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Calibration and Computation of Household Portfolio Models*

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Abstract

This paper discusses calibration and numerical solution of a wide range of household portfolio models. We illustrate the main conceptual, technical, and computational issues that arise in the context of household portfolio choice, and explore the implications of alternative modeling choices. We consider both small- and large scale optimization models under finite and infinite horizons and under two types of earnings shocks, permanent and transitory. The role of alternative preference specifications, of borrowing constraints, and of predictability of excess returns on stocks is also discussed. In the process, we explore enduring portfolio puzzles and identify new ones to be resolved in future research. These include puzzles relating to participation in the stock market and to portfolio shares conditional on participation.

JEL Classification: E2, G11.

Key Words: Precautionary Saving, Portfolio Choice, Liquidity Constraints, Preference Specifications, Buffer Stock Saving, Calibration, Computational Methods.

1 Introduction

This paper discusses calibration and numerical solution of household portfolio models in a variety of specifications. Interest in the computational approach has been generated mainly by the difficulties associated with obtaining exact analytical solutions in dynamic, intertemporal models of portfolio choice that allow for uninsurable background earnings risk. Our objective is threefold. First, to illustrate the main conceptual, technical, and computational issues that arise in this context; second to explore the portfolio implications of alternative modeling choices, isolating the individual contribution of each major factor wherever possible and understanding the main mechanisms at work; and third, to identify new and explore enduring puzzles, i.e. discrepancies between the properties of optimal portfolios and econometric findings in recent empirical studies based on household-level data, that future portfolio research should resolve.[1] As in other areas where calibration is used, the main purpose of a calibrated model is not to mimic reality but to provide understanding into the main economic mechanisms. Thus, puzzles should not be viewed as proof that existing portfolio models are irrelevant for the real world, but as an impetus to identify economic mechanisms sufficiently strong to modify the tendencies already captured in existing models.

Portfolio puzzles relate either to participation in the stock market or to portfolio composition of participants. The enduring participation puzzle is that, despite premia on equity, there is no country in the world where the majority of households hold stocks, directly or indirectly. In our calibration experiments, we do identify cases where zero stockholding is optimal. However, these typically involve a combination of low current cash on hand and constraints preventing households from borrowing. The former is unlikely to explain persistent zero stockholding by vast segments of the population; the latter may understate the true borrowing potential of most households. We report on recent work suggesting that fixed costs of entry and/or participation in the stock market could contribute significantly to explaining the participation puzzle.

We uncover three portfolio composition puzzles. One arises from the tendency of models to imply that it is optimal for small savers to hold all of their assets in the form of stocks so as to take advantage of the equity premium. Co-existence of positive holdings of stocks and of riskless assets is optimal only for households with large amounts of cash on hand, who can

[1] We refer in particular to the country studies by Alessie, Hochguertel and Van Soest (2000), Banks and Tanner (2000), Bertaut and Starr-McCluer (2000), Borsch-Supan and Eymann (2000), and Guiso and Jappelli (2000) that cover, respectively, household portfolios in the Netherlands, the UK, the US, Germany, and Italy.

afford to give up the equity premium on part of their savings in exchange for reducing portfolio risk. While the tendency of large savers to hold more diversified portfolios is present in the data, there is no corresponding tendency of small savers to be fully invested in stocks. Small savers tend to hold no stocks, and those who do hold stocks tend to hold also riskless assets. What is the economic force that overrides the incentive of small savers to put all of their savings in the most high-powered vehicle, other than risk aversion, earnings risk, and borrowing constraints which are already incorporated in the calibration models?

A second participation puzzle is that, for those predicted to hold the riskless asset, the share of risky assets in total financial wealth is decreasing in cash on hand. This is consistent with the view that richer households do not need to rely as much on the wealth-generating power of the equity premium and can afford to put a larger share of their wealth in the riskless asset. Yet, in the data financial wealth and current labor income contribute positively to the portfolio share of the risky asset, conditional on participation.

A third participation puzzle relates to the prediction that the portfolio share of risky assets, conditional on holding stocks, is strongly decreasing with age. Empirical studies using household-level data find a hump-shaped age profile for the probability of participation in the stock market, but cast serious doubt on the sign and significance of age effects for portfolio composition conditional on stock market participation (see, for example, Guiso and Jappelli, 2000). In view of the strong tendency of models to yield age effects, and of financial advisors to encourage households to reduce exposure to stockholding risk as they grow older, this is perhaps an area where the ball is in the court of empirical researchers. If the absence of age effects is indeed established across countries, time periods, and estimation methods, then this will not only provide a major challenge to model builders but may also imply that more emphasis should be placed on explaining the stock market participation decision than the age pattern of portfolio shares among stockholders.

In building models to study household portfolio choice, the researcher is faced with numerous modeling choices. Should one build a partial equilibrium or a general equilibrium model? Would the (clearly unrealistic) assumption of infinite investor horizons yield a good simplifying approximation to the portfolio behavior of an important subset of the population? If one assumes finite horizons, should one build a large scale model that allows portfolio rebalancing every year, or can one obtain the key insights by solving variants of versatile, small-scale models that focus on long term career risk and consider only the changes in portfolios associated with major landmarks in the economic life of a household? What aspects of preferences and of the environment are likely to be important in shaping household saving and portfolio behavior? Answers to such questions are unlikely to be unanimous, but they should be informed. As is true for good theory, good computation should deliver the most relevant results in the least complicated way. The wide range of calibration exercises reported in this paper are intended to provide a guide not only as to how one can build and solve models but also as to the likely relevance of each complication for optimal portfolios.

In Part 2, we describe some key choices regarding model ingredients. Section 2.1 deals with preference specification, nesting expected utility and two measurable departures from it (Kreps-Porteus preferences and rank dependent utility). Habit formation is also examined as a variant of expected utility. The main purpose here is to illustrate the effects on household portfolios of preference attributes such as risk aversion, elasticity of intertemporal substitution, and excessive concern about bad outcomes. Other preference structures could also be relevant for portfolios, and it would be fruitful to explore them in future work: concern about keeping up with the neighbors, cumulative prospect theory, Hansen-Sargent risk-sensitive preferences, etc. Experimentation with preferences should be done with caution, as it tampers with one of the fundamental building blocks of an economic model. Even if one finds a preference structure that explains portfolio behavior, this structure ultimately needs to be validated with reference to other types of behavior, unless we believe that the economy is populated by Jekylls and Hydes.

Section 2.2 deals with modeling the economic environment of the household, potentially one with market incompleteness and frictions. We discuss modeling of nondiversifiable labor income risk, borrowing and short-sales constraints, and fixed costs of entry and participation in the stock market. Earnings risk is essential if one is to study precautionary motives for asset holding. Quantity constraints on borrowing have important effects on the structure of portfolios and on the extent to which these can be adjusted in response to earnings risk. Fixed costs may hold the key to explaining the limited household participation in the stock market documented in country studies. In discussing their effects on portfolios, one is not only interested in the individual role of each factor, but also in the powerful portfolio effects of their interaction.

Part 3 of the paper describes calibration. This entails approximating continuous stochastic processes that govern labor incomes and stock returns with discrete processes, choosing values for preference parameters, and examining sensitivity of portfolios to them. This is followed, in Part 4, by a description of a generic household portfolio model, whose different variants we explore in the rest of the paper. Our focus is on household behavior for given asset return and labor income processes. Although such partial equilibrium models have more modest aims than fully-fledged general equilibrium analysis, modesty may be both warranted and instructive when it comes to portfolio analysis. It seems warranted given the puzzles discussed above: how can we hope to explain both portfolio behavior and asset returns when we find it difficult to account for portfolio behavior given the historical processes of asset returns? It is instructive, because it provides an important building block that can be used in future general equilibrium models, either long-horizon or overlapping generations models. Indeed, we discuss some promising early examples of general equilibrium setups towards the end of the paper.

Part 5 deals with the nature of optimal household portfolios and with how to compute them using different variants of the model in Part 4. We derive policy functions for consumption and portfolio components in terms of cash on hand, and we examine how these are influenced by preferences and the economic environment. We start, in Section 5.1, with methods appropriate for versatile, three-period models that can be used to study a great variety of specifications at small computational cost. Because of the nature of computational algorithms, solution is easiest in the absence of borrowing constraints. We exploit these features to study the implications of expected utility and of departures from expected utility maximization for portfolios and for precautionary effects. We also show how risk aversion and positive correlation between stock returns and earnings risk can alter portfolios. We then discuss effects of income-based borrowing limits.

This provides a link to large-scale models with many periods and portfolio rebalancing in each, since these assume that households cannot engage in short sales of either stocks or bonds. The computational algorithms for such models are actually helped by short sales constraints, as these limit the range of admissible solutions for portfolio holdings. We first consider infinite horizons, in Section 5.2, and we describe solution methods based either on computation of value functions or on iteration between first-order conditions. We use the latter to compute solutions. We also explain derivation of time series moments for household portfolios, either through simulation over time or through the time invariant distribution of cash on hand. From an economic viewpoint, this section highlights the role of risk aversion, earnings risk, positive covariance between stock returns and earnings shocks, as well as of entry costs in determining stockholding levels and total wealth holdings. Section 5.3 explores a finite-horizon, life-cycle variant. This part ends with a brief discussion of the prospects for building general equilibrium models with aggregate uncertainty. We then offer some concluding remarks.

We wish to state at the outset that we cannot and do not do justice to the full array of existing computational algorithms and approaches to solving intertemporal models of household choice under uncertainty. In order to explore variants that differ only in the sense relevant to each section, we had to write and run numerous computer programs. We cannot claim that they are the only possible ways to solve such models. We describe them and use them because we know them best, and we are reasonably confident that they do not yield materially different solutions from other existing methods in the literature. Wherever possible, we also refer to papers by other authors who follow different techniques than ours. We hope to offer enough information to the readers so that they can experiment with their own models and algorithms.

2 Modeling Choices

2.1 Preferences

A popular saying among economists is that, for each desired result, there is a preference structure that will justify it. Although this view discourages some from experimenting with alternative preference assumptions, we share the view that some exploration of flexible preference forms can be fruitful, as long as their performance is validated with reference to different aspects of behavior. Our preference specification is based on Epstein and Zin (1989). A household is assumed to maximize in each period t recursive utility U_t of the form:

$$U_t = W(C_t, \mu(U_{t+1}|I_t)) \tag{1}$$

where W is an "aggregator function". Utility is a function of current consumption and of some certainty equivalent of next period's uncertain utility, based on current information, I_t. We assume that the aggregator function is:

$$W(C_t, \mu(U_{t+1}|I_t)) = \left[(1-\beta)C_t^\zeta + \beta\mu_t^\zeta\right]^{\frac{1}{\zeta}}, 0 \neq \zeta < 1 \tag{2}$$

or

$$W(C_t, \mu(U_{t+1}|I_t)) = \left[(1-\beta)\ln C_t + \beta \ln \mu_t\right], \zeta = 0 \tag{3}$$

where $\mu_t(\cdot)$ is an abbreviation for $\mu(\cdot|I_t)$. Our proposed functional form for $\mu_t(\cdot)$ nests alternative preference specifications:

$$\mu(U_{t+1}|I_t) = \left[f_t\left(U_{t+1}^\alpha\right)\right]^{\frac{1}{\alpha}}, 0 \neq \alpha < 1 \tag{4}$$

or

$$\ln \mu \left(U_{t+1}|I_t\right) = f_t \left(\ln U_{t+1}\right), \alpha = 0 \tag{5}$$

where f_t is a linear operator that utilizes information available in period t. The definition of f_t will vary depending on preference type. Suppose that the household chooses at time t some control variable h_{it}, where i indexes control variables (e.g., asset levels). The first order conditions for utility maximization are of the form:

$$C_t^{\zeta-1}\frac{\partial C_t}{\partial h_{it}} + \beta \left[f_t \left(U_{t+1}^{\alpha}\right)\right]^{\frac{\zeta}{\alpha}-1} f_t \left[U_{t+1}^{\alpha-\zeta} C_{t+1}^{\zeta-1}\left(\frac{\partial C_{t+1}}{\partial h_{it}}\right)\right] = 0, \forall i, t. \tag{6}$$

Expected utility (EU) is obtained under two restrictions: (i) $\alpha = \zeta$; and (ii) $f_t \equiv E_t$, i.e., the linear operator f_t is the mathematical expectation operator conditional on information in period t. A variant of EU allows for habit formation, i.e. for a stock of habits to affect current utility; ceteris paribus, for a higher habit level, higher consumption will be necessary to achieve the same utility.[2] With "external" habit formation, an individual's habit depends on the history of aggregate consumption (this is Abel's (1990) "catching up with the Joneses" formulation or Duesenberry's (1949) "relative income" model). The felicity function is usually specified as $\frac{(C_t-H_t)^{\alpha}-1}{\alpha}$ where H is the level of the habit. Defining the surplus consumption ratio as $SUR_t = \frac{C_t-H_t}{C_t}$, it is straightforward to show that the local curvature of the utility function equals $\frac{1-\alpha}{SUR_t}$ and is increasing in the level of the habit. Campbell and Cochrane (1998) show that in recessions, the agent requires a higher return to hold the claim to the risky asset, rationalizing a higher equity premium. The first order condition now becomes

$$C_t^{\alpha-1} SUR_t^{\alpha-1} (\frac{\partial C_t}{\partial h_{it}}) + \beta E_t C_{t+1}^{\alpha-1} SUR_{t+1}^{\alpha-1}\left(\frac{\partial C_{t+1}}{\partial h_{it}}\right) = 0, \forall i, t.$$

In the "internal" habit formulation, the habit is determined by past individual consumption (Constantinides, 1990), and current decisions affect the utility from future consumption.

