individuals imply heterogenous choices for wealth accumulation. Naturally, then, the importance of increased earnings risk or earnings inequality in creating higher wealth inequality has long been noted in the literature (see e.g. Castaneda *et al.* (2003), De Nardi *et al.* (2016) and Benhabib *et al.* (2017)). However, earnings risk is not homogeneous (see e.g. Meghir and Pistaferri (2004) and Chang and Kim (2006)) nor are mean earnings the same between different groups in the population (see, e.g. Heathcote *at al.* (2010) and Blundell and Etheridge (2010)).

Both of these considerations imply differences between economic agents with respect to their earnings processes which may have a critical bearing on wealth inequality. An important heterogeneity in this respect is associated with educational attainment which can proxy differences in ability and skills at the beginning of working life. These differences effectively represent permanent distinctions between workers and are fundamental determinants of an individual's level of earnings and exposure to earnings risk.[1] They thus create *ex ante* heterogeneity which can contribute to wealth inequality.[2]

[1]Analysis of the importance of skills and education for inequality in a historical context can be found in Goldin and Katz (2008). Several studies have also documented differences in earnings risk between skilled and unskilled groups associated with university education (see e.g. Castro and Coen-Pirani (2008) and Hagedorn *et al.* (2016) for the US, as well as Angelopoulos *et al.* (2017) for Great Britain).

[2]Differences in initial conditions on assets do not affect inequality in this class of models in a stationary equilibrium that is characterised by a unique invariant distribution. However, *ex ante* differences in e.g. skill, preferences, or access to markets, which capture predetermined and permanent differences between the agents, do affect the equilibrium and thus potentially can affect inequality.

1.1 Empirical facts

To further motivate our focus on *ex ante* heterogeneity relating to differences in university education, Table 1 summarises some key empirical facts relating to earnings and wealth inequality in Great Britain (GB). We use data on household annual earnings post policy (i.e. net labour income) from the Wealth and Asset Survey (WAS) and the British Household Panel Survey (BHPS) as well as household net worth data from the WAS. For each measure of inequality reported in Table 1, we show the average statistic from the cross sectional samples over the entire time period for which they are available.[3]

Table 1: Income and Wealth Inequality in Great Britain

	$\frac{Y_u}{Y_b}$	Gini	$\frac{sd}{mean}$	$\frac{mean}{median}$	VarLog
		WAS Net Labour Income (waves 3-5)			
Uni	-	0.296	0.614	1.131	0.340
Non-Uni	-	0.267	0.517	1.101	0.264
Total	1.535	0.299	0.619	1.145	0.321
		BHPS Net Labour Income (1991-2008)			
Uni	-	0.279	0.621	1.100	0.360
Non-Uni	-	0.279	0.527	1.087	0.307
Total	1.569	0.297	0.609	1.111	0.343

	$\frac{a_u}{a_b}$	Gini	$\frac{sd}{mean}$	$\frac{mean}{median}$	top 10%
		WAS Net Worth (waves 1-5)			
Uni	-	0.661	2.217	2.038	0.483
Non-Uni	-	0.731	2.232	2.871	0.507
Total	2.270	0.720	2.432	2.398	0.519

Notes: (i) WAS waves 1-5 correspond to years 2006-08, 2008-10, 2010-12, 2012-14 and 2014-16 respectively; (ii) household net income data in WAS is only available in waves 3-5; (iii) since a significant proportion of the population have negative wealth, we use the top 10% measure of inequality for wealth instead of the VarLog measure used for income; (iv) $\frac{Y_u}{Y_b}$ and $\frac{a_u}{a_b}$ are the ratios of mean net labour income, Y, and net worth, a, between university (u) and below university (b) groups.

The first observation by looking at column 1 in Table 1 is that net labour income and wealth are on average higher for the university educated group,

[3] These datasets, as well as variable definitions and sample selection are discussed in detail in Section 3 and Appendix A. Further statistics obtained from analysing these data are shown in Sections 4 and 5, while in Appendix A we also report the wealth distribution statistics for each wave in the data.

implying a positive relationship between mean earnings and mean wealth. Second, columns (2-4) show that wealth inequality is significantly higher than net labour income inequality. Finally, comparing the university and non-university educated groups in columns 2-4, we can see that while net labour income inequality is generally higher for the university educated group, its wealth inequality is lower.[4] In other words, the group with the higher net labour income inequality has lower wealth inequality which, as pointed out above, is the opposite of what might be generally expected.

In an attempt to reconcile larger within-group earnings inequality for the university educated with smaller within-group wealth inequality, this paper addresses the following questions: (i) Can the benchmark incomplete markets model as in Aiyagari (1994), when extended to allow for *ex ante* heterogeneity capturing the differences in earnings processes between the two education groups, deliver predictions regarding wealth inequality that cohere with the data? (ii) Since higher earnings inequality, *ceteris paribus,* tends to create higher wealth inequality, what transmission mechanism might reverse this expectation for the university and below university educated? (iii) Does this transmission mechanism contribute to better explaining wealth inequality at the aggregate level?

1.2 Approach and key findings

To address the questions posed above, we specify an otherwise standard Aiyagari (1994) model with state-dependent (Markovian) stochastic earnings processes and let households belong to one of two groups. These groups differ in their earnings processes, both in the state-space and in the transition matrix for idiosyncratic earnings shocks. Using recent advances in theoretical research (see, e.g. Acikgoz (2018)), this model can be shown to have a well-defined stationary equilibrium with a unique invariant wealth distribution for each type of household.[5]

We calibrate the model to GB, using earnings data from the WAS and BHPS. We find that the model predicts wealth inequality both within and between the university and non-university educated groups that is consistent with the data. In particular, the university educated group has significantly lower within group wealth inequality than the non-university educated group, despite having more persistent and volatile stochastic earnings processes. Using the BHPS earnings data, which are typically used in studies of earnings

[4] As we shall see in Section 3, the variance of the idiosyncratic component of net labour income is also higher for the university educated group.

[5] Acemoglu and Jensen (2015) have also provided a general proof of existence of equilibrium for this class of models.

inequality and dynamics in the U.K., the model effectively matches the difference in the wealth Ginis between the two groups that are observed in the data and predicts a mean wealth ratio that is also very close to the data. The model also predicts a Gini wealth inequality index of about 0.63 and, although it still under-predicts the extent of income inequality at the very top end (top 1%), it produces very good predictions for the remaining distribution, especially up to the top 5%.

The main reason why *ex ante* heterogeneity matters for wealth inequality is that the differences in the processes for earnings between the groups create a pecuniary externality that affects between and within-group inequality via aggregate savings and the interest rate. In particular, earnings differences, both in terms of mean earnings and idiosyncratic uncertainty, imply different asset supply functions. The equilibrium interest rate is determined by the per capital asset supply function, which is higher (lower) than the asset supply functions for the university (non-university) educated. In other words, the savings of each group move the market interest rate away from the equilibrium level that would be consistent with the asset supply of each group. Consequently, households in the non-university and university educated groups lower and raise their savings respectively. This in turn implies that within group wealth inequality is increased for the non-university and decreased for the university educated, conditional on the earnings shocks that the households in each group receive.[6]

The pecuniary externality channel is strong enough to create wealth inequality effects so that the two groups are ranked in the reverse order in terms of wealth inequality, compared with the ranking implied by the earnings risk. In other words, while we may expect the university group to have higher wealth inequality because they face higher earnings inequality, they are able to accumulate more wealth due to the pecuniary externality which allows them to self-insure and thus reduce within group wealth inequality.[7] Therefore, *ex ante* heterogeneity in this framework generates a between-group pecuniary externality, which is critical in generating the ranking of within group wealth inequality between the two groups seen in the data.

[6] The importance of pecuniary externalities implicit in the benchmark model with *ex ante* identical agents, for the efficiency properties of general equilibrium, has been pointed out in the literature (see e.g. Greewald and Stiglitz (1986) and Davilla *et al.* (2012)). Here we examine pecuniary externalities arising from *ex ante* skill heterogeneity, and focus on their implications for wealth inequality, as opposed to efficiency.

[7] Higher mean earnings lead to higher mean savings and hence also contribute to lower inequality for the university educated. However, our analysis later implies that higher mean earnings alone are not sufficient to explain the difference in wealth inequality between the two groups observed in the data.

The pecuniary externality channel also works as an amplification mechanism which increases aggregate wealth inequality. In particular, compared with the model with *ex ante* identical agents it increases the Gini index by 2.3 units.[8] We show that the pecuniary externality mechanism is stronger the bigger the difference in mean earnings between the two groups and the more earnings uncertainty the larger mean earnings group faces relative to the lower mean earnings group. It is thus important that the mean earnings gap works in the same direction as the earnings risk gap.

The literature has explored several extensions to the benchmark model which can contribute to improving its predictions regarding wealth inequality (see e.g. the reviews in Quadrini and Rios-Rull (2015) and Krueger *et al.* (2016)). For example, the literature has analysed factors relating to work effort and occupational choice (e.g. Quadrini (2000)), bequests and inheritance motives (e.g. De Nardi (2004)) and differences in preferences (e.g. Krusell and Smith (1998)). While *ex ante*, permanent differences between the agents regarding their productive capacity, along the lines of this paper, have also been considered (see e.g. Castaneda *et al.* (1998), Quadrini (2000) and Guvenen (2006)), these have not always been shown to help the model improve its predictions regarding inequality (see e.g. De Nardi (2015) and Krueger *et al.* (2016)). In general, a large part of the interest in the literature has been in mechanisms that can improve the predictions of the model regarding the upper tail of the wealth distribution. In this paper, we analyse a situation where *ex ante* heterogeneity creates a pecuniary externality that increases wealth inequality across the main part of the distribution.

The rest of the paper is organised as follows. We first present the model and data/calibration in Sections 2 and 3 respectively. The model is discussed in some detail to formally introduce the economic environment and clarify the economic quantities used later. We next examine the quantitative implications of the model. We first evaluate the predictions of the model with respect to between and within group wealth inequality in Section 4. We then study the pecuniary externality mechanism which is at the heart of the nexus between *ex ante* skill heterogeneity and wealth inequality. Following this, we analyse in Section 5 the predictions of the model regarding aggregate wealth inequality and the contribution of the pecuniary externality channel in this direction. Finally, we present our conclusions in Section 6 and we provide Appendices including details relating to the data as well as the computational algorithm.

[8] The increase in the Gini by 2.3 units is equivalent to that effect on wealth inequality of raising the variance of the estimated earnings process by nearly 50%.

2 Economic Environment

We compute the long-run *stationary equilibrium* of an economy that is populated by a continuum of infinitely lived agents (households) distributed on the interval $I = [0, 1]$ with measure φ. Time is discrete and denoted by $t = 0, 1, 2, ...$There are two types of households, university educated households (Uni), which belong to a set $I^u \subset I$ and households that have a level of education below university (Non-Uni), which belong to a set $I^b \subset I$, such that $I^u \cup I^b = I$ and $I^u \cap I^b = \emptyset$. The proportions of university and non-university educated households are given respectively by $n^u \equiv \int_{I^u} i\varphi(di)$ and $n^b \equiv \int_{I^b} i\varphi(di) = 1 - n^u$. Therefore, there is *ex ante* heterogeneity in the population determined by the education level of the household, which is assumed to be given.[9]

All households have exogenous labour supply and derive utility from consuming one good that can be acquired by spending either labour income or accumulated savings. Households are identical in their preferences. However, their labour income depends on their education level, since it determines their skill set and thus affects their productivity. In particular, households' predictable earnings component differs, reflecting their different education. This implies that the two groups of households face different effective wage rates. In addition, each household is subject to idiosyncratic shocks, which affect labour income, by determining the residual, unpredictable earnings component. Households draw idiosyncratic shocks independently from a Markov chain which differs for university and non-university educated households. Both the state-space and the transition matrix differ across the two household types, implying that the level of labour income and the size and persistence of productivity shocks differ for each household type, reflecting different opportunities and earnings risk.

There is a single asset in the economy implying that households cannot fully insure themselves against shocks to labour income. In a stationary equilibrium, aggregate quantities are constant. In what follows we present the problem for a "typical" university educated household, denoted by the superscript u, and the problem for a "typical" non-university educated household, denoted by the superscript b.

[9]The earnings and wealth data in the U.K. refer to households whose head is University educated or not. At the age of 25, which is the minimum age for heads of households in our sample, the education level is predetermined.