Kreps-Porteus preferences (KP) disentangle the effects of risk aversion from those of the elasticity of substitution. Under KP, the linear operator f_t in equation (6) is still the expectations operator E_t, as under EU, but the risk aversion parameter α is no longer tied to the intertemporal elasticity

[2]See Ryder and Heal (1973), Sundaresan (1989), Constantinides (1990); and Deaton (1992) for a recent overview.

parameter ζ. Departures from expected utility are measured by the difference between the elasticity of substitution used in the KP model and the value used in the EU model, namely the inverse of risk aversion (see Haliassos and Hassapis, forthcoming).

Under EU or KP preferences, households assign to each state a weight equal to its probability of occurrence. A literature pioneered by Quiggin (1982) and Yaari (1987) argues in favor of specifying weights that depend on the desirability ranking of each state. A simple example involves only two states, "bad" and "good", occurring with probabilities p and 1-p, respectively. Under rank-dependent utility, the bad state obtains a weight of p^γ, where $\gamma < 1$, and the good state obtains 1-p^γ. Given that both p and γ are below unity, this results in overweighting of the bad state relative to expected utility. When more than two states exist, the formula for assigning a weigth w_j to the state ranked jth is

$$w_j = \left(\sum_{i=1}^{j} p_i \right)^\gamma - \left(\sum_{i=1}^{j-1} p_i \right)^\gamma \tag{7}$$

where i indexes states of the world. When $\gamma = 1$, these reduce to $w_j = p_j$, as in EU and KP models. When $\gamma < 1$ and $\alpha = 1$, i.e., the degree of relative risk aversion is equal to zero, we have a version of Yaari's "Dual Theory of Choice".

The desirability ranking of states of the world that involve different labor income and asset return realizations can change, often repeatedly, as we vary the level of risky asset holdings. For example, in the absence of any other risk, including labor income risk, high stock returns are preferred when stockholding is positive, but low returns are preferred when the household has a short position in stocks. Whenever the desirability ranking changes, the weights attached to each state need to be recomputed using equation (7). Adjustment of weights alters the objective function, generating a point of nondifferentiability of indifference curves at each level of risky asset holdings where a switch in desirability rankings occurs. Epstein and Zin (1990) and Haliassos and Bertaut (1995) had suggested that this property might help resolve the participation puzzle because of a kink at zero stockholding, where the household switches objective functions. Haliassos and Hassapis (forthcoming) have since shown that kinks do not occur at zero stockholding in the presence of labor income risk, and that such preferences cannot resolve the participation puzzle in the absence of other frictions and imperfections. Further improvements in predicted portfolio shares, conditional on participation, could result from experimentation with rank dependent utility or with more flexible forms of expected utility.

2.2 Market Frictions and Imperfections

Portfolio literature to date suggests that preferences alone are unlikely to resolve the stock market participation puzzle and may even have trouble accounting for the limited level of stockholding conditional on participation. Both tasks are facilitated, however, when certain types of market frictions and imperfections are incorporated in portfolio models. In this section, we describe such complications that have important portfolio consequences: labor income risk, borrowing constraints, and stock market participation costs.

2.2.1 Nondiversifiable Labor Income Risk

Background labor income risk is nondiversifiable because of moral hazard and adverse selection considerations. Analytical solutions for portfolio models with labor income risk are available for linear, quadratic, and exponential felicity, all of which have known questionable properties for consumption and portfolios. Preferences displaying constant relative risk aversion require solution via computational methods. We adopt the following exogenous stochastic process for income of household i:

$$Y_{it} = P_{it} U_{it} \tag{8}$$

$$P_{it} = G_t P_{it-1} N_{it} \tag{9}$$

This process, first used in a nearly identical form by Carroll (1992)[3], is decomposed into a "permanent" component, P_{it}, and a transitory component, U_{it}, where P_{it} is defined as the labor income that would be received if the white noise multiplicative transitory shock U_{it} were equal to its mean of unity. Assume that the $\ln U_{it}$, and $\ln N_{it}$ are each independent and identically (Normally) distributed with mean $-.5 * \sigma_u^2$, $-.5 * \sigma_v^2$, and variances σ_u^2, and σ_v^2, respectively. The lognormality of U_{it} and the assumption about the mean of its logarithm imply that

$$EU_{it} = \exp(-.5 * \sigma_u^2 + .5 * \sigma_u^2) = 1 \tag{10}$$

and similarly for EN_{it}. The log of P_{it}, evolves as a random walk with a deterministic drift, $\mu_g = \ln G_t$, assumed to be common to all individuals. Given these assumptions, the growth in individual labor income follows

$$\Delta \ln Y_{it} = \ln G_t + \ln N_{it} + \ln U_{it} - \ln U_{it-1}, \tag{11}$$

[3]Carroll (1992, 1997) assumes a very small probability (usually 0.5 percent) of an unemployment state with zero labor income.

where the unconditional mean growth for individual earnings is $\mu_g - .5 * \sigma_v^2$, and the unconditional variance equals $(\sigma_v^2 + 2\sigma_u^2)$. The last three terms in (11) are idiosyncratic and average to zero over a sufficiently large number of households, implying that per capita aggregate income growth is given by $\ln G_t$. Individual labor income growth has a single Wold representation that is equivalent to the MA(1) process for individual income growth estimated using household level data (MaCurdy [1982], Abowd and Card [1989], and Pischke [1995]).[4] An alternative specification with less persistent income shocks, not examined in this paper, has been proposed by Hubbard et al. (1994, 1995) and is explained in Haliassos and Michaelides (2000; HM from now on).[5]

2.2.2 Borrowing and Short-sales Constraints

In portfolio models, borrowing needs arise not only for current consumption but also for investment in assets with an expected return premium. Thus, constraints on borrowing could in principle limit investment in premium assets or even preclude participation in certain asset markets. Three types of borrowing constraints that can have important portfolio consequences are borrowing limits, interest rate wedges between borrowing and lending rates, and downpayment requirements for major durables purchases.

In portfolio models incorporating N assets, a general form of borrowing limits is

$$\sum_{i=1}^{N} b_{it} A_{it} \geq 0 \forall t \tag{12}$$

where $0 \leq b_{it} \leq 1$. This allows short sales of any asset provided that certain collateral requirements are met, which depend on the asset used as collateral. The most frequently used quantity constraint in existing portfolio studies imposes no-short-sales restrictions on each asset:

$$A_{it} \geq 0 \forall i, t. \tag{13}$$

It is also possible to incorporate borrowing limits that depend on household labor income, perhaps as a signal of the household's ability to meet repayment schedules:

$$-B_t \leq k Y_t, \quad k \geq 0 \tag{14}$$

[4] Although these studies generally suggest that individual income changes follow an MA(2), the MA(1) is found to be a close approximation.

[5] Portfolio effects of such processes in a variety of small-scale models are derived by Bertaut and Haliassos (1997), and Haliassos and Hassapis (1998, forthcoming, and 2000).

where B_t is the amount of riskless asset (bond) holding in period t and the negative of this is borrowing at the riskless rate.[6] Interest rate wedges and down payment requirements are not examined here, but they are explained in HM.

2.2.3 Stock Market Participation Costs

A promising avenue for explaining the stock market participation puzzle is fixed costs for entering the stock market, possibly coupled by subequent recurring costs for continued participation. Some such costs may be direct, e.g., brokerage or membership fees. Others may involve the value of the household's time devoted to keeping up with developments in the stock market and to monitoring brokers and financial advisors. Value-of-time considerations imply costs proportional to household income. Whatever the objective size of such entry and participation costs, what matters for participation decisions is how they are perceived by the household. Misperceptions, ignorance, and even prejudice can further contribute to inertia.

Rather than attempting to calibrate such unobservable costs, one can compute the minimum size of entry and participation costs required to keep a household with given characteristics out of the stock market. Consider the simplest case of a ticket fee, which applies only to first-time investors. If we denote the value function associated with participating in the stock market by V_s and the value function when using solely the bond market by V_B, the threshold ticket fee that would make a household indifferent between participating and not participating is a function of a state variable like cash on hand, $K(X)$, such that

$$V_S(X - K(X)) = V_B(X) \tag{15}$$

Value functions are monotonic in the state variable and therefore the value functions can be inverted to derive the cost $K(X)$. This function must be greater than zero, since the investor has the right (but not the obligation) to participate in the equity market. Using methods described below, one can determine the distribution of cash on hand in the population if households

[6] The consequences of such constraints have been empirically investigated by Ludvigson (1999) in the context of a single asset model. The saving and portfolio effects of varying the constraint tightness parameter k have been analyzed computationally by Haliassos and Hassapis (1998). In view of accumulating evidence that lenders are unwilling to extend credit to households with highly variable income because of their high probability of default, an interesting extension would be to link borrowing limits to the variability of earnings.

only use bonds as a saving vehicle. This distribution also represents the possible outcomes of cash on hand for a given household over time. One can then compute the maximum level of X that any household is likely to experience, $\widehat{X}$, as that which satisfies $\Pr(X \leq \widehat{X}) = 1$. Then a level of costs equal to $K(\widehat{X})$ would ensure that nobody participates in the stock market, with the marginal investor being indifferent between participating or not. The lower the levels of such ceilings, the more plausible are entry costs as explanations of the participation puzzle.

3 Calibration

Once the various components of the model have been chosen, the researcher needs to calibrate parameter values and to approximate continuous stochastic processes, such as asset returns and labor incomes, using discrete approximations. Calibration of parameter values is normally based on empirical estimates, where these are available. Even when they are, but especially when they are not available, it is instructive to examine the sensitivity of solutions to a range of parameter values.

A simple (binomial) method to approximate a continuous stochastic process is to postulate two possible outcomes, a "high" and a "low" realization, such that their mean and variance match those of the original stochastic process. In small-scale models (including overlapping generations models), each period is thought of as lasting twenty to thirty years. While riskless rates are simply compounded over this longer interval, risky annual returns can be converted to a binomial process first, which can then be used to compute the mean and variance of multi-year compounded returns. Similarly, any continuous stochastic process for labor incomes can be simulated over a twenty-year period to derive the relevant moments to be matched by a binomial model.

More generally, a discrete approximation of $I = \int_a^b f(x)w(x)dx$ where $w(x)$ is a probability density function, can be found by considering N states and using $\sum_{i=1}^{i=N} \omega_i f(x_i)$. The quadrature nodes $\{x_i : i = 1, ..., N\}$ lie in the domain of x and the quadrature weights $\{\omega_i : i = 1, ..., N\}$ are chosen appropriately so as to make the approximation of $\int fw$ a "good" one.[7] Gauss-Hermite quadrature is often used to evaluate numerically the integral over a function of a normal variable.[8] Tauchen (1986) showed that for univariate problems, a discrete approximation of the underlying random variable over 10

[7]For a more detailed discussion of the practical issues involved in the numerical evaluation of a definite integral, see Chapter 7 in Judd (1998).

[8]For $N = 10$, the quadrature nodes and the quadrature weights are given in Judd (1998, Table 7.4).

points, for instance, works well in practice. Deaton and Laroque (1995) follow a similar procedure by replacing a standard normal variable with N discrete points $Z = (Z_1, ..., Z_N)$. The Z_i are chosen by first dividing the support of the normal distribution into N equiprobable intervals and then finding the conditional means within each interval. For $N = 10$, the 10 values are given by ($\pm$1.75498333, $\pm$1.04463587, $\pm$0.67730694, $\pm$0.38649919, $\pm$0.12599747).[9] Tauchen and Hussey (1991) show how to extend these methods to evaluate expectations of functions of random variables that follow a Markov chain.[10] HM provide more detail, including methods for handling serially dependent processes.

When using discretization methods, a function is evaluated at, say, 100 grid points. It will often be necessary to interpolate the function at points not on the grid. There are two common procedures, linear interpolation and cubic splines (see Judd, 1998, chapter 6). Linear interpolation works well in many portfolio problems where policy functions are well approximated by a piecewise linear specification. Cubic splines are continuously differentiable and have a non-zero third derivative, thus preserving the prudence feature of the utility function. The existence of a second derivative can also be a useful attribute when estimating the model with maximum likelihood, for instance.

Disastrous states of the world that result from the confluence of adverse realizations of random economic variables, such as labor incomes and stock returns, can have substantial effects on optimal portfolios even when they have small probability of occurrence. This is obviously true in rank-dependent utility models, where utility in bad states receives a weight disproportionate to its probability of realization, but also in expected-utility or Kreps-Porteus frameworks under constant relative risk aversion, because marginal utility tends to infinity as consumption tends to zero. Inclusion of such states will induce households to choose portfolios that will not lead to a very low level of consumption even in the small-probability disastrous state. In practice, this means limiting both the extent of borrowing and the exposure to stockholding risk (see Carroll, 1997, on saving effects of zero unemployment income; and Rietz, 1988, on the equity premium).[11]

[9] Assigning a probability of one tenth for each of these nodes, gives a mean equal to zero and standard deviation equal to .964, whereas if the Gauss-Hermite quadrature is used (with $N = 10$), the mean is again zero but the standard deviation is exactly one. In some instances (especially when estimation is involved) this approximation error is worth paying if a matrix programming language like GAUSS is being used.

[10] Burnside (1999, pp. 106-107) provides an excellent discussion of the Tauchen and Hussey (1991) proposal and its relationship to the method described in the text.

[11] Although this approach is potentially powerful and does away with the need to consider credit market frictions in the form of quantity constraints, it still requires assumptions regarding the institutional and legal framework. For example, would it be possible for house-

4 A Model of Household Portfolio Choice

Consider now the problem of a household that lives for T periods, where T can be either finite or infinite. Household preferences are represented using the general Epstein-Zin formulation in Section 2.1. In the first period of life, the household is faced with the recursive problem of choosing a sequence of bond and stock holdings, $\{B_t, S_t\}_{t=0}^{T}$, in order to maximize lifetime utility, U_0:

$$MAX_{\{B_t,S_t\}_{t=0}^{T}} \left[U_0 = W(C_t, \mu(U_{t+1}|I_t))\right], \tag{16}$$

In each period t, the household consumes C_t and chooses a portfolio of bonds and stocks to hold for one period, given the cash on hand, X_t, available to it in the current period:

$$C_t + B_t + S_t \leq X_t \tag{17}$$

In finite-life variants, the household is assumed to have no bequest motive, and thus to consume all cash on hand in the last period, i.e. $C_T = X_T$. Portfolio income in $t+1$ is determined by portfolio composition chosen in t, by the random gross return on stocks, $\widetilde{R}_{t+1}$, and by the constant return on bonds, R_f. The excess return on equity is assumed i.i.d., except in Section 5.2.2 where we discuss mean reversion of stock prices. Annual labor income follows the specification in 2.2.1. Formally, cash on hand evolves as follows:

$$X_{t+1} = S_t \widetilde{R}_{t+1} + B_t R_f + Y_{t+1} \tag{18}$$

for given initial cash on hand, X_0. Consumption plans must satisfy the usual nonnegativity conditions

$$C_t \geq 0, \quad \forall\, t \tag{19}$$

In some model variants, we consider borrowing constraints in the form of no-short-sales restrictions (13):

holds to choose not to repay their loans in such unlikely disastrous states? Alternatively, would it be possible for them to buy unemployment insurance to cover (at least partially) these unlikely events instead of modifying their entire portfolio to accommodate those states? If such unemployment insurance does not exist, then portfolio effects continue to arise from a market failure even though we have not imposed borrowing constraints.

$$B_t \geq 0, \quad S_t \geq 0 \tag{20}$$

The most general form of first-order conditions in $t = 1, ..., T-1$ for choice of B_t and S_t, respectively, are:

$$-C_t^{\zeta-1} + \beta \left[f_t\left(U_{t+1}^{\alpha}\right)\right]^{\frac{\zeta}{\alpha}-1} f_t\left[U_{t+1}^{\alpha-\zeta} C_{t+1}^{\zeta-1} R_f\right] = \lambda_B, \tag{21}$$

and

$$-C_t^{\zeta-1} + \beta \left[f_t\left(U_{t+1}^{\alpha}\right)\right]^{\frac{\zeta}{\alpha}-1} f_t\left[U_{t+1}^{\alpha-\zeta} C_{t+1}^{\zeta-1} \widetilde{R}_{t+1}\right] = \lambda_S, \tag{22}$$

where the Lagrange multipliers λ_B and λ_S are zero when short-sales constraints are either not imposed or not binding. For expected-utility variants, f_t is the expectations operator E_t, and $\alpha = \zeta$, thus yielding the familiar set of conditions for CRRA preferences.

5 Solution

In this Section, we derive and discuss solutions to variants of the basic household portfolio model. We examine three-period models, infinite-horizon, and multi-period finite-horizon models, under various specifications of exogenous labor income and asset return processes, as well as market imperfections.

5.1 A Small-scale Model Variant

Let us start with an end-of-period, three-period model ($T = 3$), which could also be used as a module within general equilibrium, overlapping generations models of portfolio choice. At the end of the first two twenty-year time periods, the household consumes and chooses portfolios to hold over the second half of working life and during retirement, respectively. At the end of the retirement period, it consumes all cash on hand. Such models are solved either by constrained-optimization routines in software such as MATLAB or GAMS, or by solving the full nonlinear equation system. Since solutions are indexed by time period, state of the world, and history of past states, it is easy to handle cases where current policy is not only a function of realized cash on hand but also of prior portfolio composition (e.g., because of differential transactions costs, or capital gains taxation).

Annual labor incomes follow the specification described in Section 2.2.1. First period income is the present value of labor incomes received between ages 21 and 40, and it is known prior to consumption or portfolio decisions. Starting from unity (a normalization), annual incomes grow exponentially at a known annual rate $\mu_g - .5 * \sigma_v^2$,with $\mu_g = 0.03$ and $\sigma_v = 0.08$. This rate is equal to the unconditional mean growth for individual annual earnings when earnings are stochastic. When second-period incomes (from age 41 to 60) are assumed nonstochastic, they are derived by extrapolation of this process for the next twenty years. When they are assumed stochastic, we set $\sigma_u = 0.1$, and $\sigma_v = 0.08$ to simulate 20,000 twenty-year sequences of annual labor incomes and compute the mean and variance of their present values. Our high- (low-) income state equals this expected value plus (minus) one standard deviation.[12] Third-period (retirement) income is assumed nonstochastic. To compute the twenty-year present value, annual retirement income is set to 70% of the annual labor income that would be obtained in the last year of working life if annual labor incomes were growing at $\mu_g - .5 * \sigma_v^2$ up to that point.[13]

The benchmark levels of preference parameters are set at $(\rho, \delta, \gamma) = (2, 0.05, 0.5)$, where ρ is relative risk aversion, δ is the annual rate of time preference, and γ is the degree of overweighting of inferior states in rank dependent preferences (see Section 2.1). The intertemporal elasticity of substitution, σ, is equal to the inverse of relative risk aversion in expected utility models, but it is set at 0.5 in non-expected utility specifications.[14] The annual riskless rate is set at 0.02, and the annual equity premium at 0.042, with standard deviation equal to 0.18. [15]

First-period policy functions can be derived by solving the problem for a grid of first-period cash on hand and plotting solutions for real consumption, real stock holdings, and real bond holdings against cash on hand, all normalized by current labor income. Fig. 1 shows such policy functions for an expected utility specification with risk aversion of 3 and without borrowing constraints, while the first panel of Table 1 reports numerical results for a selected subset of the grid of normalized cash on hand.[16] In the absence of borrowing constraints, the model implies that it would be optimal for young

[12]Results are reported in terms of that level of annual labor income which, if received every year, would yield the same present value. This facilitates comparison with levels of annual incomes used elsewhere in the paper.

[13]The labor income levels used in our runs are $[y1, y2h, y2l, y3] = [1.2826, 2.7793, 1.7639, 1.9908]$. Models with income certainty set $y2h = y2l = 2.2716$.

[14]Note that our benchmark Expected Utility specification with $\rho = 3$ is identical to a Kreps Porteus specification with $\rho = 3$ and $\sigma = \frac{1}{3}$. (See Section 2.1).

[15]The high and low twenty-year rates of return on stocks used are 5.2375 and −0.5768, respectively.

[16]First-period income is set at 1.2826 because of the normalization described in the previous subsection.

expected-utility maximizers to hold stocks even at very low levels of normalized cash on hand. This finding is at variance with observed behavior of most young households, and is an illustration of the stock market participation puzzle. As shown in Haliassos and Bertaut (1995), the theoretical result arises because stocks dominate bonds in rate of return and they have zero covariance with the marginal utility of consumption at zero stockholding. At low levels of cash on hand, it is optimal for such young households who expect their labor income to grow over time to borrow at the riskless rate, so as to enhance consumption and to purchase stocks that offer an equity premium. Since borrowing is devoted both to consumption and to stockholding, the net financial worth of these households is negative, and this explains the negative portfolio shares of stocks in Table 1. The marginal propensity to consume out of initial cash on hand is less than one, and households with higher initial resources tend to borrow less and to invest more in stocks.

In addition to the participation puzzle, the model illustrates the three portfolio composition puzzles described in the Introduction. The model implies that it is optimal for poorer households to hold only stocks in positive net amounts (portfolio specialization puzzle), to enrich their portfolios with positive net holdings of riskless assets only if their initial cash on hand exceeds a certain threshold (portfolio coexistence puzzle), and for those with positive net worth to have decreasing portfolio share of stocks as a function of initial cash on hand (decreasing portfolio share puzzle).[17] These puzzles occur, despite a modest perceived equity premium of 4.2%. As will be seen below, they are surprisingly robust to augmenting the scale of the model through extensions in the household's horizon and in the number of states of the world.

It may not be obvious how these theoretical predictions can be reconciled with the usual results of static two-asset models, surveyed in Gollier (2000). In static models, the investor is given a positive amount of initial wealth to allocate between risky and riskless assets and usually chooses a portfolio share of risky assets between zero and one, even in the presence of background labor income risk. Fig. 2 and Table 2 shed light on the apparent conflict, by showing how the first-period share of risky assets in financial net worth varies with normalized cash on hand in the three-period model. In tracing this policy rule, we keep constant the process governing future labor income. Thus, we vary the ratio of initial cash on hand to human wealth. As can be seen in the Figure, the risky portfolio share is particularly sensitive to such

[17] Specifically, households hold positive financial net worth when their initial cash on hand is a bit less than 2.5 times their initial labor income in this calibration, and they start investing positive amounts in stocks and in bonds when it is about triple their labor income.

variation. This suggests that the main source of difference between static and dynamic portfolio models lies in the type of question each asks. Static models postulate a wealth-allocation problem, often with no future labor income, implying a large (in the limit, infinite) ratio of initial resources to human wealth. Indeed, when normalized cash on hand is roughly greater than 3, even the three-period dynamic model predicts an optimal portfolio share between zero and one, as in the static model. By contrast, dynamic computational models focus on young households with future earnings potential but with little or no inherited assets. Fig. 2 shows that for a young household with no initial wealth, who would have normalized cash on hand equal to one, the dynamic model predicts negative financial net worth and positive demand for stocks.

In small-scale models, second-period consumption and asset holdings in each state can be plotted against the corresponding level of second-period cash on hand. This gives us a visual impression of the subset of second-period policy functions relevant for each state. Figure 2 shows solutions for the "best" state 1 that involves high labor incomes and high stock returns and the "worst" state 4 that involves the corresponding low realizations. Although the second-period consumption function has a lower intercept than that for the first, comparison with Fig. 1 shows that its marginal propensity to consume (MPC) is higher, because of the shorter remaining lifetime. Similarly, the bond holding function has a higher intercept but also a higher slope than in the first period. Since first- and second-period policy functions are quite similar in shape, we focus on policy functions for the young in the remainder of this section.