2.1 Households

Households have different skill levels ζ^h, $h = u, b$. Denote the idiosyncratic component of labour income of a typical household $h = u, b$ at time t by s_t^h, so that labour income is given by $w\zeta^h s_t^h$, where w is an average wage rate. Therefore, the idiosyncratic earnings shock s_t^h contains shocks that may affect the individual's work hours in a time period and/or her productivity.[10] The idiosyncratic earnings shock follows a Markov chain. We follow Acikgoz (2018) and assume that the stochastic process $\left(s_t^h\right)_{t=0}^{\infty}$ satisfies `Assumption 1`, where note that part (b) implies that the Markov chain has a unique invariant distribution, with probability measure that we denote by ξ^h.[11]

`Assumption 1`

(a) The process s_t^h is an m-state Markov chain with state space S^h and transition matrix Q^h. The state space $S^h = [\overline{s}_1^h, \overline{s}_2^h, ..., \overline{s}_m^h]$ is ordered according to $\overline{s}_1^h > 0$, $\overline{s}_{j+1}^h > \overline{s}_j^h$, $j = 1, ..., m-1$ and has the natural σ-algebra $\mathcal{S}^h$ made up of all subsets of S^h. The elements of the transition matrix Q^h are denoted $\pi^h\left(s_{t+1}^h|s_t^h\right) = Pr(s_{t+1}^h = \overline{s}_{j'}^h|s_t^h = \overline{s}_j^h)$.

(b) The Markov chain is *irreducible* and *aperiodic*, i.e. there exists a $k_0 \in \mathbb{N}$ such that $\left[\pi^h\left(s_{t+1}^h|s_t^h\right)\right]^{(k)} > 0$ for all $\left(s_{t+1}^h, s_t^h\right) \in S^h$ and $k > k_0$. Moreover, $\pi^h\left(\overline{s}_1^h|\overline{s}_1^h\right) > 0$.

Households' earnings shock s_t^h is observed at the beginning of period t. They also receive interest income from accumulated assets ra_t^h, and use their income for consumption and to invest in future assets, subject to the budget constraint for each $h = u, b$:

$$c_t^h + a_{t+1}^h = (1+r)\,a_t^h + w\zeta^h s_t^h, \tag{1}$$

where $c^h \geq 0$, $a_t^h \geq -\phi^h$ and $-\phi^h < 0$ denotes a borrowing limit on the household. The set comprising a_t^h is defined as $\mathcal{A}^h = [-\phi^h, +\infty)$. The prices (interest rate r and wage rate w) are assumed to be fixed and non-random quantities. This holds if the household's actions take place in a stationary equilibrium, which is defined below. Households assess consumption streams

[10] Examples include the quality of the match between employer and employee, health shocks, or changes in personal circumstances.

[11] On notation. For any set D in some n-dimensional Euclidean space $\mathbb{R}^n$, $\mathcal{B}(D)$ denotes the Borel $\sigma-$algebra of D. The set of probability measures on the measurable space $(D, \mathcal{B}(D))$, is denoted by $\mathcal{P}(D)$.

with an intertemporal discount factor $\beta \in (0,1)$, using a per period utility function $u(c_t^h)$, which satisfies the following assumption:

`Assumption 2`

The function $u : [0,+\infty) \to \mathbb{R}$ is bounded, twice continuously differentiable, strictly increasing and strictly concave. Furthermore, it satisfies the conditions $\lim_{c\to 0} u_c(c) = +\infty$, $\lim_{c\to\infty} u_c(c) = 0$ and $\lim_{c\to\infty} \inf -\frac{u_{cc}(c)}{u_c(c)} = 0$.

The assumptions imposed on the utility function are typically employed in the literature of partial equilibrium income fluctuation problems (see e.g. Miao (2014, ch. 8)) and in the literature relating to incomplete markets with heterogeneous agents in general equilibrium ((see e.g. Aiyagari (1994) and Acikgoz (2018))). The assumption that $\lim_{c\to\infty} \inf -\frac{u_{cc}(c)}{u_c(c)} = 0$ implies that the degree of absolute risk aversion tends to zero as consumption tends to infinity.[12]

The interest rate and wage rate are taken as given and satisfy $r > -1$ and $w > 0$. Moreover, as has been shown (see e.g. Aiyagari (1994), Miao (2014, ch. 8) and Acikgoz (2018)), a necessary condition for an equilibrium with finite assets at the household level in this class of models is that $\beta(1+r) < 1$. Borrowing limits are imposed following e.g. Aiyagari (1994), i.e. assets must satisfy:

$$\begin{aligned} &a_t^h \geq -\phi^h, \text{ where} \\ &\phi^h = \min\left[\gamma, \frac{\bar{s}_1^h \zeta^h w}{r}\right], \text{ if } r > 0 \text{ or} \\ &\phi^h = \gamma, \text{ if } r \leq 0, \end{aligned} \tag{2}$$

and $\gamma > 0$ is arbitrary parameter, capturing an *ad hoc* debt limit. This restriction implies that even if the financial markets have the power to confiscate all of the income of the household, they would never lend so much that the household reaches an asset position where its lifetime labour income (assuming the worst earnings shock is always realised) was not sufficient to repay debt. This requires that $-r\phi^h + w\zeta^h \bar{s}_1^h \geq 0$. Hence, if the household is at the borrowing limit and receives the worst case labour income shock, it always has at least one option to have non-negative consumption, by borrowing again the maximum possible. The various assumptions on prices are summarised below.

[12]Boundedness is not needed for equilibrium (see Acikgoz (2018)). In the calibration and computation below we will use a CRRA utility function which is not bounded below. However, we will work there with a compact set for assets, needed for computation, which, given the continuity of the utility function, implies boundedness.

`Assumption 3`

Assume that $(1+r)>0$, $w>0$ and $\beta(1+r)<1$.

The problem of the typical household $h=u,b$ is summarised as follows. For given values of (w,r) that satisfy `Assumption 3` and given initial values $(a_0^h,s_0^h)\in\mathcal{A}^h\times S^h$, the household chooses plans $\left(c_t^h\right)_{t=0}^{\infty}$ and $\left(a_{t+1}^h\right)_{t=0}^{\infty}$ that solve the maximisation problem:

$$V^h(a_0,s_0)=\max_{\left(c_t^h,a_{t+1}^h\right)_{t=0}^{\infty}} E_0\sum_{t=0}^{\infty}\beta^t u(c_t^h), \tag{3}$$

subject to (2), where $\beta\in(0,1)$, $c_t^h\geq 0$ is given by (1), $u(c_t^h)$ satisfies `Assumption 2` and s_t^h satisfies `Assumption 1`. To obtain the dynamic programming formulation of the household's problem, let $v^h\left(a_t^h,s_t^h;w,r\right)$ denote the optimal value of the objective function starting from asset-earnings state $\left(a_t^h,s_t^h\right)$ and given the interest and wage rate. The Bellman equation is:

$$\begin{aligned}&v^h\left(a_t^h,s_t^h;w,r\right)=\\&=\max_{\substack{a_{t+1}^h\geq-\phi^h\\ c_t^h\geq 0}}\{u(c_t^i)+\beta\sum_{s_{t+1}^h\in S^h}\pi^h\left(s_{t+1}^h|s_t^h\right)v^h\left(a_{t+1}^h,s_{t+1}^h;w,r\right)\}.\end{aligned} \tag{4}$$

In this case, we aim to find the value function $v^h\left(a_t^h,s_t^h;w,r\right)$ and the policy functions $a_{t+1}^h=g^h\left(a_t^h,s_t^h;w,r\right)$ and $c_t^h=q^h\left(a_t^h,s_t^h;w,r\right)$, which generate the optimal sequences $\left(a_{t+1}^{*h}\right)_{t=0}^{\infty}$ and $\left(c_t^{*h}\right)_{t=0}^{\infty}$ that solve (3).[13] Standard dynamic programming results imply that the policy functions exist, are unique and continuous.

Following e.g. Stokey *et al.* (1989, ch. 9), we define $\Lambda^h\left[(a,s),A\times B\right]:\left(\mathcal{A}^h\times S^h\right)\times\left(\mathcal{B}\left(\mathcal{A}^h\right)\times\mathcal{S}^h\right)\to[0,1]$, for all $a\times s\in\mathcal{A}^h\times S^h$, $A\times B\in\mathcal{B}\left(\mathcal{A}^h\right)\times\mathcal{S}^h$, to be the transition functions on $\left(\mathcal{A}^h\times S^h\right)$, induced by the Markov processes $\left(s_t^h\right)_{t=0}^{\infty}$ and the optimal policies $g^h\left(a_t^h,s_t^h\right)$. In particular, the transition function is given by:

$$\Lambda^h\left[(a,s),A\times B\right]=\begin{Bmatrix}\Pr\left(s_{t+1}^h\in B|s_t^h=s\right),\text{ if } g^h(a,s)\in A\\ 0,\text{ if } g^h(a,s)\notin A\end{Bmatrix}. \tag{5}$$

Given `Assumptions 1-3`, Proposition 5 in Acikgoz (2018) implies that the Markov process on the joint state-space $\left(\mathcal{A}^h\times S^h\right)$ with transition matrix Λ^h

[13]In what follows, we suppress the explicit dependence of the value and policy functions on aggregate prices to simplify notation.

has, for each $h = u, b$, a unique invariant distribution denoted by $\lambda^h (A \times B)$. Furthermore, Proposition 6 in Acikgoz (2018) implies that assets for the typical household tend to infinity when $\beta(1 + r) \to 1$. Moreover, Theorem 1 in Acikgoz (2018) implies that the expected value of assets using the invariant distribution is continuous in the net interest rate, r.

2.2 Firm

A representative firm operates the technology to transform accumulated assets from the households to capital to be used in production and an aggregate constant returns to scale production function, using as inputs the average (per capita) levels of capital K and employment L. The production function is given by $F(K, L)$ and is assumed to satisfy the usual Inada conditions. In particular, F is continuously differentiable in the interior of its domain, strictly increasing, strictly concave and satisfies: $F(0, L) = 0$, $F_{KL} > 0$, $\lim_{K \to 0} F_K(K, L) \to +\infty$ and $\lim_{K \to \infty} F_K(K, L) \to 0$. The capital stock depreciates at a constant rate $\delta \in (0, 1)$. The firm takes the interest and wage rate as given and chooses capital and employment to maximise profits, which gives the standard first order conditions, defining factor input prices equal to the relevant marginal products:

$$w = \partial F(K, L)/\partial L, \tag{6}$$

$$r = \partial F(K, L)/\partial K - \delta. \tag{7}$$

2.3 General equilibrium

We define a stationary recursive equilibrium following e.g. Ljungqvist and Sargent (2012, ch. 18), Miao (2014, ch. 17) and Acikgoz (2018). Aggregation over the households can be obtained by using the methods discussed e.g. in Acemoglu and Jensen (2015). The versions of the Strong Law of Large Numbers delivered by these methods (see e.g. Uhlig (1996) and Al-Najjar (2004)) imply that: (i) at the aggregate level idiosyncratic uncertainty is cancelled out, so that aggregate outcomes are fixed (non-random) quantities; and (ii) the invariant distribution at the household level also gives the proportion of households at the cross-sectional level. Aggregation implies the following market clearing conditions:

$$\begin{array}{l} K = \int_I a_t^i \varphi\left(di\right) \\ L = \int_{I^u} \zeta^u s_t^i \varphi\left(di\right) + \int_{I^b} \zeta^b s_t^i \varphi\left(di\right) = \\ \;\; = n^u \zeta^u \sum_{j \in S^u} \overline{s}_j^u \xi^u \left(\overline{s}_j^u\right) + n^b \zeta^b \sum_{j \in S^b} \overline{s}_j^b \xi^b \left(\overline{s}_j^b\right). \end{array} \tag{8}$$

We define the distribution of households over the joint state-space, for $h = u, b$. Given individual asset holdings a_t^i and exogenous shocks s_t^i at period t by household, $i \in I^h$, the joint distribution over asset accumulation and shocks across households for each household type, $\overline{\lambda}_t^h \in \mathcal{P}\left(\mathcal{A}^h \times S^h\right)$ is given by:

$$\overline{\lambda}_t^h (A \times B) = \varphi\left(i \in I^h : \left(a_t^i, s_t^i\right) \in A \times B, A \times B \in \mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h\right). \tag{9}$$

The measure $\overline{\lambda}_t^h (A \times B)$ gives the fraction of households whose asset holdings and shocks at period t lie in the set $A \times B$. Using this, we can define the stationary recursive equilibrium as follows.