Although Kreps-Porteus (KP) or Quiggin (Q) preferences (see Section 2.1) have small effects on policy functions for consumption, Table 1 shows that rank-dependent utility dramatically lowers stockholding (see also Fig. 4), borrowing, and the portfolio share of stocks in absolute value.[18] In unreported calibrations, we found that the size and sign of differences in stockholding predicted by an EU and a KP model depend on the relationship between risk aversion and the inverse of the elasticity of substitution in the KP model.[19] Positive correlation between earnings shocks and stock returns

[18]See Haliassos and Hassapis (forthcoming) for the solution method for Quiggin models that involve kinks of the indifference curves at points not known a priori. The straight line in Fig. 4 is due to such a kink. A complete set of graphs is in HM.

[19]When risk aversion is larger than the inverse of the intertemporal elasticity of substitution (as in Fig. 4, where risk aversion is $3 > \frac{1}{0.5}$), KP preferences imply lower stockholding than EU preferences. When risk aversion is smaller than the inverse, KP preferences actually enhance stockholding at a given level of normalized cash on hand. When the two are equal, the KP and EU model obviously coincide.

enhances the correlation between stock returns and consumption, thus making stockholding less desirable. Recent empirical research suggests that such correlation is relevant especially for highly educated households (see Heaton and Lucas, 1999 and Davis and Willen (1999)). We have found that even combining positive correlation of 0.3, risk aversion of 8 and Q preferences does not justify zero stockholding in this model (see HM).

Precautionary effects are derived by comparing the above policy functions with those for an identical model that removes earnings risk and ensures labor incomes equal to the values that were expected when earnings risk was present. Table 3 presents precautionary effects on wealth, stocks, and bonds normalized by current labor income, for risk aversion of 3 and uncorrelated labor incomes and stock returns. Figures 5 and 6 plot precautionary wealth and effects on stockholding, respectively. Precautionary wealth is a decreasing function of initial cash on hand for all three preference specifications. This accords with intuition: since the marginal propensity to consume out of initial cash on hand is less than one, households with higher initial resources hold a larger amount of total wealth relative to future labor income and are able to accommodate future earnings shocks with a smaller precautionary buffer. Households that are particularly concerned about utility in the worst state (Q preferences) accumulate a larger precautionary wealth buffer than their EU counterparts with the same level of normalized cash on hand. Figure 6 confirms that normalized stockholding under EU or KP preferences is discouraged by the presence of uncorrelated background risk, but less so for households with higher initial resources. Although KP preferences yield larger precautionary responses in wealth and in stockholding than EU preferences in our benchmark calibration, we found in unreported calibrations that this ranking is reversed when risk aversion falls short of the inverse of the intertemporal elasticity of substitution. Even when reversals between EU and KP rankings were found, Q preferences continued to yield larger precautionary effects than either KP or EU. Thus, weighting of bad states by more than their probability of occurrence can exert considerable influence on precautionary portfolio behavior. Indeed, Table 3 and Fig. 6 show that households with Q preferences accumulate so large precautionary wealth buffers that they end up holding more stocks as well as more riskless assets (or less riskless borrowing).[20]

Haliassos and Hassapis (1998) derive effects of income-based and collateral constraints. They compute precautionary effects as differences between models with and without earnings risk, when both incorporate borrowing constraints. They find that binding borrowing constraints of either type

[20]The peak is a consequence of the kink in the model with risky income (see HM).

reduce precautionary effects on wealth relative to what would have been observed in the absence of constraints, and can reduce or even reverse precautionary effects on stockholding. Such findings suggest that populations which contain a sizeable proportion of borrowing-constrained households are likely to exhibit small or insignificant effects of earnings risk on wealth and on risky asset holdings.

5.2 A Large-scale, Infinite-horizon Model

The wealth of information provided by small-scale models comes at some cost, namely that the number of equations increases rapidly as we add time periods, states of the world, and constraints (the dimensionality issue). Large-scale models adopt computational shortcuts that sacrifice some information but yield solutions for a much larger number of periods and states of the world. The remainder of this paper is devoted to large-scale models that assume expected utility maximization, constant relative risk aversion preferences, and short sales constraints on bonds and on stocks. We first consider the limiting case of an infinite planning horizon, by setting $T \to \infty$ in the Model of Section 4 (see Ramsey, 1926, and Barro, 1974, for motivation).[21]

5.2.1 A Solution Method Based on Euler Equations

We describe here an approach to solving based on the first-order conditions for bonds and stocks. An alternative approach, based on the value function, is described below in Section 5.3.1. Analytical first order conditions for bonds and for stocks respectively can be written as follows:

$$U'(C_t) = \frac{1+r}{1+\delta} E_t U'(C_{t+1}) + \lambda_B \tag{23}$$

and

$$U'(C_t) = \frac{1}{1+\delta} E_t \left[U'(C_{t+1}) \, \widetilde{R}_{t+1} \right] + \lambda_S \tag{24}$$

where λ_B and λ_S refer to the Lagrange multipliers for the no short sales constraints. Recalling the budget constraint $C_t = X_t - B_t - S_t$,where X_t is cash on hand, a binding short sales constraint on bonds, implies that $C_t = X_t - S_t$ since bond holdings are zero. Similarly, a binding constraint on short sales of stock implies $C_t = X_t - B_t$. The Deaton (1991) solution can be generalized to allow for portfolio choice by writing the two Euler equations as:

[21] This first section follows closely the analysis in Haliassos and Michaelides (1999).

$$U'(C_t) = MAX\left[U'(X_t - S_t), \frac{1+r}{1+\delta}E_tU'(C_{t+1})\right] \quad (25)$$

and

$$U'(C_t) = MAX\left[U'(X_t - B_t), \frac{1}{1+\delta}E_t\widetilde{R}_{t+1}U'(C_{t+1})\right]. \quad (26)$$

Given the nonstationary process followed by labor income, we normalize asset holdings and cash on hand by the permanent component of earnings P_{it}, denoting the normalized variables by lower case letters (Carroll, 1992). Defining $Z_{t+1} = \frac{P_{t+1}}{P_t}$ and taking advantage of the homogeneity of degree $(-\rho)$ of marginal utility implied by CRRA preferences,

$$U'(x_t - s_t - b_t) = MAX\left[U'(x_t - s_t), \frac{1+r}{1+\delta}E_tU'(c_{t+1})Z_{t+1}^{-\rho}\right] \quad (27)$$

and

$$U'(x_t - s_t - b_t) = MAX\left[U'(x_t - b_t), \frac{1}{1+\delta}E_t\widetilde{R}_{t+1}U'(c_{t+1})Z_{t+1}^{-\rho}\right]. \quad (28)$$

The normalized state variable x evolves according to

$$x_{t+1} = (s_t\widetilde{R}_{t+1} + b_tR_f)Z_{t+1}^{-1} + U_{it+1} \quad (29)$$

where the last term is the ratio of labor income in period $t+1$ to its permanent component, namely the transitory earnings shock. We use the identity $c_{t+1} = x_{t+1} - b_{t+1} - s_{t+1}$ where both b_{t+1} and s_{t+1} will be functions of x_{t+1} to substitute out c_{t+1} on the right hand sides of (27) and (28). Given that conditions (30) and (31) below are satisfied, we can solve simultaneously for $\{s(x), b(x)\}$. Starting with any initial guess (say $s(x) = .1 * x$ and $b(x) = .1 * x$), we use the right hand side of the first Euler equation to get an update for b and continue doing so until b converges to its time invariant solution b_1^* (see Deaton (1991)). We then use the second Euler equation with b_1^* taken as given, to find the solution for the time invariant optimal s, call it s_1^*. We now have two updated functions $\{s_1^*, b_1^*\}$; the process can be repeated until these functions converge to their time invariant solutions.

In order for the algorithm to work, the Euler equations (27) and (28) must define a contraction mapping. Based on Deaton and Laroque (1992), sufficient conditions for a contraction mapping are

$$\frac{1+r}{1+\delta} E_t Z_{t+1}^{-\rho} < 1 \tag{30}$$

for (27) and

$$\frac{1}{1+\delta} E_t \widetilde{R}_{t+1} Z_{t+1}^{-\rho} < 1 \tag{31}$$

for (28). If these conditions hold simultaneously, there will exist a unique set of optimum policies satisfying the two Euler equations. It can be shown that, under a positive equity premium, these conditions translate into

$$\frac{\mu_r - \delta}{\rho} + \frac{\rho}{2}\sigma_n^2 < \mu_g + \mu_n. \tag{32}$$

Impatience must now be even higher than in a corresponding single-asset model to prevent the accumulation of infinite stocks (see HM). Note that a high expected earnings growth profile, μ_g, can guarantee that the individual will not want to accumulate an infinite amount of stocks or bonds but would rather borrow, expecting future earnings increases. Also, if the rate of time preference exceeds the expected stock return, more risk averse (higher ρ) individuals will not satisfy the convergence conditions. We set the rate of time preference, δ, equal to 0.1, and the constant real interest rate, r, equal to 0.02. Carroll (1992) estimates the variances of the idiosyncratic shocks using data from the *Panel Study of Income Dynamics*, and our baseline simulations use values close to those: 0.1 percent per year for σ_u and 0.08 percent per year for σ_v. We set the mean aggregate labor income growth rate, denoted μ_g, equal to 0.03, and we consider various coefficients of relative risk aversion that meet sufficient condition (32) for the existence of a contraction mapping.

5.2.2 Policy Functions and Time-Series Results

Figures 7, 9, and 10 show respectively consumption, stock holdings, and bond holdings, normalized by the permanent component of income, as functions of similarly normalized cash on hand. Figure 8 plots the share of financial wealth held in the risky asset for different levels of cash on hand for relative risk aversion coefficients equal to 6, 7, and 8. Fig. 7 shows that, at levels of normalized cash on hand below a cutoff x^* (typically around 97% of the permanent component of labor income), the household wants to borrow but is bound by both short sales constraints (Figs. 9 and 10). Its stockholding is zero, as a result. This suggests that a combination of short-sales constraints on both assets and low current resources can provide a reason for not participating in the stock market, but only for those who have no other savings.

These Figures also illustrate two of the three portfolio composition puzzles discussed in the Introduction. Figure 7 demonstrates the portfolio specialization puzzle of Heaton and Lucas (1997). It shows that it is optimal for households with normalized cash on hand above x^* to start saving exclusively in stocks. Haliassos and Michaelides (1999) argue that this happens because, under no stockholding and no correlation between earnings and stock returns,

$$\frac{1}{1+\delta}E_t\left[U'(C_{t+1})\right]\ E_t\left[\widetilde{R}_{t+1}-R_f\right]=\lambda_B-\lambda_S. \tag{33}$$

Given nonsatiation and an equity premium, the left hand side of (33) is positive, i.e. $\lambda_B > \lambda_S$. Thus, households in the neighborhood of x^* would like to borrow risklessly to consume and invest in stocks that offer an equity premium and have zero covariance with consumption.[22] Prevented from borrowing, they devote all saving to stocks. Changes in the degree of risk aversion, rate of time preference, perceived size of (positive) equity premium, or even habit persistence, cannot reverse this result.

Fig. 8 shows that, for those predicted to hold the riskless asset, the share of risky assets in total financial wealth is decreasing in cash on hand. Richer households do not need to rely as much on the wealth-generating power of the equity premium and can afford to put a larger share of their wealth in the riskless asset. Yet, country studies consistently find that both financial wealth and current labor income contribute positively to the portfolio share in the risky asset, conditional on holding stocks.

Fig. 9 shows that normalized stock holdings are increasing in risk aversion at levels of normalized cash on hand that justify saving, while Fig. 8 shows that the portfolio share remains unaffected by risk aversion over a range of cash on hand. This surprising result is due to a conflict between risk aversion and "prudence" in the presence of binding short sales constraints. Since prudence is positively related to risk aversion, households want to increase their net wealth when cash on hand is above x^* (Fig. 7), but none of this increase comes from changes in realized borrowing, which is still at zero because of the binding short sales constraint (Fig. 10). Their desire to increase wealth

[22] Recall that this was also a finding of the unconstrained small-scale model above.

dominates their motive to reduce exposure to stockholding risk, leading to increased stockholding for higher degrees of risk aversion. Interestingly, we have found in unreported calibrations that this feature persists even in a model which assumes that there are no permanent earnings shocks but allows for transitory shocks to earnings.