`Definition of Stationary Recursive Equilibrium`

For $h = u, b$, a *Stationary Recursive Equilibrium* is aggregate stationary distributions $\overline{\lambda}^h (A \times B)$, policy functions $a_{t+1}^h = g^h\left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathcal{A}^h$, $c_t^h = q^h\left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathbb{R}_+$, value functions $v^h\left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathbb{R}$, and positive real numbers $K, w(K), r(K)$ such that

1. The firm maximises its profits given prices, so that $(w(K), r(K))$ satisfy:

$$w(K) = \partial F(K, L)/\partial L, \tag{10}$$
$$r(K) = \partial F(K, L)/\partial K - \delta. \tag{11}$$

2. The policy functions $a_{t+1}^h = g^h\left(a_t^h, s_t^h\right)$ and $c_t^h = q^h\left(a_t^h, s_t^h\right)$ solve the households' optimum problems in (4) given prices and aggregate quantities, and the value functions $v^h\left(a_t^h, s_t^h\right)$ solve equations (4).

3. $\lambda^h (A \times B)$ is a stationary distribution:

$$\lambda^h (A \times B) = \int_{\mathcal{A}^h \times S^h} \Lambda^h \left[(a, s), A \times B\right] \lambda^h (da, ds), \tag{12}$$

   for all $A \times B \in \mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h$, where $\Lambda^h\left[(a, s), A \times B\right] : \left(\mathcal{A}^h \times S^h\right) \times \left(\mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h\right) \to [0, 1]$ are transition functions on $\left(\mathcal{A}^h \times S^h\right)$ induced by the Markov process $\left(s_t^h\right)_{t=0}^{\infty}$ and the optimal policy $g^h\left(a_t^h, s_t^h\right)$.

4. When $\lambda^h (A \times B)$ describe the cross-section of households at each date, i.e. $\overline{\lambda}^h (A \times B) = \lambda^h (A \times B)$, markets clear. In particular, asset market clears, that is the cross-section average value of K is equal to the average of the households' decisions:

$$\begin{aligned} K = n^u \textstyle\int_{\mathcal{A}^u \times S^u} g^u (a, s)\, \overline{\lambda}^u (da, ds) + \\ + n^b \textstyle\int_{\mathcal{A}^b \times S^b} g^b (a, s)\, \overline{\lambda}^b (da, ds). \end{aligned} \tag{13}$$

Secondly, the labour market clears:

$$L = n^u \zeta^u \int_{\mathcal{A}^u \times S^u} s^u(a,s)\, \overline{\lambda}^u(da, ds) + \\ + n^b \zeta^b \int_{\mathcal{A}^b \times S^b} s^b \overline{\lambda}^b(da, ds) = 1, \tag{14}$$

and the goods market clears, which, using factor input market clearing, implies:

$$F(K,1) - \delta K = \\ = n^u \int_{\mathcal{A}^u \times S^u} q^u(a,s)\, \overline{\lambda}^u(da, ds) + n^b \int_{\mathcal{A}^b \times S^b} q^b(a,s)\, \overline{\lambda}^b(da, ds). \tag{15}$$

Following standard arguments (commonly used in this class of models since Aiyagari (1994)), it is straight forward to show that continuity of the asset supply and demand functions at the aggregate level with respect to the interest rate as well as the limit properties of supply and demand for assets, imply that a general equilibrium exists.[14] A more general proof of existence of equilibrium for this class of models can be found in Acemoglu and Jensen (2015). To compute the general equilibrium solution of the model we implement a standard numerical algorithm which is summarised in Appendix B.

3 Data and Calibration

We calibrate the model to British data, at an annual frequency, and estimate the parameters relating to the Markov processes for the idiosyncratic shocks for the university and non-university groups of households using data from the British Household Panel Survey (BHPS) and the Wealth and Assets Survey (WAS). Regarding WAS, we make use of the 3rd to 5th waves which are the only waves that contain measures of net labour income at the household level. We use annual net labour income as the relevant quantity to calibrate the earnings processes, as this measure coheres well to earnings in the model. Waves 3-5 cover the period between 2010 and 2016 (see Appendix A for details). Regarding BHPS, we use all available periods, 1991-2008, to obtain annual net labour income measures. We then evaluate the predictions of the model regarding wealth inequality against the WAS wealth data.

[14] For details on a proof that can be applied here see Acikgoz (2018), Theorem 1. Further note that continuity of mean assets with respect to the interest rate, for each type of household, also implies continuity for the weighted average between households.

3.1 Wealth inequality

The WAS is a longitudinal survey for GB reporting information on earnings, income, the ownership of assets (financial assets, physical assets and property), pensions, savings and debt, as well as on socio-economic characteristics of the respondents over five waves between 2006 and 2016.[15] The sample corresponds to the households included in the wave, but the interviews in each wave are carried over a two year period, with the respondents providing information for the year of the interview.

An important feature of WAS is that it uses a 'probability proportional to size' method of sampling cases. This means that the probability of an address being selected is proportional to the number of addresses within a given geographic area, with a higher number of addresses being selected from densely populated areas. The design of WAS recognizes the fact that wealth is highly skewed, with a small proportion of households owning a large share of the wealth. Thus, WAS over-samples addresses likely to be in the wealthiest 10% of households at a rate three times the average. Moreover, the large overall sample size (around 20,000 households) provides robust cross-sectional estimates. These features ensure both good coverage of the very wealthy and more precise estimates of overall household wealth. However, as in similar surveys, the very rich (e.g. Forbes 400) are not included and this can affect the estimates of the top 1%.

Households are defined as the family or group of individuals who live in the same residence. The head is defined as the member of the household in whose name the accommodation is owned or rented, or is otherwise responsible for the accommodation. We select household heads between 25-59 years of age. We use household net worth as our measure for wealth. It is the sum of assets minus debt for all household members.[16] Net worth also admits a substantial proportion of the population which have negative current wealth. Details on the wealth data are in Appendix A, which includes key statistics summarising the wealth distributions for all five waves in Table A1.

3.2 Earnings dynamics

Household net labour income is our main measure of income that we use to estimate the extent and persistence of idiosyncratic earnings uncertainty

[15] The WAS does not provide information for Northern Ireland.

[16] We do not add pension wealth to our measure of net-worth. This allows us to maintain comparability with the infinite horizon incomplete markets literature that generally excludes pension wealth. Further note that pension wealth is highly imputed in WAS.

since wealth inequality is measured using household-level data.[17] We estimate the parameters pertaining to idiosyncratic earnings uncertainty for the whole sample and also separately for the university and non-university educated groups. We report estimates and model calibrations using data from both BHPS and WAS.

3.2.1 BHPS and WAS data

The BHPS is a comprehensive longitudinal study for GB, covering 1991 to 2008.[18] It includes information for up to 5000 households on annual earnings and other sources of income for individuals and households, as well as on socio-economic characteristics of the respondents. BHPS was replaced in 2010 by a new panel data survey, Understanding Society (USoc), which however does not include information on annual earnings, but only for monthly earnings. The latter are likely to underestimate annual earnings risk, especially by not capturing unemployment risk very well. We therefore focus on BHPS and annual earnings, although we do briefly report results from using USoc below. We also make use of the auxiliary dataset Derived Current and Annual Net Household Income Variables (DCANHIV), compiled by Bardasi *et al.* (2012), which contains derived data on household disposable income. The Bardasi *et al.* (2012) dataset tracks the same individuals/households for the same time periods as the BHPS i.e. 1991-2008. The BHPS dataset has been widely used to measure and estimate earnings inequality and earnings risk in the literature (see e.g. Blundell and Etheridge (2010), Etheridge (2015) and Cappellari and Jenkins (2014)).

The WAS dataset, as explained above, reports annual earnings and information to construct net labour income, although the households are interviewed every two years. The WAS data complement usefully the BHPS-based series, as they refer to similar quantities in different time periods, obtained from different surveys and different samples. In Appendix A, we report more information on the net labour income series obtained from the BHPS and WAS datasets.

We define net labour income as gross household labour income for employment or self employment minus taxes and national insurance contributions, plus social benefits and private transfers. For the BHPS, we can obtain data on the components of household income from the DCANHIV dataset, while

[17] Note that in what follows, net labour income and earnings are used synonymously.

[18] Data on Northern Ireland are available from 1997 via the additional BHPS sub-sample European Community Household Panel Survey. Moreover, boost samples for Scotland and Wales are available after 1999. However, we focus on Great Britain since the WAS data refers to Great Britain only.

for the WAS these are readily provided by the survey.

Moreover, in the two datasets we harmonise the definition of the head of household as it is defined in the previous section.[19] Furthermore, following the literature, we keep those households where the head is between 25-59 years and reports non-zero net labour income, and we further trim the top and bottom 0.25% of observations of each distribution in each year, to avoid extreme cases or possible outliers in recorded income (see e.g. Blundell and Etheridge (2010)).

3.2.2 Mincer regressions

Following the literature (see e.g. Mincer (1974), Meghir and Pistaferri (2004), and Blundell and Etheridge (2010)) household net labour income is composed of: (i) a component capturing aggregate conditions common to all households; (ii) a predictable part at the household level (capturing observed characteristics of the household); and (iii) an element capturing idiosyncratic shocks. We denote the natural logarithm of the measure of income in period t as $y^h_{i,t}$, for $h = u, b$, and assume that it follows the process:

$$\begin{aligned} y^h_{i,t} &= \beta^h_{0,t} + \beta^h_{1,t} ED_i + \beta^h_{2,t} HSIZE_i + \beta^h_{3,t} MAR_i + \\ &+ \beta^h_{4,t} GENDER_i + f_t(AGE_i) + \varepsilon^h_{i,t}, \end{aligned} \tag{16}$$

where t refers to years for BHPS data and to waves for WAS data; $\beta^h_{0,t}$ measures effects that are common to all households; ED_i is an education level dummy and captures the income difference between agents with some qualifications and with those with no qualifications (it applies to non-university educated group only); $HSIZE_i$ is number of members living in the household, while, MAR_i, and $GENDER_i$ are dummies for the marital status and sex of the head of the household respectively; $f_t(AGE_i)$ denotes a cubic polynomial in AGE_i of the head; and $\varepsilon^h_{i,t}$ is the unobserved idiosyncratic component. We run least squares regressions using equation (16) for each year/wave separately to capture potential time effects.[20]

3.2.3 Idiosyncratic shocks

We next retain the residuals $\varepsilon_{i,t}$ for each t as a proxy for the unobserved component of $y_{i,t}$ and, following the literature, assume that they are determined

[19] In fact, we follow the definition of the head of household set out in the BHPS.

[20] Each wave in the WAS lasts for 24 months and is split across 3 different years. For example, wave one began in July 2006 and completed in July 2008. Thus, for WAS data we have two more dummy variables capturing the year effects within each wave.

by an exogenous $AR(1)$ process (see e.g. Chang and Kim (2006)):

$$\varepsilon^h_{i,t+1} = \rho^h \varepsilon^h_{i,t} + \mu^h_{i,t+1}, \tag{17}$$

where $\left|\rho^h\right| < 1$ and $\mu^h_{i,t}$ is a white noise process with variance $\left(\sigma^h_\mu\right)^2$. We further assume that the $AR(1)$ process is covariance-stationary with a zero mean and variance $\left(\sigma^h_\varepsilon\right)^2 = \frac{\left(\sigma^h_\mu\right)^2}{1-\left(\rho^h\right)^2}$.

Following Chang and Kim (2006, 2007), we estimate (17) via OLS. We summarise the results for the Uni, Non-Uni and the whole sample in Table 2. For the WAS results, since the correlations reported in Table 2 are calculated over a two-year period, they must be transformed to an annual basis to cohere with the annual calibration of the model. To ensure that the annual residual income has the same persistence as the bi-annual residual income, we take the square root of the estimated persistence of (17). Moreover, following Krueger *et al.* (2016), we set the cross-sectional distribution of the residuals at annual frequency to be equal to the cross-sectional distribution of the residuals at bi-annual frequency i.e. we assume that the unconditional variance of $\varepsilon_{i,t}$ is the same at both annual and bi-annual frequencies.

Table 2 shows that for both datasets, the estimated variance of shocks to net labour income for the Uni group is higher than that for the Non-Uni group. The differences between σ_ε and ρ for Uni and Non-Uni are very similar for the two the datasets, albeit marginally bigger for the BHPS data.[21] Finally, to approximate the (17) by a discrete state-space process, we apply the Rouwenhorst (1995) method to build a Markov chain with 9-states (see e.g. Kopecky and Suen (2010) and Krueger *et al.* (2016)).

Table 2: Markov Process Parameters

	Uni	Non-Uni	Pooled
	WAS (waves 3-5)		
σ_ε	0.5260	0.4921	0.5038
ρ	0.8326	0.7988	0.8137
	BHPS (1991-2008)		
σ_ε	0.5099	0.4645	0.4687
ρ	0.8349	0.7974	0.8053

[21] We also used monthly net labour income data from USoc to estimate the same earnings processes. The differences between the two groups in the implied labour income risk in this case are higher.

3.3 Model parameters

The model parameters that do not relate to the Markov chains are summarised in Table 3. Regarding preferences, we set $\beta = 0.97$ which implies an equilibrium interest rate of around 2 percent per year. Following the literature we use a CRRA utility function:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{18}$$

and set $\sigma = 1.5$, which is the mid-point of values typically employed in calibration studies for the U.K. (see also Harrison and Oomen (2010) who econometrically estimate $\sigma = 1.52$). The *ad hoc* borrowing limit is calibrated to $\gamma = 0.80$ for the WAS calibration and $\gamma = 0.81$ for the BHPS calibration, so that we match, in equilibrium, the percentage of indebted agents (i.e. those with negative net-worth) in WAS wealth estimates. These shares are approximately 18.5%, 10% and 22% for the pooled sample, university and non-university educated respectively. The annual depreciation rate is set to $\delta = 0.1$ (see, e.g. Faccini *et al.* (2011) and Harrison and Oomen (2010)). We use a Cobb-Douglas production function with constant returns to scale with respect to its inputs:

$$Y = AK^{\alpha}L^{1-\alpha}. \tag{19}$$

We normalise $A = 1$ and set α to 0.3 (see, e.g. Faccini *et al.* (2011) and Harrison and Oomen (2010)). The value of n_u is set to 0.3 based on information on the percentage of university educated households from WAS, BHPS and USoc datasets. Finally, we make use of the ratio of the predicted earnings components between the two groups (i.e. ζ^u/ζ^b in Table 3) and of the normalisation of aggregate labour supply to one, to obtain ζ^u and ζ^b for the two calibrations.

Table 3: Model Parameters

β	σ	δ	A	α	n_u	γ (BHPS)	γ (WAS)	ζ^u/ζ^b (BHPS)	ζ^u/ζ^b (WAS)
0.97	1.50	0.10	1.00	0.30	0.30	0.81	0.80	1.47	

4 Group Wealth Inequality

We first examine wealth inequality within and between the groups of university and non-university educated and analyse the mechanism by which the pecuniary externality associated with savings contributes to the differences in wealth accumulation and inequality. We summarise the data and model predictions for key statistics of wealth inequality in Table 4, following standard practice in the choice of these statistics, see e.g. Quadrini and Rios-Rull (2015) and Krueger *et al.* (2016). We complement this Table by Figure 1, which provides a graphical representation of the wealth distributions using the quintile measures of the proportion of total wealth owned by households in the relevant quintile (the first column) and the Lorenz curves (the second column). We also report summary measures of wealth inequality at the aggregate level in the last rows of Table 4 to contextualise the discussion on within and between group wealth inequality.[22]

4.1 Data

The first two columns in Table 4 summarise wealth distributions in the data. As with Table 1, we present the averages across the five waves of WAS. The main pattern shown, as already noted, is that households whose head is university educated has lower wealth inequality than households whose head is not university educated. This can be seen in Table 4 by comparing the wealth distributions as approximated by the quintile statistics, wealth ownership at the upper tail and the Gini indices.

Table 4: Wealth distributions by group

	WAS Data		BHPS Calibration		WAS Calibration	
	Uni	Non-Uni	Uni	Non-Uni	Uni	Non-Uni
Q1 share	-0.006	-0.015	-0.009	-0.036	-0.010	-0.034
Q2 share	0.037	0.003	0.055	0.041	0.053	0.040
Q3 share	0.101	0.075	0.141	0.139	0.139	0.136
Q4 share	0.205	0.226	0.261	0.269	0.261	0.270
Q5 share	0.663	0.712	0.553	0.587	0.557	0.588
T 90-95%	0.136	0.153	0.140	0.148	0.141	0.150
T 95-99%	0.191	0.205	0.150	0.159	0.152	0.161
T 1%	0.155	0.148	0.056	0.060	0.057	0.060
Gini	0.661	0.731	0.566	0.630	0.572	0.626
a_u/a_b	2.270		2.148		1.984	
Gini Total	0.720		0.629		0.624	

Note: "WAS Data" refers to the average statistics over waves 1-5.

The quintile shares suggest a relatively smaller concentration of wealth in the lower three quintiles and a relatively higher concentration of wealth in the upper two quintiles for the non-university educated. Given the implied

[22]The model's predictions regarding aggregate inequality will be discussed in more detail in the next section.

spread between the lower and upper parts of the wealth distributions, all of these observations suggest that wealth inequality is higher for the non-university than for the university educated groups, which is confirmed by the summary Gini measures. Further note that the group of university educated has higher wealth on average, compared with the non-university educated, i.e. the relative wealth ratio, a_u/a_b, is at 2.27 on average across the five waves of data.

4.2 Model predictions

The next four columns in Table 4 summarise the predictions of the model with *ex ante* heterogeneity, presented in Section 2 and calibrated in Section 3, which we denote as the EHM model. This is done for both calibrations using BHPS and WAS net labour income data. Note that while the BHPS calibration implies an average wealth ratio of Uni to Non-Uni households predicted by the model about 2.15, which is consistent with between group wealth inequality in the data, the WAS calibration predicts relatively lower between group wealth inequality, at about 1.98.

Importantly, the model coheres with key properties of within group wealth inequality for the two groups, for either calibration. In particular, the model predicts higher wealth inequality for the Non-Uni group relative to the Uni group. This result can be seen by comparing the Gini indices, but is more comprehensively demonstrated by examining the relative rankings of the measures of wealth ownership for the two groups. The model predictions track those in the data. When the quintile shares are higher in the data for the Uni group (the Q1, Q2 and Q3 shares), they are also higher in the model. Whereas, when the quintile measures are higher in the data for the Non-Uni group they are also higher in the model. Overall, the model predicts a Gini index for the non-university educated that is 6.4 units higher than the respective index for the university educated for the BHPS calibration, which is very close to the data (7 unit difference). As with between group inequality, the predicted Gini gap between the two groups is smaller with the WAS calibration, at 5.4 units, but still close to the data.[23]

The EHM model's predictions regarding the extent of wealth inequality relative to the data are close for both groups, with the exception of the predictions for the top 5%, and especially the top 1%, where the models significantly underestimate wealth inequality, as is well known in the literature. The first column in Figure 1 shows the wealth distribution approximated by the quintile shares for the BHPS calibration in Table 4. Both show that

[23] We will examine further the differences between the two calibrations below.

the model magnitudes are similar to the data for both groups. The second column of Figure 1 suggests that the level of predicted inequality within each group is lower compared with the data, reflecting that overall the EHM model quantitatively under-predicts the extent of wealth inequality This can also been seen by referring to the Gini index implied by the model for the aggregate economy in the last row of Table 4.

[Figure 1]

In contrast to the WAS data, both model calibrations predict slightly higher wealth concentrations for the top 1% of the Non-Uni relative to the Uni groups. However, a closer look at each of the WAS waves shows that the wealth concentration ranking for the top 1% is not the consistent over all the waves. For example, in the first three waves, wealth ownership by the top 1% is higher for the Non-Uni while it is higher for the Uni in the last two waves.[24] In contrast, the ranking of the remaining statistics between the two groups in Table 1 does not change over the waves. Importantly, both calibrations give rise to predictions regarding the relative ranking of the group wealth concentrations in the top percentiles below the top 1% (i.e. the shares owned by the top 90-95% and 95-99%) that are very similar to the data.

As discussed in the Introduction, it is a well known finding that standard incomplete markets models do not match the extent of wealth inequality in the data. We investigate the EHM model's potential in delivering improved predictions with respect to overall inequality in more detail in Section 5. For now, we focus on the main result in Table 4, which is the model's ability to generally capture the relative rankings of wealth inequality between the two groups.

4.3 The importance of group interactions

Considering our findings in the previous sub-section, we next investigate the mechanism in the EHM model that leads to the correct ranking of wealth inequality for the Uni and Non-Uni groups. To do so, we study the general equilibrium and investigate the effect of group interaction on aggregate outcomes, and in particular on asset accumulation and the wealth distributions within each group. This allows us to show that differences in savings between groups (due to differences in earnings processes) can create (via the

[24] For example, the ratios of the Non-Uni top 1% to the Uni top 1% for Waves 1-3 are 1.029, 1.13 and 1.06 respectively. Whereas the corresponding ratios for Waves 4-5 are 0.812 0.873 respectively.

market mechanism) a pecuniary externality which works to reverse the effect of differences in earnings processes on wealth inequality.

4.3.1 General equilibrium

In Figure 2, we plot the asset supply curves for a typical household in both groups of university and non-university educated, based on the BHPS calibration, as well as the asset supply and demand functions for the aggregate economy.[25]

[Figure 2]

We summarise key quantitative information relating to this Figure in Table 5 under the EHM column for the BHPS calibration. In addition, we add in Table 5 key statistics that capture model predicted earnings and wealth inequality. In particular, we report the earnings inequality that is implied by the calibration in Section 3 and the wealth inequality in general equilibrium. We also repeat the same exercise for the WAS calibration and summarise results under the EHM column for the WAS calibration, but to save on space we do not repeat in this case Figure 2. The general equilibrium is obtained at the intersection point of the aggregate-level supply and demand curves for assets. Focusing on the BHPS calibration, this general equilibrium gives an interest rate of $r^* = 0.0181$ and capital stock of $a^* = 3.787$.

Table 5: Pecuniary externality and inequality per group

	BHPS Calibration			WAS Calibration		
	EHM	NI_u	NI_b	EHM	NI_u	NI_b
r^*	0.0181	0.0157	0.0197	0.0170	0.0149	0.0183
a^*	3.787			3.841		
a_u^*	6.050	5.084		5.884	5.121	
a_b^*	2.817		3.231	2.966		3.295
Wealth Gini Uni	0.566	0.597		0.572	0.596	
Wealth Gini Non-Uni	0.630		0.601	0.626		0.603
Earnings Gini Uni	0.276	0.276		0.284	0.284	
Earnings Gini Non-Uni	0.253		0.253	0.267		0.267

Notes: (i) $\frac{a_u}{a_b}$=2.27 in the data; (ii) $\frac{a_u^*}{a_b^*}$=2.15 for EHM (BHPS); iii) $\frac{a_u^*}{a_b^*}$=1.98 for EHM (WAS); (iv) $\frac{a_u^*}{a_b^*}$=1.57 for NI (BHPS); and (v) $\frac{a_u^*}{a_b^*}$=1.55 for NI (WAS).

Note that the efficient interest rate in this economy, defined as the interest rate under complete financial markets which allows the agents to eliminate

[25] Note that the group-level and aggregate-level supply and demand functions are in per capita units. Thus, they refer to mean asset supply and demand functions.

idiosyncratic risk, is given by 0.031, implying, via the asset demand function, an efficient capital stock of 3.27. Thus, in the model with *ex ante* skill heterogeneity, these quantities are reduced and increased, respectively, to r^* and a^*, implying inefficiently high asset accumulation as has been shown since Aiyagari (1994).

4.3.2 Externalities from skill heterogeneity

In Figure 3, we plot again the supply and demand curves for the EHM model based on the BHPS calibration, which provide the equilibrium (already shown in Figure 2) when the two groups interact via the market in a single economy. We complement this by plotting the asset supply curves for a typical household in each group, which capture mean asset supply per group, together with the mean asset demand curves that would apply if these two groups did not interact. In other words, we treat the two groups as separate economies, each populated with the *ex ante* identical university or non-university educated agents. We denote these as NI (non-interaction) supply and demand. The intersection points of the respective asset supply and demand curves represent the equilibrium interest rate and assets in the absence of group interaction, which are reported in Table 5 under the NI_h, $h = u, b$ columns. Note that mean assets per group imply between group wealth inequality which is significantly lower compared with the data (see notes to Table 5).

[Figure 3]

The asset supply curves for a typical household in each group in the EHM model encapsulate their optimal policy functions and thus choices for savings given aggregate outcomes under market incompleteness. Therefore, from Figure 3 and Table 5, we can see that in the EHM model the equilibrium interest rate $r^* = 0.017$ implies mean assets for the Uni group that are equal to $a^*_u = 5.884$ and for the Non-Uni group that are equal to $a^*_b = 2.966$. Hence, compared with the case where the groups' savings do not affect each other (i.e. $r^* = 0.0149 \Rightarrow a^*_u = 5.121$ and $r^* = 0.0183 \Rightarrow a^*_b = 3.295$ for BHPS), the asset supply of the other group in the general equilibrium of EHM economy, works to lower (increase) the interest rate for the Non-Uni (Uni) groups respectively. Thus, reducing (increasing) their respective incentives to save.