When we are interested in either the aggregate or the time series implications of a portfolio model, we can simulate individual life histories and optimal choices over time. In the current model, however, normalized cash on hand follows a renewal process and therefore the aggregate or individual time series implications of the model can be derived by computing the time invariant distribution of cash on hand. The method by which this can be done is explained in HM. The invariant distribution of normalized cash on hand can be used to show that mean and median bondholding are zero in the infinite horizon model. Consistent with policy functions, mean and median normalized stock holdings are not only positive, but also increasing in risk aversion. Such portfolio behavior by the more risk averse is justified, since it results in smaller standard deviation of normalized consumption, as well as in higher mean normalized consumption.

Can positive correlation between labor incomes and stock returns, which tends to lower demand for stocks, account for participation and portfolio composition puzzles? Figures 11 to 14 illustrate the effects of positive correlation equal to $0.1, 0.3$, and 0.5. For correlation of 0.3, the household is still predicted to enter the stock market first, but the range of cash on hand for which the saver is predicted to hold both stocks and bonds is considerably expanded (Fig. 13). Thus, this level of correlation is consistent both with households that do not participate in any asset market because of low resources and binding constraints (relevant to the participation puzzle), as well as with households that are better off and hold diversified portfolios (relevant to one portfolio composition puzzle). Positive correlation cannot handle the second composition puzzle identified in the previous subsection. At correlation of 0.5, we find that it will not be optimal for households to participate in the stock market for any level of cash on hand they are likely to experience, a rather extreme solution to the participation puzzle.

How plausible are such levels of correlation? Davis and Willen (1999) obtain correlation estimates ranging between .1 and .3 over most of the working life for college educated males and around $-.25$ at all ages for male high school dropouts.[23] Heaton and Lucas (1999) argue that entrepreneurial risk is positively correlated with stock returns and reaches levels around .2. These

[23] They use the Annual Demographic Files of the March Current Population Survey (CPS) to construct panel data on mean annual earnings between 1963 and 1994.

numbers appear smaller than needed to explain zero stockholding. Moreover, they come close to generating zero stockholding for college graduates or entrepreneurs who in fact tend to hold stocks, and they predict that low education households should actually be holding stocks as a hedging instrument when in fact they tend not to do so.

The positive probability of a disaster event (either in the labor income process or in the realization of a very low stock market return) might substantially affect portfolio choices. We have found that even with a small probability (.5 percent)[24] of receiving a low labor income realization (the latter set at 25 percent of mean labor income) the complete portfolio specialization in stocks result is not reversed. The result is even more robust when disaster events in stocks are allowed (complete ruin with a small probability equal to .5 percent), but a positive floor in labor income exists with positive probability. More work is needed to explore the robustness of these preliminary results for different probabilities and specifications of disaster.

Effects of Stock Market Participation Costs In this section, we report the normalized entry cost to the stock market that would make agents indifferent between entering the stock market or not participating computed in Haliassos and Michaelides (1999). For a household with rate of time preference $\delta = 0.1$ whose labor income is uncorrelated with stock returns, the threshold ranges from 4% of the permanent component of annual labor income when risk aversion is 2 to 16% when risk aversion is 8. The reason that higher costs are needed to discourage more risk averse households is the conflict between prudence and risk aversion noted above. When risk aversion rises, prudence dominates risk aversion and dictates that more wealth be accumulated in the form of stocks. This, in turn, raises the entry costs needed to prevent stockholding. Now, when permanent shocks to household labor income have correlation with stock returns equal to 0.3, the corresponding range is only from 3% to 6%, because of the reduced attractiveness of stocks. Raising the equity premium from the assumed 4.2% to 6% increases the thresholds by about 50%. Halving the rate of time preference to $\delta = 0.05$ roughly doubles the necessary fixed costs.

All in all, threshold fixed costs of entry needed to keep households out of the stock market tend to be quite small, given that they are paid only once and that we have constructed our experiment so as to overstate these costs in at least two other respects. First, we have assumed that once these costs are

[24] Carroll (1997) uses an even lower probability equal to .05 percent to endogenously generate no borrowing in the single asset version of the model.

paid, they entail the household to access to stocks over a (remaining) infinite horizon. Second, the reported level is what would be sufficient to keep all households out of the stock market, as opposed to the approximately 50% that do not undertake stockholding in the United States. Thus, the figures suggest that relatively small costs associated with information acquisition, commissions, time spent, and perhaps even inertia, could keep households out of the stock market.

Indeed, one may wonder why threshold entry costs are so small, despite assumptions designed to overstate them. As shown by the invariant distributions, the reason is that even in the absence of entry costs impatient households are likely to spend a substantial fraction of their time at levels of normalized cash on hand that justify none or very limited stockholding.[25] Since their use of the stock market will be limited in this sense, households require relatively small entry costs to be deterred from entering. The relevance of entry costs for the participation puzzle is the subject of ongoing research. A particularly troublesome feature of the data is the observed co-existence of zero stockholding with substantial holdings of essentially riskless liquid assets for some households.[26]

Stock Market Mean Reversion This Section, based closely on Michaelides (1999) highlights portfolio effects of predictability of the excess return of stocks over Treasury Bills, now considered a stylized fact in finance (see Cochrane (1999)).[27] Stock market predictability is interesting for our purposes, because it can contribute to resolution of portfolio composition puzzles by rationalizing the observed co-existence of bonds and stocks. Letting $\{r_f, r_t\}$ denote the net risk free rate and the net stock market return respectively and f_t being the factor that predicts future excess returns, we have

$$r_{t+1} - r_f = f_t + z_{t+1} \tag{34}$$

$$f_{t+1} = \mu + \phi(f_t - \mu) + \varepsilon_{t+1} \tag{35}$$

[25]The implication of the model that there is not only entry but also exit from the stock market is corroborated by the empirical findings of Bertaut (1998).

[26]See King and Leape, 1984; Mankiw and Zeldes, 1991; Haliassos and Bertaut, 1995.

[27]Other recent papers on the effects of return predictability for saving and portfolios include Barberis (1999), Brennan, Schwartz and Lagnado (1997), Campbell and Koo (1997), Campbell, Cocco, Gomes, Maenhout and Viceira (1998), Campbell and Viceira (1999), and Balduzzi and Lynch (1999). See Michaelides (1999) for an extended bibliography.

where the two innovations $\{z_{t+1}, \varepsilon_{t+1}\}$ are contemporaneously correlated. Mean reversion in the stock market is captured by the autoregressive nature of the factor (f_t) predicting stock market returns ($\phi > 0$). The autoregressive nature of the factor is captured by a ten point discretization scheme. Labelling the m factor states $i = 1, ..., m$, there are m bond and stock demand functions, one for each currently observed factor state.

Figs. 15-18 depict some of the resulting policy functions.[28] When the factor predicting stock returns follows an AR(1) process, there is an incentive for the individual to "time the stock market". A low current factor realization signifying lower future returns induces a decrease in demand for stocks and in saving relative to the i.i.d. case, in response to less favorable future investment opportunities and vice versa. When the current factor realization is above its mean, any additional demand for stocks is equal to the increase in saving since the borrowing constraint is already binding in the i.i.d. model. For such factor realizations, the complete portfolio specialization puzzle persists (Fig. 17). However, when the current factor realization is below its mean, the demand for stocks falls relative to the i.i.d. model, and so does their portfolio share, thus generating portfolio coexistence of bonds and stocks at lower levels of normalized cash on hand than in the i.i.d. case. Witness, for example, the policy functions for the third lowest factor in Figs. 17 and 18 compared to those for the i.i.d. case. Under the lowest realization of the factor, the investor stops participating in the stock market altogether because of the grim stock market prospects (Fig. 17), suggesting a further reason for stock market non-participation, namely the perception of bad prospects in the stock market.

Michaelides (1999) also shows that positive correlation between labor income innovations and stock returns increases the hedging demand for bonds. Time series moments confirm the portfolio co-existence of bonds and stocks. On the negative side, the median stockholding share (counterfactually) remains equal to one, while the volatility of stock market trading that arises from the market timing activity is very high.

[28]Calibration settings are as follows: $\delta = 0.12, r = 0.01$, $\sigma_u = 0.1, \sigma_n = 0.08$, $\mu_g = 0.03, \rho = 3$. The high discount rate is chosen to accomodate the convergence conditions $\frac{r_f + f_t - \delta}{\rho} + \frac{\rho}{2}\sigma_n^2 < \mu_g + \mu_n$ for all factor realizations. The parameters describing the evolution of stock market returns are selected from Campbell (1999, Table 2C) who reports parameter estimates for a VAR model based on annual US data between 1891 and 1994. They are $\mu = .042, \phi = .798, \sigma_z^2 = .0319, \sigma_\varepsilon^2 = .9^2 * .001$, and $\sigma_{z,\varepsilon} = -.0039$. He estimates r_f to be .0199 and $\sigma_\varepsilon = .001$. We decrease both quantities so that the convergence condition can be satisfied for all factor state realizations.

5.3 Large-scale Models with Finite Horizons

Let us now turn to large-scale portfolio models that analyze household choices over the life cycle. Such models are useful even when the properties of solutions to infinite-horizon setups are fully understood. They yield predictions on the age pattern of asset market participation and portfolio composition, based on age-earnings profiles and on factors that are likely to vary over the life cycle, such as earnings uncertainty, demographic characteristics, and constraints facing the household.

We modify the objective function (16) of the model in Section 4 to allow for a horizon of $T+1$ periods with a positive probability of death in each period:

$$MAX_{\{S_{it},B_{it}\}_{t=1}^{T}} E_1 \sum_{t=1}^{T} \beta^{t-1}\{\Pi_{j=0}^{t-1} p_j\} U(C_{it}), \tag{36}$$

subject to constraints (17), (18), (19), and (20). E_1 is the mathematical expectations operator, and $\beta \equiv \frac{1}{1+\delta}$ is the constant discount factor. The probability that a consumer/investor is alive at time $(t+1)$ conditional on being alive at time t is denoted by p_t, with $p_0 = 1$,as in Hubbard, Skinner, and Zeldes (1995). We abstract from bequests, although they can be accommodated easily.

During working years, $1 \leq t \leq T-k-1$, labor income is given by equations (8) and (9) discussed in Section 2.2.1. In the k retirement years, $T-k < t \leq T$, pension income is a fraction c of permanent income

$$Y_t = cP_t \tag{37}$$

where c lies between zero and one.

5.3.1 The Value Function Approach

This approach involves the repeated use of backward induction on the value function. Assuming constant relative risk aversion felicity, the Bellman equation associated with the problem is

$$V_t(X_t, P_t) = MAX_{\{S_t,B_t\}} \left[\frac{C_t^{1-\rho}}{1-\rho} + \beta E_t V_{t+1}\left(\left[S_t\widetilde{R}_{t+1} + B_t R_f + Y_{t+1}\right], P_{t+1}\right)\right] \tag{38}$$

where $V_t(\cdot)$ denotes the value function which depends on the age of the individual and thus has a time subscript, and the first argument of $V_{t+1}(X_{t+1}, P_{t+1})$

has been substituted using equation (18). Cocco, Gomes and Maenhout (1999) use backward induction on (38) to derive the optimal policy functions.

Considerable simplification can be obtained by utilizing the fact that the value function is homogeneous of degree $(1-\rho)$.[29] This property can be used to reduce the number of state variables from three (X_t, P_t, Age_t) to two $(x_t \equiv \frac{X_t}{P_t}$ and $Age_t)$. Instead of computing $V_t(X_t, P_t)$,we can focus on $\widehat{V}_t(x_t) \equiv V(x_t, 1)$; and in view of (38) and of the homogeneity property, this is given by

$$\begin{aligned} \widehat{V}_t(x_t) &= MAX_{\{s_t(x_t), b_t(x_t)\}} \left[\frac{c_t^{1-\rho}}{1-\rho} + \beta E_t V_{t+1} \left(\frac{X_{t+1}}{P_t}, \frac{P_{t+1}}{P_t} \right) \right] = \qquad (39) \\ &= MAX_{\{s_t(x_t), b_t(x_t)\}} \left[\frac{c_t^{1-\rho}}{1-\rho} + \beta E_t \left(\{ \frac{P_t}{P_{t+1}} \}^{1-\rho} \, \widehat{V}_{t+1}(x_{t+1}) \right) \right] \end{aligned}$$

where $s_t = \frac{S_t}{P_t}$ and $b_t = \frac{B_t}{P_t}$ are the normalized holdings of stocks and bonds respectively, and

$$x_{t+1} = \left[s_t \widetilde{R}_{t+1} + b_t R_f \right] \frac{P_t}{P_{t+1}} + U_{t+1}. \qquad (40)$$

Note that U_{t+1} is the transitory earnings shock which enters as the ratio of Y_{t+1} to P_{t+1}.