In turn, this under-accumulation (over-accumulation) of assets works to increase (decrease) wealth inequality in each group, by increasing (decreasing) the exposure to earnings variability. Therefore, the asset supply of each group creates a pecuniary externality in the financial market which affects inequality in the other group. To quantify this effect, we summarise in Table 5, wealth inequality for the two groups in these two scenaria.

Comparing the NI_h equilibria to the EHM equilibrium, the latter implies higher wealth inequality within the non-university educated, and lower wealth inequality within the university educated. The earnings Ginis in Table 5 further show that the differences in wealth inequality between NI_h and EHM for both groups are not driven by differences in earnings risk (and in within group earnings inequality). This is due to the fact that the same earnings processes are used in both the EHM and the NI_h equilibria.

The increase (decrease) in wealth inequality within the Non-Uni (Uni) in the EHM relative to the NI_h equilibria are important qualitatively. In particular, in the NI_h equilibria, both groups have effectively the same wealth inequality. This is because, on one hand, higher earnings inequality and risk tends to generate higher wealth inequality since they imply a higher spread in earnings and thus in accumulated assets. On the other hand, higher mean earnings tends to generate lower wealth inequality since they work to reduce quantitatively the importance of shocks to earnings relative to the mean. Hence, higher mean earnings for the university educated effectively offset their higher risk (and vice versa for non-university educated), so that in the NI_h equilibria, both groups end up with the same wealth inequality.[26] The pecuniary externality implied by the interaction in the financial market under *ex ante* heterogeneity in the EHM model then creates the conditions to change wealth inequality within each group, so that the wealth inequality rankings cohere with the data.

The predicted between group inequality and the differences between the within group inequalities for the two groups increase more when moving from the NI_h equilibria to the EHM equilibrium (and get closer to the data) for the BHPS versus the WAS calibration. As can be seen in Table 5, the predicted earnings inequality is slightly higher under the WAS calibration (note also from Section 3 that the variance of idiosyncratic earnings is estimated to be slightly higher in the WAS data). Given that earnings risk is thus lower under the BHPS calibration, the relative importance of the pecuniary externality channel analysed above is bigger, so that the BHPS calibration implies higher asset accumulation and wealth inequality differences between the two groups. However, and despite small quantitative differences between the WAS and BHPS calibration, the results in Tables 4 and 5 are very similar for either calibration.

[26] Indeed, if we normalise earnings in each group to be the same, then the group with higher earnings risk (the University educated) also has the higher wealth inequality.

5 Aggregate Wealth Inequality

We now turn to analyse the predictions of the model regarding wealth inequality for the aggregate economy and examine whether group interaction and the implied pecuniary externality can work as an amplification mechanism to increase overall wealth inequality. Table 6 presents aggregate wealth inequality in the WAS data (as averages over the available waves), for the EHM model and for the Aiyagari model with *ex ante* identical households, which we denote as EIM. The latter is calibrated using the pooled earnings data for the whole sample to construct the earnings processes, as discussed in Section 3, while the rest of the calibration is the same as for the EHM model.[27] Again, we show results using both the BHPS and WAS calibrations.

Regarding overall wealth inequality, the Gini index for the EHM model is about 0.63 for the BHPS calibration, compared with the Gini index in the data of about 0.72. The predictions for the share of wealth owned by each of the five quintiles are also reasonably close. They are further from the data for the top 5%, and in particular the model underpredicts the top 1% wealth share. To complement Table 6, the predicted wealth distribution from the model as approximated by the quintile measures and the Lorenz curve are also shown graphically in Figure 4 for the BHPS calibration.

Table 6: Aggregate Wealth distributions

	WAS	BHPS Calibration		WAS Calibration	
	Data	EHM	EIM	EHM	EIM
Q1 share	-0.012	-0.024	-0.028	-0.024	-0.024
Q2 share	0.009	0.041	0.048	0.042	0.048
Q3 share	0.085	0.125	0.139	0.128	0.133
Q4 share	0.208	0.256	0.268	0.254	0.267
Q5 share	0.709	0.602	0.574	0.600	0.575
T 90-95%	0.148	0.150	0.145	0.149	0.145
T 95-99%	0.206	0.172	0.155	0.169	0.157
T 1%	0.165	0.072	0.058	0.070	0.059
Gini	0.720	0.629	0.606	0.624	0.603

Note: "WAS Data" refers to the average statistics over waves 1-5.

The results in Table 6 and Figure 4 show that the benchmark EIM model generally provides weaker inequality predictions, compared with the EHM model. In particular, these results show that *ex ante* heterogeneity in the EHM increases the Gini index by 2.1-2.3 units, and it further contributes

[27]Note that for the EIM model we also recalibrated γ =0.94 (BHPS) and γ =0.815 (WAS).

to improvements in the top 5% and top 1%. The improvements are bigger for the BHPS calibration. It is also worth pointing out that the model with *ex ante* identical agents does predict higher inequality than typically found in the literature when the earnings processes are calibrated using earnings data[28]. This is the result of the higher earnings uncertainty implied using the estimates of the earnings processes in Section 3 and because we allow for borrowing. *Ex ante* heterogeneity adds to this, and improves the predictions of the model even further. Thus, *ex ante* heterogeneity appears to matter for wealth inequality at the aggregate level. In the next section we further explore why this may be the case.

[Figure 4]

5.1 Group interactions & aggregate wealth inequality

Table 7 presents key earnings and wealth inequality statistics from the data and from different model variants, focusing on the BHPS calibration.[29] In particular, we report the Uni to Non-Uni mean earnings and wealth ratios as well as the earnings and wealth Ginis for the aggregate-level economy.[30]

Table 7: Pecuniary externality and aggregate inequality

	earnings ratio	wealth ratio	earnings Gini	wealth Gini
		Data		
BHPS	1.569	-	0.297	-
WAS	1.535	2.270	0.299	0.720
		Models (BHPS calibration)		
EHM	1.499	2.148	0.282	0.629
EIM	-	-	0.255	0.606
NI	1.521	1.573	0.284	0.609

The first two models summarised in Table 7 are the model with *ex ante* heterogeneity (EHM) and the model with *ex ante* identical agents (EIM). In contrast to the EHM model, between-group earnings and wealth differences are ignored in the EIM model (see columns 1 and 2 respectively). Further

[28]The predicted Gini index of the Aiyagari (1994) model ranges roughly between 0.35 and 0.45 in the literature, depending on the earnings data used, and can be higher when the model allows for borrowing, as we do here (see e.g. the reviews in Quadrini and Rios-Rull (1997, 2015) and Krueger *et al.* (2016)).

[29]Note that similar results are obtained using the WAS calibration.

[30]Some statistics relating to the data and models in Table 7 have also been reported in previous tables, but are repeated here for convenience.

note from columns 1 and 2 that the EHM model correctly predicts a positive relationship between mean earnings and mean wealth, in that the higher income group also has higher wealth, with relative magnitudes that are similar to the data.

The final row of Table 7 summarises earnings and wealth inequality results for the aggregate economy that would be created by merging the two NI economies analysed in Section 4 (see Figure 3 and Table 5). Recall that each of these economies is populated by *ex ante* identical agents, who differ between the two economies in that they have different earnings processes, specifically those of the Uni and Non-Uni groups. Hence, this aggregate economy captures the differences in mean earnings (see column 1) and earnings inequality (see column 3) as in the EHM economy.[31] However, it abstracts from the pecuniary externality created by group interaction.

As pointed out above, comparing the wealth Ginis in the EHM and EIM rows of Table 7, *ex ante* heterogeneity implies higher wealth inequality at the aggregate level. In turn, the results for NI equilibrium suggest that these improved inequality predictions for the EHM model are largely due to the interaction between the *ex ante* heterogeneous groups and the implied pecuniary externality analysed earlier. In particular, the increased wealth inequality in EHM is not a direct implication of the higher earnings inequality in this model, because the NI and EHM models have the same earnings structure and earnings inequality and yet there are effectively no improvements in the wealth Gini coefficient for the NI relative to the EIM economy.[32] Moreover, the predicted wealth ratio between the two groups in the NI equilibrium is not as high as in the data.

Although the NI modeling captures qualitatively the relationship between mean earnings and mean wealth between groups, it underestimates it quantitatively because between group wealth inequality is smaller, for effectively the same between group earnings inequality. At the same time, as was discussed in Section 4, the NI model, by ignoring group interaction and the pecuniary externality, misses the ranking of within group wealth inequality observed in the data. Therefore, given *ex ante* earnings differences between the two groups, the pecuniary externality via group interaction is important quantitatively for the EHM model's predictions regarding aggregate wealth

[31] In fact, the earnings ratio is a bit higher in this case compared with EHM, because the wage rates also differ in the two NI economies, so that the ratio of mean labour incomes $\int_{I^u} w^u \zeta^u s_t^i \varphi(di) \, / \int_{I^b} w^b \zeta^b s_t^i \varphi(di)$ is marginally higher than the equivalent quantity in EHM ($\int_{I^u} \zeta^u s_t^i \varphi(di) \, / \int_{I^b} \zeta^b s_t^i \varphi(di)$).

[32] Note that both the NI and EHM models incorporate higher earnings inequality compared with the EIM model (see column 3) because they add realistic between group differences in earnings.

inequality. This is due to the amplification of between group wealth inequality and the amplification of within group inequality for the larger group.

To contextualise the importance of the increase in the Gini index for wealth inequality by 2.3 units that can be attributed to the pecuniary externality, we ask the following question. How much would we need to increase the standard deviation of shocks to earnings, as measured by σ_ε in Section 3, in the EIM model, so that this model generates the same wealth inequality as the EHM model? It turns out that *ceteris paribus* σ_ε would need to be increased by 47% compared to that estimated using the data, i.e. we would have to calibrate the EIM model with $\sigma_\varepsilon = 0.689$ as opposed to $\sigma_\varepsilon = 0.469$. This in turn implies a Gini earnings inequality index of 0.365. Therefore, the effect of the pecuniary externality on wealth inequality is equivalent to an increase in earnings risk (as measured by σ_ε) by 47%, or an increase in earnings inequality (as measured by Gini) by 43%.

5.2 When does *ex ante* heterogeneity matter?

Ex ante heterogeneity matters for wealth inequality mainly because it creates the conditions for the pecuniary externality to amplify the mapping from earnings differences to wealth differences. However, *ex ante* earnings heterogeneity need not necessarily increase wealth inequality, as has been noted in the literature (see e.g. Quadrini and Rios-Rull (1997) and Castaneda *et al.* (1998)) and as we further elaborate here. *Ex ante* earnings heterogeneity in this context implies differences in both mean earnings and in the transition matrices (capturing differences in earnings risk) between the two groups. The pecuniary externality is higher (and thus aggregate inequality tends to increase more due to this channel), the bigger the difference between the asset supplies of the two groups.[33] The two forms of *ex ante* heterogeneity have distinct (and potentially different) effects on the distance between the asset supply functions. In particular, a higher earnings difference tends to increase the gap, since in general equilibrium assets increase with mean productivity (see e.g. Acemoglu and Jensen (2015)).[34] Moreover, increased earnings uncertainty also increases savings, as agents increase precautionary wealth (see e.g. Acemoglu and Jensen (2015) for a theoretical analysis and Aiyagari (1994) for quantitative applications).

The differences between the Uni and Non-Uni groups regarding mean earnings and earnings risk in GB both work to increase the differences in

[33] This follows from the discussion in the previous sub-section, and we confirm it below.

[34] Also, in a partial equilibrium context, savings are typically increasing in earnings and income in this class of models (see e.g. Miao (2002) for a theoretical result that savings increase in earnings, and Aiyagari (1994) that savings increase in disposable resources).

savings. This happens because the university educated have both higher mean earnings and higher earnings uncertainty. Therefore, in the model with *ex ante* skill heterogeneity calibrated as in Section 3, both factors work in the same direction to increase the distance between the asset supply curves, and thus ultimately aggregate inequality. In fact, as we saw in Section 4, in this model, the pecuniary externality channel is strong enough to create wealth inequality effects for each group that are ranked in the reverse order compared with the ranking implied by the earnings risk. In other words, while we may expect the Uni group to have higher wealth inequality because they face higher earnings uncertainty, the pecuniary externality implies that they are able to accumulate more wealth to self-insure and thus reduce within group wealth inequality. The effects are reversed for the Non-Uni group.

What is important in the above is that the group with higher mean earnings also has higher earnings risk. Then, via the pecuniary externality, the model can also match the earnings-wealth correlation, as well as the within group wealth inequality rankings. The importance of the earnings-wealth correlation between groups for the wealth inequality implications of *ex ante* heterogeneity has already been noted in e.g. Quadrini and Rios-Rull (1997) and Castaneda *et al.* (1998), who showed that considering five groups of households based on mean earnings does not need to improve the wealth inequality predictions compared with a model with *ex ante* identical households. In their setup, earnings risk is purely unemployment risk, which is inversely related in the PSID data with the mean earnings of the five income groups. As a result, the groups with higher mean earnings also have lower earnings risk, which, as Quadrini and Rios-Rull (1997) point out, implies that the model cannot match a positive earnings-wealth correlation, because the groups with lower risk have less precautionary savings.

5.2.1 Counterfactuals

To illustrate the workings of the two components of *ex ante* earnings heterogeneity in this setup and demonstrate that they can work to offset each other, we consider two counterfactuals in Figure 5, which are also summarised in Table 8 (middle rows). The first subplot of this Figure repeats Figure 2 to facilitate comparison.

[Figure 5 here]

In the second subplot (labelled Reversed Earnings), we leave the differences in the transition matrices between the Uni and Non-Uni groups as they are in Table 2 and increase mean earnings for the Non-Uni group so that its asset supply moves to the right. Specifically, we calibrate the relative difference in mean earnings so that the asset supply curves become the same. This is obtained by effectively changing the mean earnings premium in favour of the Non-Uni group. Thus, if mean earnings for the Non-Uni group are increased sufficiently, relative to the Uni group, the increased incentive that the latter groups has for savings due to their more uncertain income is offset so that the two asset supply curves are the same.

In the third subplot (labelled Reversed Risk), we examine the role of increased earnings uncertainty for the Non-Uni group and/or of reduced earnings uncertainty for the Uni group, in the form of increases/decreases in the variances of the estimated earnings process. In this case we calibrate the relative difference in the variances of the earnings processes between the two groups so that the asset supply curves become the same. This requires that the variance of the Non-Uni group increases by 33.5%.

Table 8: Counterfactuals

	wealth ratio	wealth Gini
	Models (BHPS Calibration)	
EHM	2.148	0.629
EIM	-	0.606
	Counterfactuals EHM (BHPS Calibration)	
Reversed earnings	1.000	0.596
Reversed risk	1.000	0.608
	Counterfactuals (USoc Calibration)	
EHM	2.915	0.644
EIM	-	0.606

Hence, in both counterfactuals, the group with the higher mean earnings now faces lower earnings risk. In these two equilibria (in the second and third subplots in Figure 5), the *ex ante* heterogeneity is such that there is no pecuniary externality, since the asset supply curves are effectively the same. In other words, one form of *ex ante* heterogeneity has offset the effects of the other, to eliminate the externality via the interest rate. In both these cases, predicted wealth inequality is reduced back to the levels of the EIM model with *ex ante* identical agents (i.e. compare the wealth Ginis in rows 3 and 4 with row 2). Therefore, elimination of the pecuniary externality has eliminated the increase in the Gini index from the model with *ex ante* identical agents to the model with *ex ante* skill heterogeneity. These results complement previous analysis in Quadrini and Rios-Rull (1997) and Castaneda *et al.* (1998) and show that there can be *ex ante* skill heterogeneity without an increase in wealth inequality, which in turn may explain why the role of *ex ante* heterogeneity has not been explored much in the literature.

The previous two counterfactual calibrations are potentially closer to situations where earnings risk is measured by unemployment risk, since lower skilled workers can have higher unemployment risk (e.g. probability of losing their job) as well as lower mean earnings.[35] The opposite situation can arise if earnings risk did not include unemployment risk. For example, using monthly earnings data from Understanding Society, which likely underestimates unemployment risk, gives a higher spread of earnings inequality between university and non-university educated as was discussed earlier. As a result, when these earnings estimates are used to calibrate the model, by working via the pecuniary externality channel, they exaggerate the spread in inequality between the Uni and Non-Uni groups, resulting in Gini indices for the two groups that differ by nearly 14 units. Moreover, they exaggerate between group wealth inequality, as summarised in the penultimate row of Table 8. As a result of the higher between group inequality and the bigger increase in the within wealth inequality for the group of non-university educated, this calibration incorporates a stronger amplification mechanism for aggregate wealth inequality, as can be seen by comparing the EHM and EIM Gini indices for this calibration in the last two rows of Table 8.

6 Conclusions

This paper developed an incomplete markets model with state dependent (Markovian) stochastic earnings processes and *ex ante* heterogeneity corresponding to being university educated or not. We allowed the two groups to differ in their earnings processes, both in the state-space and in the transition matrix for idiosyncratic earnings shocks. Using the British Household Panel Survey and the Wealth and Assets Survey for Great Britain to estimate the earnings processes, we found that this model predicted wealth inequality both within and between the university and non-university educated groups that was consistent with the data. Moreover, the model predicted wealth inequality which was closer to that in the British data than the benchmark model with *ex ante* identical agents. Our analysis showed that *ex ante* heterogeneity in this framework generated a between-group pecuniary externality that works to bring the model regarding wealth inequality closer to the data.

In this framework, *ex ante* heterogeneity affects wealth inequality because the differences in the earning processes between the groups imply interest rate

[35] For instance, in the U.S., job separation and unemployment rates are higher for less skilled workers. In particular, the unemployment rates are 2.6% and 8.4% for skilled and unskilled workers respectively. Moreover, the separation rate are 0.97% and 3.78% for the skilled and unskilled workers respectively (see e.g. Hagedorn *et al.* (2016)).

externalities. Earnings differences, both in terms of mean earnings and idiosyncratic uncertainty, imply that the savings of each group move the market interest rate away from the level that would be the equilibrium outcome consistent with the asset supply of each group. The equilibrium interest rate is determined by the aggregate asset supply function, which is higher (lower) than the asset supply functions for university and non-university groups respectively. Consequently, households in the non-university and university educated groups lower and raise, respectively, their savings. Therefore, within group wealth inequality is increased and decreased, respectively, while between group inequality and overall wealth inequality is increased.

The between-group pecuniary externality is due to both factors defining *ex ante* heterogeneity, i.e. the differences in mean earnings and in the transition matrices, and we found both of these to be important quantitatively. For the pecuniary externality to improve the model's predictions on wealth inequality, both factors need to work in the right direction. In particular, the mechanism is stronger the bigger the difference in mean earnings and the more uncertainty the higher earnings group faces. However, it is possible for the uncertainty channel to offset the mean earnings channel, thus eliminating the pecuniary externality and the gains it provides in terms of improving the inequality predictions of the model. Therefore, *ex ante* heterogeneity is less likely to matter in situations where the lower earnings groups also have relatively higher earnings risk. In contrast, the pecuniary externality will have a stronger effect on wealth inequality if the lower earnings groups face relatively lower earnings risk.

7 Appendix A

The WAS and BHPS data sets employed in this paper refer to the free "End User Licence" versions of the datasets (i.e. WAS: SN-7215 and BHPS: SN-5151). Additionally, the BHPS Derived Current and Annual Net Household Income Variables datatset (DCANHIV) that we use is DCANHIV: SN-3909. The counterfactual in Table 8 relating the USoc data uses Understanding Society: Waves 1-7, 2009-2016, SN: 6614.

The WAS started in July 2006 with a first wave of interviews carried out over two years to June 2008. The WAS interviewed approximately 30,500 households including 53,300 adult household members in Wave 1. The same households were approached again for a Wave 2 interview between July 2008 and June 2010. In this wave 20,170 households responded (around 70% success) including 35,000 adult household members. Waves 3-5 covered the periods between July and June for the years 2010-12, 2012-14 and 2014-

16 respectively. After Wave 2, due to sample attrition, the WAS started implementing boost samples in each wave to keep the number of interviewed households around 20,000 and 35,000-40,000 adult household members.

The BHPS is a longitudinal study for the U.K. running from 1991 to 2008. As a panel data survey, the BHPS tracks individuals across households over time. In the first wave, the BHPS achieved a sample size of around 5000 households (10,000 adult interviews) or a 65% response rate. After the first wave, due to sample attrition, the sample size shrank slightly. For example, in 2000 it achieved around 4200 complete interviews or a 75% response rate (see Taylor *et al.* 2010). The DCANHIV is a supplement to the set of derived income variables in the official BHPS release which focus on gross income (see Bardasi *et al.* (2012)).

The BHPS was replaced in 2009 by a new panel data survey, Understanding Society. This survey is a large longitudinal survey which follows approximately 40,000 households in the U.K.. Data collection for each wave takes place over a 24-month period and the first wave occurred between January 2009 and January 2011. Note that the periods of waves overlap, but the individual respondents are interviewed around the same time each year. Thus, there is no respondent who is interviewed twice within a wave or a calendar year (see Knies (2017)).

7.1 Demographics (WAS)

1. **Head of the Household**: We define the head of household as the principal owner or renter of the property, and, when there is more than one head, the eldest takes precedence. This follows the reference person definition in BHPS. We use of the following variables: (HhldrW), (HiHNumW), (DVAGEw) and/or (DVAge17w).

2. **Education level**: There are two educational attainment variables in the WAS. The first is the TEAw, which is the age that the individual completed education. The second is the EdLevelw which is a derived variable of the education level and represents the highest educational level that respondent has achieved. EdLevelw provides three categories: (i) degree level or above; (ii) below degree qualifications (iii) no qualifications. The TEAw has the disadvantage that it cannot distinguish the type of qualification that the respondent had achieved. Moreover, 33% of the TEAw observations of working-age adults have either missing values or partial answers. Thus, we choose to work with the EdLevelw which is a derived variable and has only 2,942 missing values, i.e. around 2.7% of working-age adult observations. However,

using EdLevelw, we note that there are respondents for whom educational attainment changes in a way that indicates misreporting. For example, for some respondents, there is an increase of educational attainment just for one wave and then a return back to the previous level of education in subsequent waves. Thus, we have chosen to make some corrections to the educational level when a respondent's educational attainment changes. In particular, if we observe a respondent for all the 5 waves, we replace her educational attainment with the level that was reported the most times across the 5 waves. We follow a similar procedure if a respondent changes her educational attainment just once. In particular, we require the respondents being present in the sample for at least 3 waves and we use the most commonly recorded education level across waves. These corrections were applied to 4,873 observations out of 107,320 total amount observations of adult respondents (around 4.5%) and only half of these 4,873 observations correspond to a head of a household. Despite these corrections, the results are very similar when they are not made.

3. **Age**: WAS provides the age variable after the third wave (DVAGEw). If the exact age is not available WAS provides age bands and we make use of the 5 years bands (DVAge17w).

4. **Marital Status**: *De facto* marital status of the head or his/her partner. (HRPDVMrDfW)

5. **Year**: WAS provides the year of interview only from the fourth wave and on. However, since the interviews are conducted every two years around the same month for each respondent, we use the variable YearW to find the year of the interview in waves 1-3. For example, a household interviewed by the WAS in 2014, if present in previous waves, must have been interviewed again in 2012 (wave 3), 2010 (wave 2) and 2008 (wave 1). We work similarly for those who were interviewed in 2013 and 2012. For the respondents for which we do not have the year of interview (e.g. those absent from wave 4), we set their year of the interview to be in the middle of the interview periods when most of interviews are conducted. Every wave spans three different years, for example, wave 1 starts in 2006 and finishes in 2008. Thus, we set the year for respondents with no information for date of interview to be 2007. We work in a similar fashion for the rest of the waves.

7.2 Demographics (BHPS)

1. **Head of the Household**: We use the BHPS definition of the head of household. The head of household is defined as the principal owner or renter of the property, and, where there is more than one head, the eldest takes precedence. (wHOH).

2. **Education level**: BHPS includes very detailed information on educational attainment. We have used the variable wQFEDHI (where the prefix w denotes wave). To examine the potential heterogeneity of earnings risk in the main text, the sample is split into degree holders and non-degree holders. The former are the individuals who hold either a Higher Degree or 1st Degree, while the latter are the individuals who hold either Higher National Certificate/Diploma or teaching qualifications or A-levels/AS level/Highers or GCSE/O level/other qualification or they have no qualifications. To create the dummy variables for the regression in equation (4), we use the following education categories: (a) high education includes those with a higher or first degree (b) other higher/diploma/teaching/nursing; (c) intermediate education includes those with A-levels or equivalent; (d) GCSE/O level and (e) no qualifications.