Backward induction produces the value functions, $\widehat{V}_t(x_t)$, and the policy functions, $b_t(x_t)$ and $s_t(x_t)$, for each period. In the last period and without a bequest motive, $c_T = x_T$ and the value function corresponds to the indirect utility function $\widehat{V}_T(x_T)$. To compute the policy rules and the value function for the previous period $T-1$, the set of admissible values for the decision variables is discretized using equally spaced grids and noting that the short sales constraints (20) bound b_t and s_t from below at zero. For each given level of cash on hand (which is also discretized), the optimal levels of decision variables are chosen by evaluating the value function at all possible pairs (b_t, s_t) and picking the maximands. This grid search is intended to avoid choosing local optima. Expectations of random variables are taken using quadrature methods, and interpolation is used to evaluate the value function for points not on the grid (see Section 3). Once $\widehat{V}_{T-1}(x_{T-1})$ is thus computed, the procedure is iterated backwards to the beginning of working life.

[29]Merton has shown that the value functions for problems with HARA felicity functions inherit the functional form of the felicity function. Homogeneity follows from the same arguments as in proposition 4 and lemma 1 in Koo (1995). Viceira (1998) uses a similar normalization (dividing by the level of earnings).

5.3.2 The Euler Equation Approach

The model can also be solved using the first-order conditions (23) and (24) in their normalized form (27) and (28) respectively, recognizing that policy functions are age-dependent under finite horizons. Equations (27) and (28) comprise a system with two unknowns, $s(x_t)$ and $b(x_t)$, once a functional form for $c_{t+1}(x_{t+1})$ is given. In the absence of a bequest motive, $c_T = x_T$, and the functional form is determined for period T. For $t < T-1$, the policy function $c_{t+1}(x_{t+1})$ is determined numerically, as a set of consumption levels each of which corresponds to a grid point for normalized cash on hand. Using $c_{t+1}(x_{t+1})$, we can begin solving simultaneously this system of Euler equations using backward induction.[30] The proposed algotrithm takes the following form: (1) Given an initial guess about $s(x_t)$, find $b(x_t)$ from (27) using a standard bisection algorithm.[31] (2) Given $b(x_t)$ from (1), find $s(x_t)$ from (28) using the bisection algorithm. (3) If the maximum of the absolute differences between the initial $s(x_t)$ and its update from (2) is less than a convergence criterion (say .0001), then the policy functions for normalized bonds and stocks are determined. The policy function for normalized consumption can also be determined using $c_t = x_t - b_t - s_t$. We repeat for period $t-1$, until we reach the first period of life.

5.3.3 Policy Function Results

Figs. 19-22 report normalized consumption and the share of wealth in stocks both during retirement and working life.[32] They confirm that, for parameter configurations that respect the contraction mapping condition, the backward recursion converges to the infinite horizon solution derived earlier using a different method. Policy rules for the younger agents (age 25 in Figs. 21-22) suggest that infinite horizon models are a good approximation to the behavior of the younger segment of the population.

The low level of prudence ($\rho = 3$) and the equity premium continue to generate complete portfolio specialization in stocks during working life (Fig. 22), illustrating that the puzzle is not unique to the infinite horizons model. Fig. 21 illustrates how saving rises (consumption drops) as one ages

[30] Two questions arise: (a) Do solutions for $\{s(x_t), b(x_t)\}$ that satisfy (27) and (28) exist? (b) Are these solutions unique? If we assume that c_{t+1} is an increasing function of cash on hand, then one can easily show that given $s(x_t)$ the right hand side of (27) is decreasing in $b(x_t)$ while the left hand side is increasing in $b(x_t)$ guaranteeing existence and uniqueness of a solution for $b(x_t)$ from the bond Euler equation. The argument works in exactly the same fashion for $s(x_t)$ by symmetry (given $b(x_t)$ now).

[31] For more details on bisection, see Judd (1998, pp. 147-150).

[32] Calibration settings are $\delta = 0.1, r = 0.02, \sigma_u = 0.1, \sigma_v = 0.08, \mu_g = 0.03, \rho = 3, c = .7$.

once normalized cash on hand exceeds a certain threshold (this is saving for retirement, see Gourinchas and Parker (1999)). Moreover, older people tend to enter the stock market at lower levels of normalized cash on hand than the young (Fig. 22), because they have a higher saving rate than their younger counterparts who anticipate growing labor income.

Given age-specific policy functions, the evolution of wealth over the life cycle can be analyzed either via simulation or by using the transition distribution of normalized wealth in the economy. Figs. 23-26 report the average normalized consumption, bond and stock holdings of 10,000 simulated life histories under two alternative expected rates of income growth ($g = 0.1$ and $g = 0.3$), and two degrees of relative risk aversion ($\rho = 3$ and $\rho = 5$). High impatience ($\delta = 0.1$) results in very low wealth accumulation, and consumption is very close to mean normalized labor income during working life (equal to one) and very close or equal to the normalized pension benefit (0.7) during retirement. Higher prudence generates higher wealth accumulation (compare Fig. 24 to 23, and Fig. 26 to 25), while a higher expected income growth rate acts as a higher discount factor and reduces weatlh accumulation (compare Fig. 25 to 23, and Fig. 26 to 24). Not surprisingly, the highest wealth accumulation in the panel is observed when prudence is highest and income is expected to grow at the lower rate (Fig. 26). The portfolio specialization puzzle is evident in these simulations which imply that wealth accumulation essentially occurs through holdings of stock.

5.4 Towards General Equilibrium Models

The small-scale models we described could in principle be embedded in general equilibrium, overlapping generations models, in which each generation lives for three periods. A pioneering overlapping generations model of this type has been proposed by Constantinides, Donaldson, and Mehra (1998), in order to study the likely effects of borrowing constraints on the equity premium. In their model, as in ours, the young are faced with earnings risk in the second period of their lives, and they would like to borrow in order to invest in stocks and thus take advantage of the equity premium. The middle-aged know that their income in old age will depend on their holdings of stocks and bonds, and they choose to hold positive amounts of both. A borrowing constraint on the young prevents them from financing stockholding through borrowing. This lowers the aggregate demand for stocks in the economy and raises the equity premium relative to what would be obtained in the absence of borrowing constraints. The result shows that equity premia are likely to be higher when "junior can't borrow" and highlights the importance of undestanding the portfolio behavior of young households.

The large-scale models we examined are candidate building blocks for a different type of general equilibrium models. Aiyagari (1994) and Huggett (1993) were the first to study the equilibrium interest rate in heterogeneous agents models with labor income uncertainty and borrowing constraints but without aggregate uncertainty. Den Haan (1996), Rios-Rull (1996) and Krussell and Smith (1998) extend this line of research by solving general equilibrium models with aggregate uncertainty. In models with heterogeneous agents and aggregate uncertainty, the wealth distribution becomes an endogenously evolving state variable. Given that this is an infinite dimensional state variable, the problem becomes intractable. A potential solution is to approximate the evolution of this distribution with its important moments. Consistency of rational expectations would then require that agents' expectations about the future evolution of the weatlh distribution moments materialize in reality. In a non-linear model, however, there is no guarantee that the predicted evolution of the wealth distribution will match the actual wealth distribution, and there is no obvious choice of which moments should be followed. Surprisingly, Krussell and Smith find that (in their first order Markov equilibrium) the mean of the wealth distribution and the aggregate productivity shock are sufficient statistics for the evolution of the wealth distribution, in the sense that the actual values of the wealth distribution are very close to the predictions agents use to solve their individual problems. Storeslettern, Telmer and Yaron (1998) analyze the implications of the Krussell-Smith economy for the equity premium using similar techniques and find that their model can explain part of the equity premium.

6 Concluding Remarks

This paper has presented a set of computational techniques that can be used to solve models of household portfolio choice, the main features of solutions, and key puzzles when solutions are confronted with data in empirical studies. Household portfolios is an exciting area of research with many still unanswered questions. Most existing models have explored implications of our standard choice models for portfolio choices between risky and riskless financial assets. Although this literature has contributed to our understanding of key factors influencing portfolio choice, it is fair to say that we do not yet have models that can account simultaneously for the limited incidence of stockholding in the population, the pattern of observed stockholding among different age, education, and other relevant demographic groups, and for the positive (concave?) relationship between risky portfolio shares and cash on hand typically observed in country data. This is an important agenda for future portfolio research.

A second fruitful avenue for future research is to study the portfolio interaction between financial and real assets such as private businesses and housing. As Carroll's (2000) paper on the rich illustrates, private businesses are a key portfolio component for wealthy households, and an important determinant of their labor income. Housing is important for all households (though less so for the rich) both as a portfolio component and as a source of housing services. The dual role of real assets, their usefulness as collateral for loans, and their lumpiness that often necessitates accumulation of downpayments or startup funds make them potentially fascinating objects of future analysis.

A third area for potential research lies in the analysis of tax effects on portfolios (see Poterba, 2000). One needs to explore the portfolio effects of the tax treatment of labor income, interest and dividend income, capital gains, bequests and inheritances. Particularly exciting from a computational perspective is the analysis of capital gains taxation, in view of the requirement that capital gains be taxed at realization and of special tax provisions for capital gains bequeathed to descendants, such as "step up of basis" clauses. A fourth dimension deals with privatization of pension systems, new types of retirement accounts and their tax treatment, as well as with the implications of such schemes for the rest of a household's portfolio.

From a methodological perspective, a challenge for future portfolio research is to move from partial-equilibrium to general-equilibrium models in an effort to account simultaneously for portfolio puzzles and for asset return puzzles, such as the equity premium. This move will be much smoother if it takes place after we gain a clear understanding of optimal portfolios involving each of the various asset types discussed above. When asset returns are also successfuly endogenized, the stage will have been set for computer simulations of artificial economies with optimizing agents in which portfolio behavior and the wealth distribution play a key role. Perhaps this is as close as we are likely to get to controlled experiments in Economics.

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Table 1: First-Period Policy Functions for Asset Holdings Under Alternative Preference Specifications (Three-period Model, Risk Aversion = 3)

Normalized Cash on Hand	Expected Utility			Kreps-Porteus Preferences (σ = 0.5)			Quiggin Preferences (σ = 0.5, γ = 0.5)		
	Stock to income ratio	Bond to income ratio	Stocks as a share of net worth	Stock to income ratio	Bond to income ratio	Stocks as a share of net worth	Stock to income ratio	Bond to income ratio	Stocks as a share of net worth
0.19	0.12	-1.01	-0.13	0.11	-1.05	-0.12	0.05	-0.93	-0.05
0.39	0.13	-0.94	-0.16	0.12	-0.98	-0.14	0.05	-0.85	-0.07
0.58	0.15	-0.87	-0.20	0.14	-0.92	-0.18	0.06	-0.78	-0.08
0.78	0.16	-0.80	-0.25	0.15	-0.85	-0.21	0.07	-0.70	-0.10
0.97	0.17	-0.73	-0.31	0.16	-0.79	-0.26	0.07	-0.63	-0.13
1.17	0.19	-0.66	-0.39	0.18	-0.72	-0.32	0.08	-0.55	-0.17
1.36	0.20	-0.59	-0.51	0.19	-0.66	-0.40	0.09	-0.48	-0.22
1.56	0.21	-0.52	-0.70	0.20	-0.59	-0.51	0.09	-0.40	-0.29
1.75	0.23	-0.45	-1.02	0.21	-0.52	-0.68	0.10	-0.33	-0.43
1.95	0.24	-0.38	-1.75	0.23	-0.46	-0.97	0.10	-0.25	-0.71
2.14	0.25	-0.31	-4.86	0.24	-0.39	-1.56	0.11	-0.18	-1.71
2.34	0.27	-0.23	8.10	0.25	-0.32	-3.41	0.12	-0.10	6.62
2.53	0.28	-0.16	2.37	0.26	-0.26	43.08	0.12	-0.02	1.23
2.73	0.29	-0.09	1.44	0.28	-0.19	3.21	0.13	0.05	0.71
2.92	0.31	-0.02	1.06	0.29	-0.12	1.74	0.14	0.13	0.51
3.12	0.32	0.05	0.85	0.30	-0.05	1.22	0.14	0.21	0.39
3.31	0.33	0.13	0.72	0.31	0.01	0.96	0.14	0.29	0.32
3.51	0.35	0.20	0.63	0.32	0.08	0.80	0.14	0.37	0.27
3.70	0.36	0.27	0.57	0.34	0.15	0.69	0.14	0.46	0.23
3.90	0.37	0.35	0.52	0.35	0.22	0.62	0.14	0.54	0.20

Notes: Initial cash on hand consists of labor income and initial net financial assets. Cash on hand and asset holdings are normalized by current labor income. Solutions are shown for equal increments of normalized cash on hand and for relative risk aversion equal to 3. For non-expected utility specifications, elasticity of intertemporal substitution is set at 0.5. For the Quiggin specification of rank-dependent utility, the gamma parameter is set at 0.5 (see text).