3. **Age**: The BHPS provides the age variable consistently in all waves. (wAGE)

4. **Marital Status**: Marital status of the head of the household. (wMASTAT)

5. **Year**: Note that for the BHPS measures, the period of observation refers to an annual cycle starting in September.

7.3 Definition of wealth (WAS)

1. **Net property wealth**:[36] is the sum of all property values minus the value of all mortgages and amounts owed as a result of equity release. (HPROPWW)

2. **Net financial wealth**: is the sum of the values of formal and informal financial assets, plus the value of certain assets held in the names of children, plus the value of endowments purchased to repay mortgages, less the value of non-mortgage debt. The informal financial assets exclude very small amounts (less than £250) and the financial liabilities

[36] All monetary values are expressed in 2012 prices as measured by CPIH.

are the sum of current account overdrafts plus amounts owed on credit cards, store cards, mail order, hire purchase and loans plus amounts owed in arrears. Finally, money held in Trusts, other than Child Trust Funds, is not included. (HFINWNTW_sum)

3. **Net Worth:** is the sum of the net property wealth and net financial wealth.

Table A1: Wealth Inequality in Great Britain

	Gini	$\frac{sd}{mean}$	$\frac{mean}{median}$	top 10%	$\frac{a_u}{a_b}$
			WAS (wave 1)		
Uni	0.644	1.948	1.846	0.460	
Non-Uni	0.702	1.972	2.073	0.480	2.085
Total	0.696	2.121	2.000	0.492	
			WAS (wave 2)		
Uni	0.632	1.697	1.798	0.442	
Non-Uni	0.714	1.983	2.404	0.481	2.148
Total	0.699	1.977	2.140	0.487	
			WAS (wave 3)		
Uni	0.655	1.995	1.997	0.476	
Non-Uni	0.733	2.488	2.619	0.507	2.247
Total	0.718	2.385	2.301	0.516	
			WAS (wave 4)		
Uni	0.691	2.854	2.267	0.522	
Non-Uni	0.748	2.315	3.410	0.530	2.499
Total	0.742	3.048	2.733	0.555	
			WAS (wave 5)		
Uni	0.685	2.359	2.281	0.514	
Non-Uni	0.761	2.400	3.849	0.538	2.372
Total	0.742	2.628	2.817	0.547	

7.4 Definition of net income (WAS)

1. **Household net labour income**: is equal to household total annual earnings, plus social benefits, plus annual transfers income minus taxes and NI contributions. Transfer income can contain the following: (i) income from government training; (ii) educational grants; (iii) redundancies; and (iv) one-off income from relatives or friends.

The derivation of the variable is the sum of the following variables: DVNISEW_aggr, DVNIEMPW_aggr, DVTotAllBenAnnualW_aggr, DVoiNfrAnnualw3_aggr, DVoiNegAnnualW_aggr, DVoiNgt AnnualW _aggr, DVoiNrrAnnualW_aggr.

7.5 Definition of net income (BHPS)

1. **Household net labour income**: is obtained from the DCANHIV dataset (Bardasi *et al.* 2012) and is defined as household net labour earnings plus benefits, plus private transfers. It is equal to household total annual earnings, plus social benefits, plus annual transfers income minus taxes, NI contributions. Private transfers income totals all receipts from other transfers (including education grants, sickness insurance, maintenance, foster allowance and payments from TU/Friendly societies, from absent family members). Social benefits income totals all receipts from state benefits including national insurance retirement pensions. Household Annual Net Labour Income=Net Labour Income (wHHYRLN) + Private Transfers (wHHYRRT) + Public Benefits (wHHYRRB)+pension contributions(wYRCONTR).

7.6 Sample selection (WAS)

Tables A2 shows the various sample selection steps. The household heads must be between 25-59 years of age, have full information for the relevant demographic information and their household earnings should be reported and not imputed. Moreover, we trim the top and bottom 0.25% of observations of net labour income distribution in each year. In Table A3 we show the various sample selection steps we followed to estimate the net income risk from the WAS dataset. The steps are similar to Table A2 with the main difference being that we keep only the survivors, i.e. those households who are in the survey for all the 5 waves and for whom we have longitudinal sampling weights. A further step is that we require the households to be observed with positive incomes for at least two consecutive periods.

Table A2: WAS Sample selection, household observations per selection step

selection step	Uni	Non-Uni	Total
1. Whole sample of households			110,963
2. Drop households with missreported age variable			110,937
3. Drop households with duplicate hh grid numbers			110,910
4. Keep if heads' age $\geq$25, $\leq$60			59,457
5. Drop if no or misreported head's educational info	17,490	41,056	58,546
6. drop if earnings of household members are imputed	17,037	40,235	57,272
Average net worth obs per wave	3,407	8,047	11,454
(the below steps are only for the calculations of income inequality)			
7. keep only waves 3, 4 & 5	9,131	19,356	28,487
8. drop if net income is zero	8,965	19,059	28,024
9. drop top and bottom 0.25% of observations	8,926	18,970	27,896
Average net income obs per wave	2,975	6,324	9,299

Table A3: WAS Sample selection 2, household observations per selection step

selection step	Uni	Non-Uni	Total
1. Whole sample of households			110,963
2. Drop households with missreported age variable			110,937
3. Drop households with duplicate hh grid numbers			110,910
4. Keep if heads' age $\geq$25, $\leq$60			59,457
5. Drop if no or misreported head's educational info	17,490	41,056	58,546
6. drop if earnings of household members are imputed	17,037	40,235	57,272
7. keep only waves 3, 4 & 5	9,131	19,356	28,487
8. keep only if households are survivors	2,953	5,690	8,643
9. drop if net income is zero	2,878	5,583	8,461
10. drop top and bottom 0.25% of observations	2,866	5,559	8,425
11. keep households with at least one net income dif.	2,676	5,064	7,740
12. drop households changing skill groups	2,494	4,902	7,396
Average net income obs per wave	831	1,634	2,465

7.7 Sample selection (BHPS)

Our sample selection for BHPS, reported in Table A4, is similar to the one applied to the WAS dataset for the earnings inequality estimations (steps 1-12 in Table A3). In particular, the household heads must be between 25-59 years of age, report non-zero labour income and their household earnings should be reported and not imputed. Moreover, the head must not have missing values for region and educational attainment. Furthermore, we trim the top and bottom 0.25% of observations of net labour income distribution in each year, to avoid extreme cases or possible outliers in recorded income that may affect results. A further step is that we require the households to be observed with positive incomes for at least 8 observations through the years. As in the WAS, we exclude Northern Ireland.

Table A4: Households and household members BHPS

selection step	Uni	Non-Uni	Total
1. Whole sample			130,974
2. Drop proxy & non-full interviews			128,348
3. Original sample			82,355
4. Full interview of all members in household			74,602
5. Keep if heads' age $\geq$25, $\leq$60			46,850
6. Drop if no head's educational info			46,443
7. Drop if head's region missing	7,974	38,435	46,409
8. Drop if head's marital status missing	7,972	38,434	46,406
9. Drop if gross labour income is missing or imputed	6,391	30,026	36,417
10. Drop if net labour income is zero	6,338	29,813	36,151
11. drop top and bottom 0.25% of observations	6,332	29,686	36,018
12. keep households with at least 8 observations	4,316	19,363	23,679
Average net income obs per year	240	1,076	1,316

8 Appendix B

The computational algorithm described below is based on the "canonical" approach (see also Ljungqvist and Sargent (2012, ch. 18) and Miao (2014, ch. 17.1)).

`Computational algorithm`

1. Guess a value for $K = K_j$ from a domain $\left[K^{\min}, K^{\max}\right]$ and calculate $r\left(K_j\right)$, $w\left(K_j\right)$.
2. Solve the "typical" households' problem to obtain $g^h\left(a_t^h, s_t^h\right)$, for $h = u, b$.
3. Use $g^h\left(a_t^h, s_t^h\right)$ and the properties of the Markov processes $\left(s_t^h\right)$ to construct the transition functions $\Lambda_{K_j}^h$. Using $\Lambda_{K_j}^h$, calculate the stationary distributions λ^h.
4. Using λ^h, compute the average value of capital $K_j^* = n^u \int_{\mathcal{A}^u \times S^u} g^u\left(a, s\right) \lambda^u\left(da, ds\right) + n^b \int_{\mathcal{A}^b \times S^b} g^b\left(a, s\right) \lambda^b\left(da, ds\right)$.
5. If $\left|K_j^* - K_j\right| < e$, where e is a pre-specified tolerance level, a stationary equilibrium has been found. If not, go back to step 1, update and repeat until convergence.

To implement this algorithm we first choose $K^{\min} = -0.85$. As discussed in more detail in the calibration section, we choose this value to match the percentage of indebted agents in WAS. We then let $K^{\max} = 50$, which implies that, in the solution, the probability of asset holdings greater than 40 is less than $3.1 * 10^{-4}$. We discretise $\left[K^{\min}, K^{\max}\right]$ by allowing for 1000 points. We have found that the obtained wealth distribution is robust to increasing $K^{\max}$ up to 100 and to decreasing it down to 40.

A useful theoretical result noted in Section 2 is that λ^h is the unique invariant distribution for the typical household $h = u, b$. As discussed, however, the computed stationary general equilibrium need not be unique. To check whether more than one equilibria exist, we solve the problem of the household in Step 2 and compute the invariant cross-sectional distribution and mean of asset supply in Steps 3-4, for a range of interest rates consistent with the model, and examine whether asset demand and supply intersect more than once.

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Figure 1: Quintle Shares and Lorenz Curves of the Wealth Distribution by Group

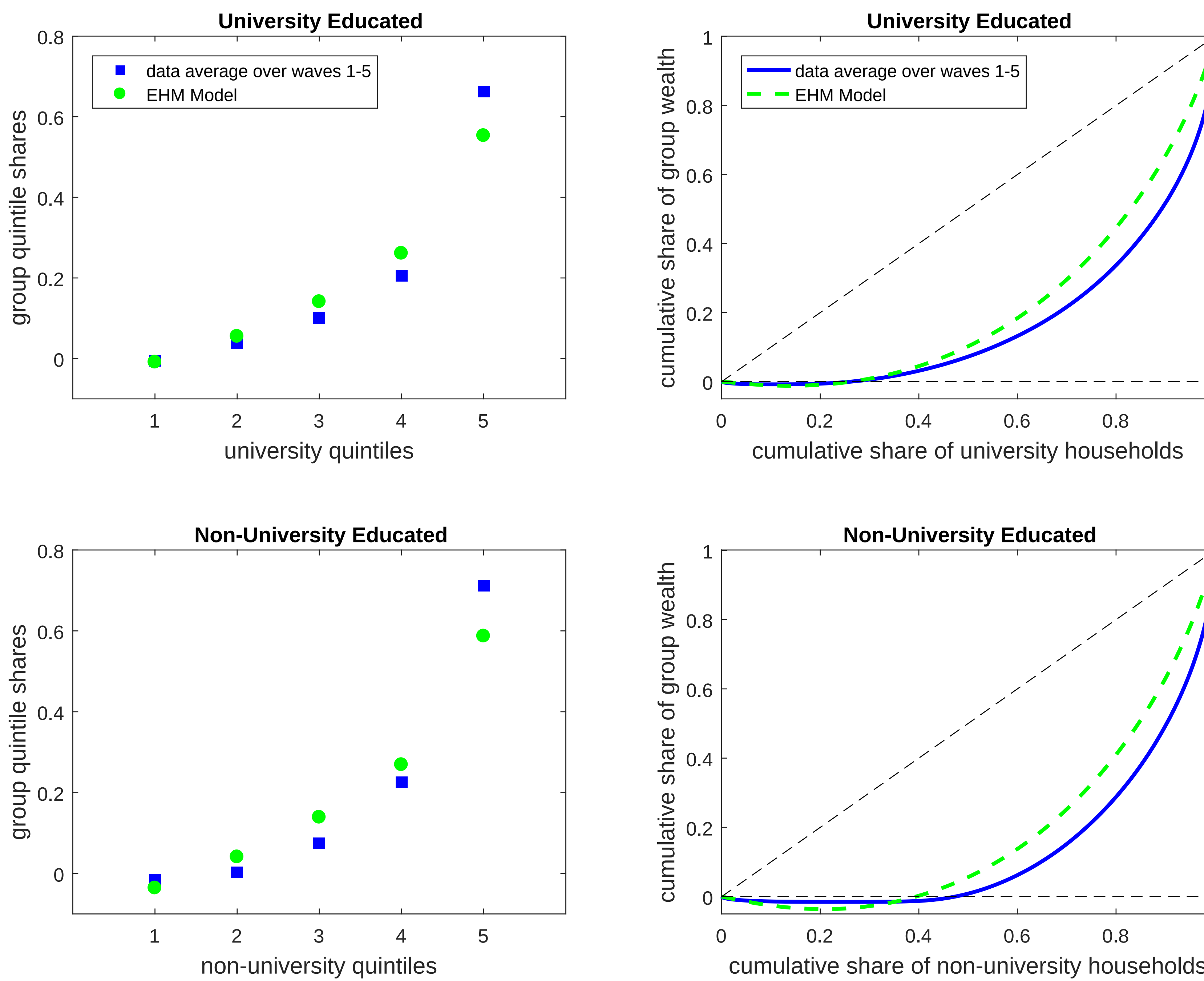

Figure 2: General Equilibrium

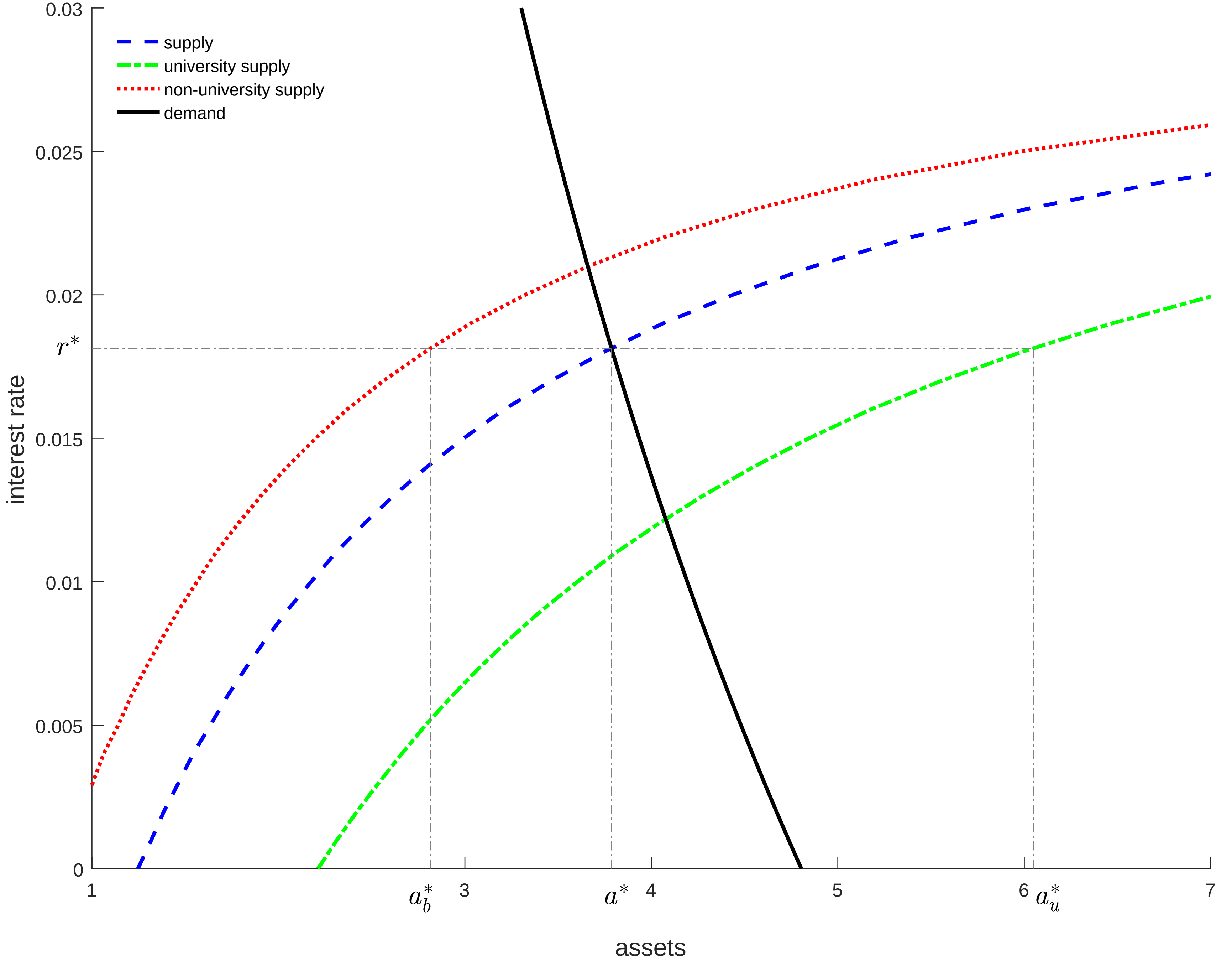

Figure 3: Externalities From Skill Heterogeneity

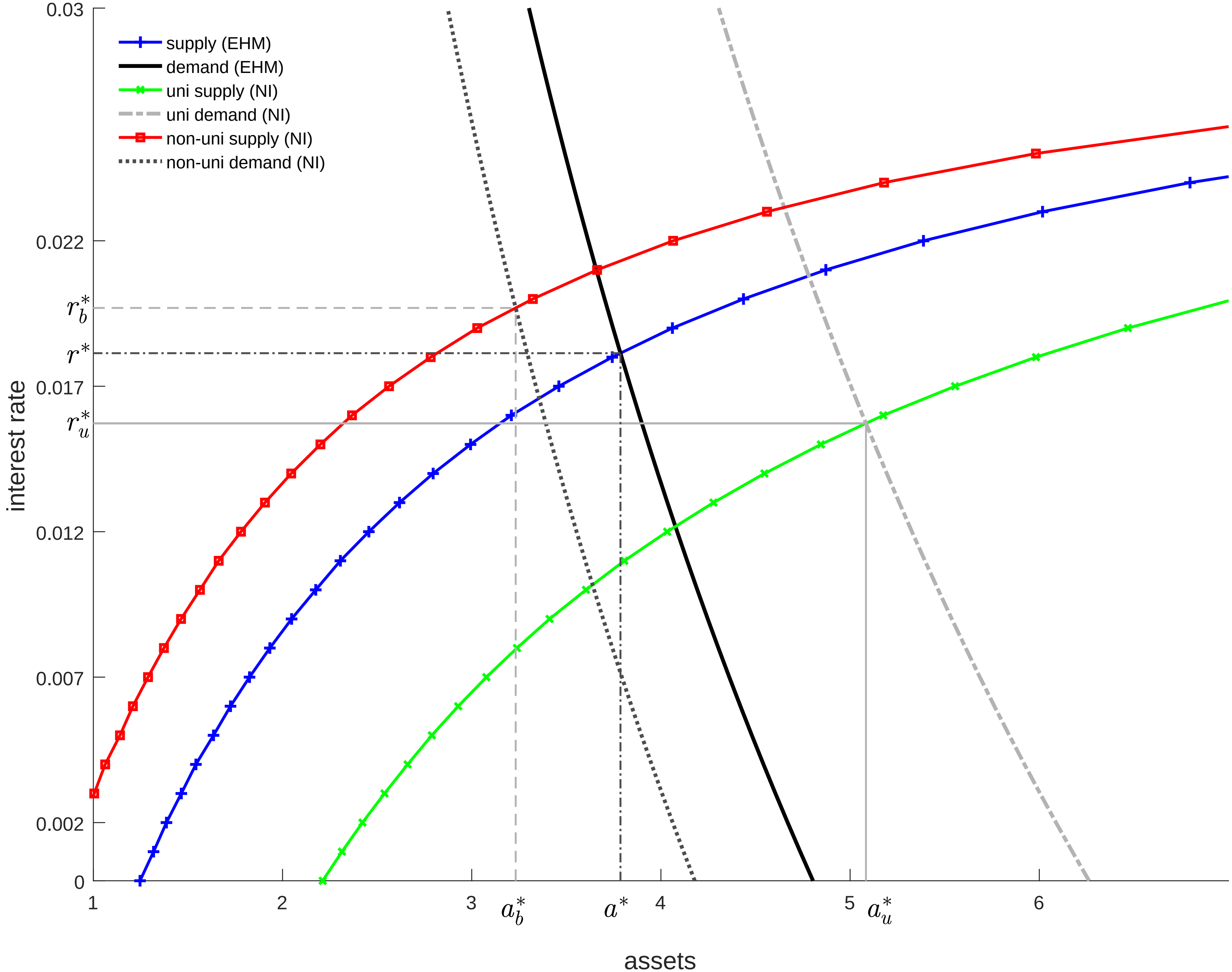

Figure 4: Lorenz Curves of the Wealth Distributions

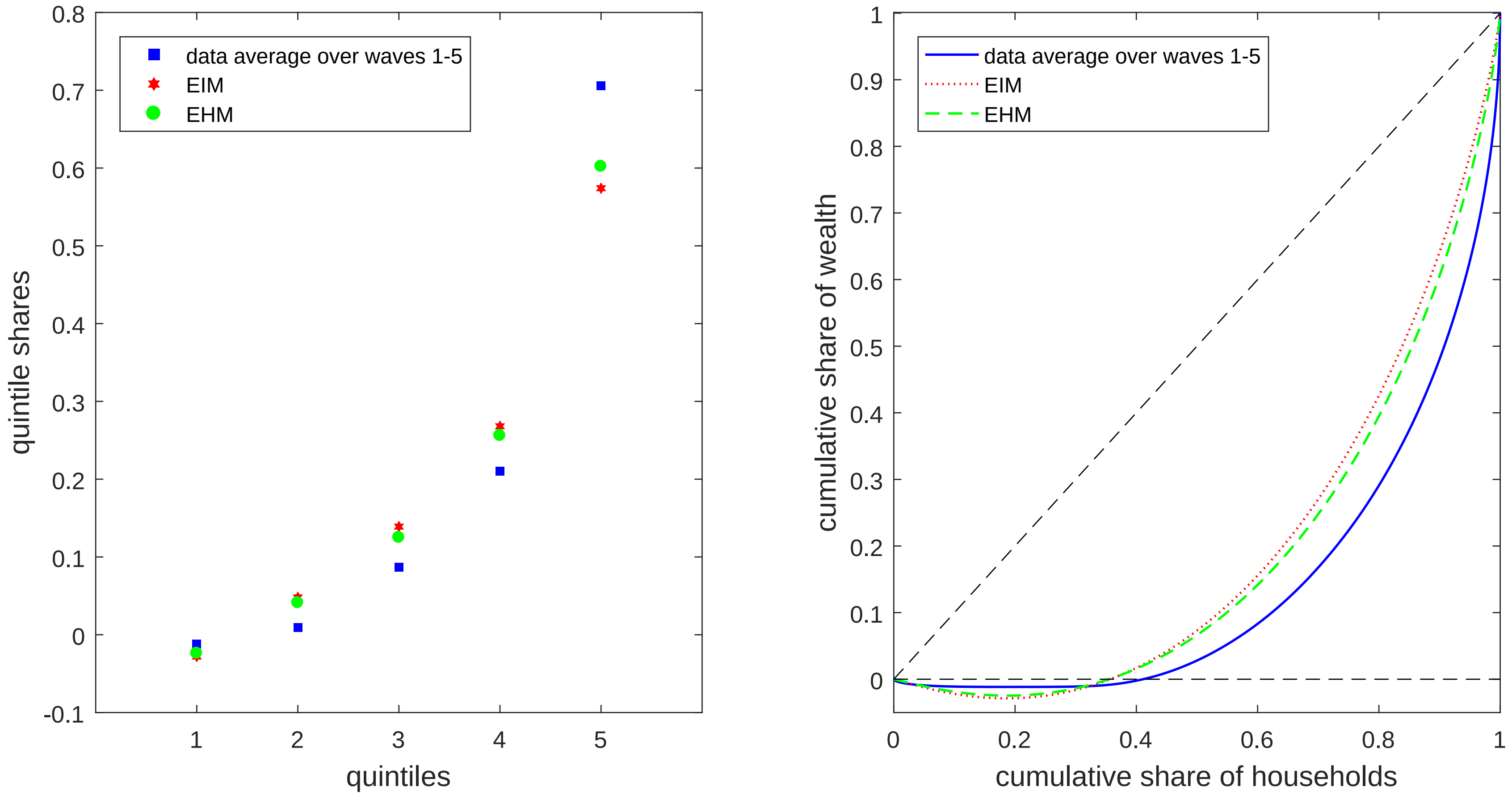

Figure 5: Changes in Earnings and Risk