Table 2: First-period Portfolio Shares of Risky Assets
For Various Levels of Initial Normalized Cash on Hand
(Three-period Expected-Utility Model, Risk Aversion = 3)

Normalized Cash on Hand	*Share of risky assets*	*Normalized Cash on Hand*	*Share of risky assets*
0.39	-0.16	5.07	0.37
0.78	-0.25	5.46	0.34
1.17	-0.39	5.85	0.32
1.56	-0.70	6.24	0.30
1.95	-1.75	6.63	0.29
2.34	8.10	7.02	0.28
2.73	1.44	7.41	0.27
3.12	0.85	7.80	0.26
3.51	0.63	8.19	0.25
3.90	0.52	8.58	0.25
4.29	0.45	8.97	0.24
4.68	0.40	9.36	0.23

Notes: Initial cash on hand consists of labor income and initial net financial assets. Cash on hand is normalized by first-period labor income, and it is shown at equal increments. The share of risky assets in the model is the share of stocks in financial net worth.

Table 3: First-Period Precautionary Effects on Normalized Wealth, Stocks, and Bonds Under Alternative Preference Specifications (Three-period Model, Risk Aversion = 3)

Normalized Cash on Hand	Expected Utility			Kreps-Porteus Preferences ($\sigma = 0.5$)			Quiggin Preferences ($\sigma = 0.5, \gamma = 0.5$)		
	Effects on wealth to income ratio	Effects on stock to income ratio	Effects on bond to income ratio	Effects on wealth to income ratio	Effects on stock to income ratio	Effects on bond to income ratio	Effects on wealth to income ratio	Effects on stock to income ratio	Effects on bond to income ratio
0.19	0.072	-0.019	0.091	0.086	-0.018	0.104	0.117	-0.002	0.118
0.39	0.068	-0.018	0.086	0.081	-0.017	0.098	0.114	0.000	0.115
0.58	0.064	-0.018	0.081	0.077	-0.016	0.093	0.112	0.001	0.111
0.78	0.060	-0.017	0.077	0.073	-0.016	0.089	0.109	0.002	0.107
0.97	0.057	-0.016	0.073	0.069	-0.015	0.084	0.107	0.004	0.103
1.17	0.054	-0.015	0.069	0.066	-0.015	0.080	0.105	0.005	0.100
1.36	0.051	-0.015	0.066	0.063	-0.014	0.077	0.103	0.007	0.096
1.56	0.049	-0.014	0.063	0.060	-0.014	0.073	0.102	0.009	0.093
1.75	0.047	-0.014	0.060	0.057	-0.013	0.070	0.100	0.010	0.090
1.95	0.045	-0.013	0.058	0.055	-0.013	0.067	0.098	0.012	0.087
2.14	0.043	-0.013	0.055	0.053	-0.012	0.065	0.097	0.014	0.083
2.34	0.041	-0.012	0.053	0.050	-0.012	0.062	0.096	0.015	0.080
2.53	0.039	-0.012	0.051	0.049	-0.011	0.060	0.094	0.017	0.077
2.73	0.038	-0.011	0.049	0.047	-0.011	0.058	0.093	0.019	0.074
2.92	0.036	-0.011	0.047	0.045	-0.011	0.056	0.092	0.020	0.072
3.12	0.035	-0.011	0.046	0.043	-0.010	0.054	0.090	0.016	0.074
3.31	0.034	-0.010	0.044	0.042	-0.010	0.052	0.088	0.011	0.077
3.51	0.033	-0.010	0.043	0.041	-0.010	0.050	0.087	0.006	0.080
3.70	0.032	-0.010	0.041	0.039	-0.009	0.049	0.085	0.002	0.083
3.90	0.031	-0.009	0.040	0.038	-0.009	0.047	0.084	-0.003	0.087

Notes: Initial cash on hand consists of labor income and initial net financial assets. Cash on hand and precautionary effects on asset holdings are normalized by current labor income. Solutions are shown for equal increments of normalized cash on hand and for relative risk aversion equal to 3. For non-expected utility specifications, elasticity of intertemporal substitution is set at 0.5. For the Quiggin specification of rank-dependent utility, the gamma parameter is set at 0.5 (see text).

Figure 1: First-Period Policy Functions for Consumption, Stock- and Bond Holding
as a Function of Cash on Hand (Expected Utility Specification)
Risk Aversion = 3

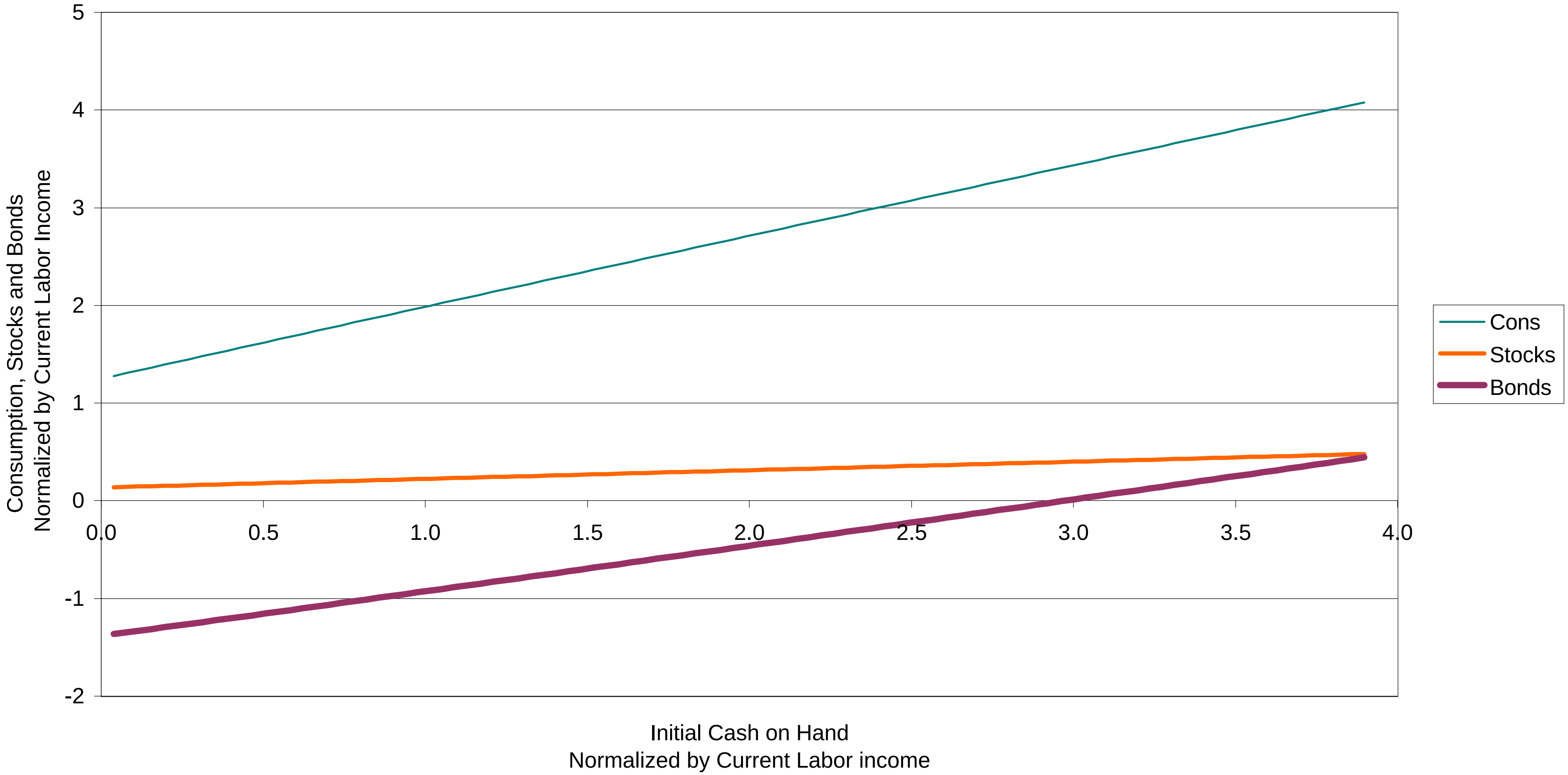


Figure 2: First-period Share of Net Wealth in Risky Assets
as a Function of Initial Cash on Hand
Risk Aversion = 3

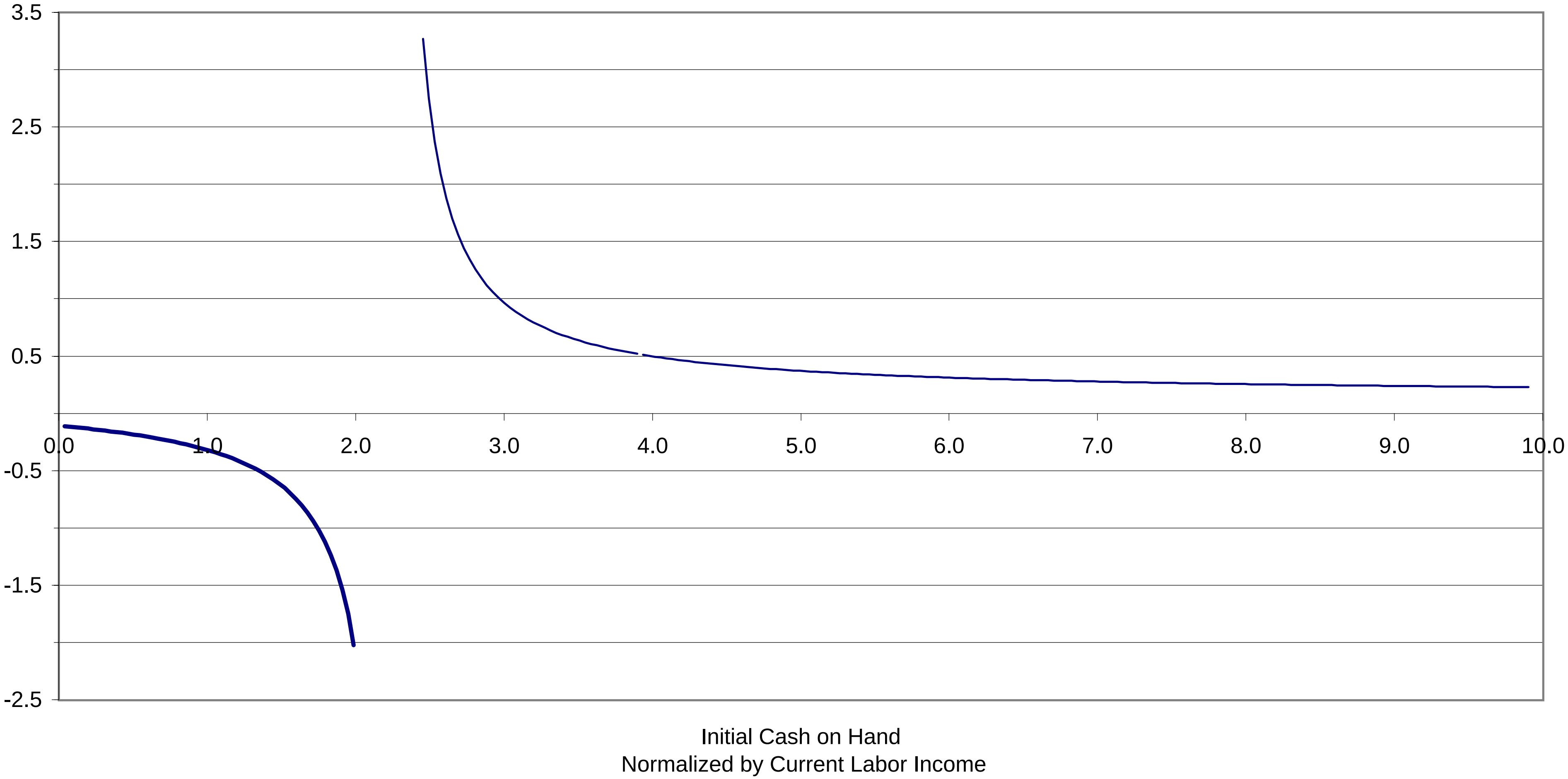


Figure 3: Second-Period Policy Functions for Consumption, Stockholding, and Bondholding
(Expected Utility Specification)
Risk Aversion = 3

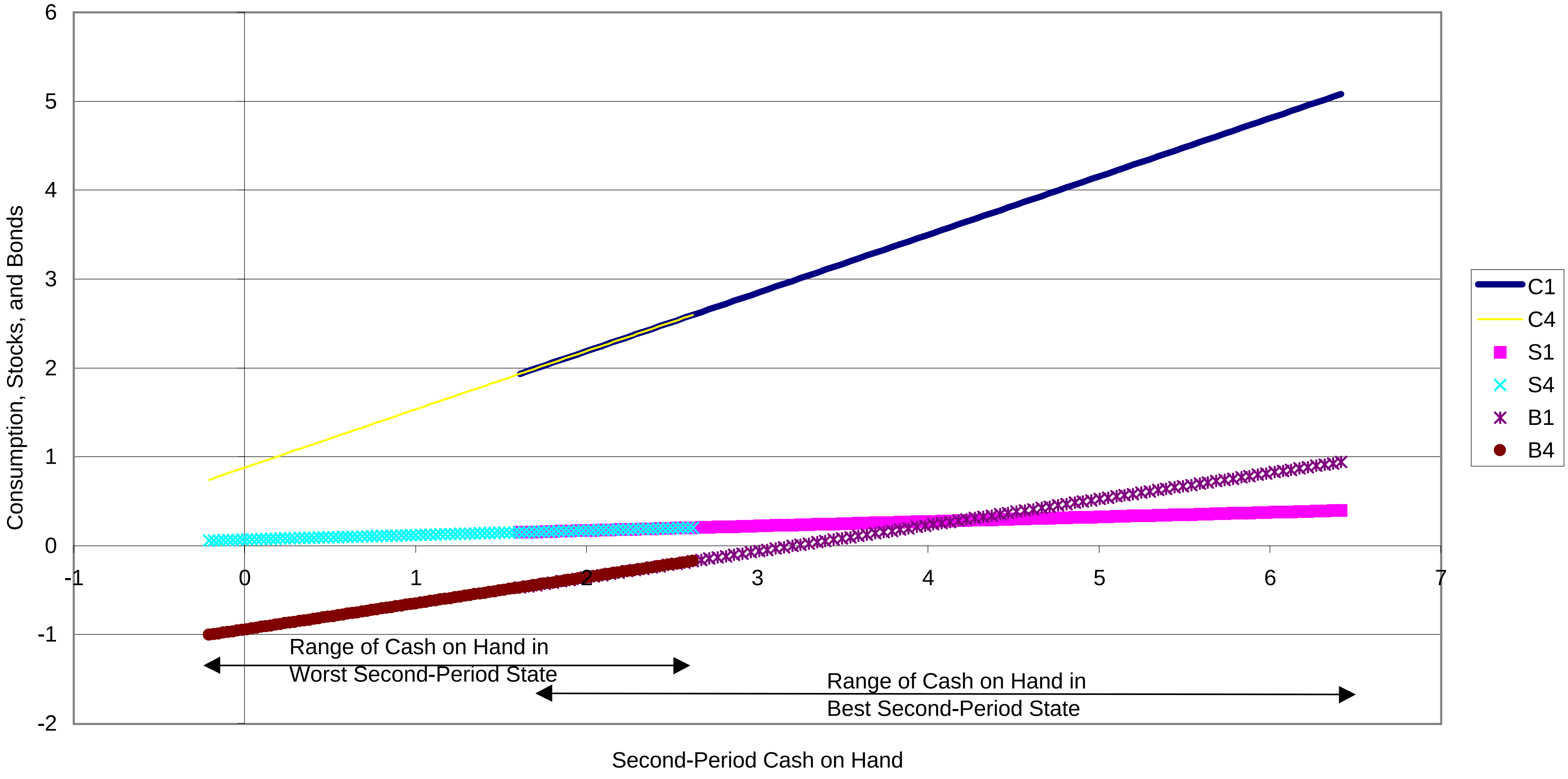
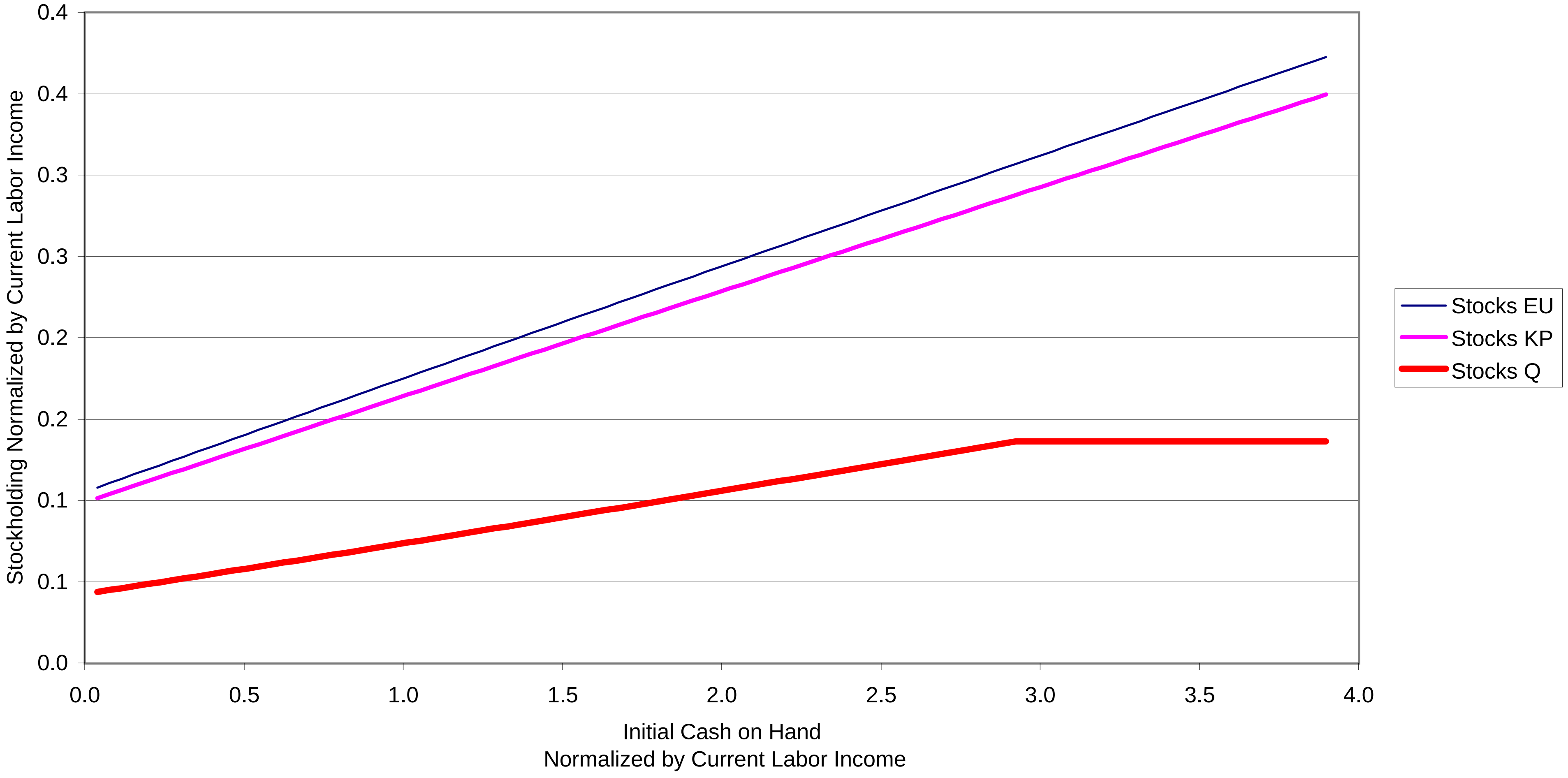


Figure 4: Policy Functions for Stockholding Under Alternative Preference Specifications:
Expected Utility (EU), Kreps-Porteus (KP), and Quiggin (Q) Preferences
Risk Aversion = 3



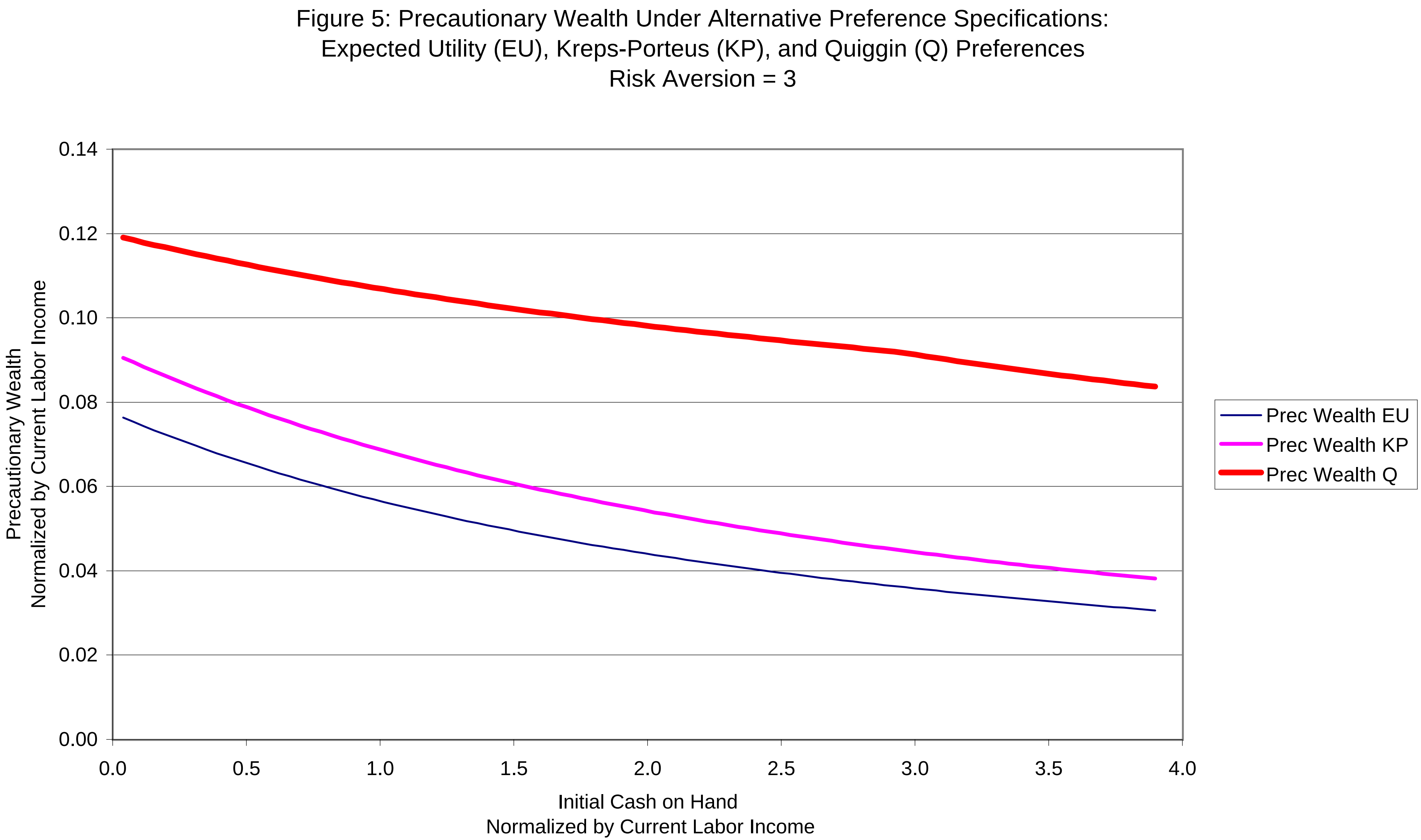


Figure 5: Precautionary Wealth Under Alternative Preference Specifications:
Expected Utility (EU), Kreps-Porteus (KP), and Quiggin (Q) Preferences
Risk Aversion = 3
Precautionary Wealth
Normalized by Current Labor Income
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Initial Cash on Hand
Normalized by Current Labor Income
Prec Wealth EU
Prec Wealth KP
Prec Wealth Q

Figure 6: Precautionary Effects on Stockholding Under Alternative Preference Specifications:
Expected Utility (EU), Kreps-Porteus (KP), and Quiggin (Q) Preferences
Risk Aversion = 3

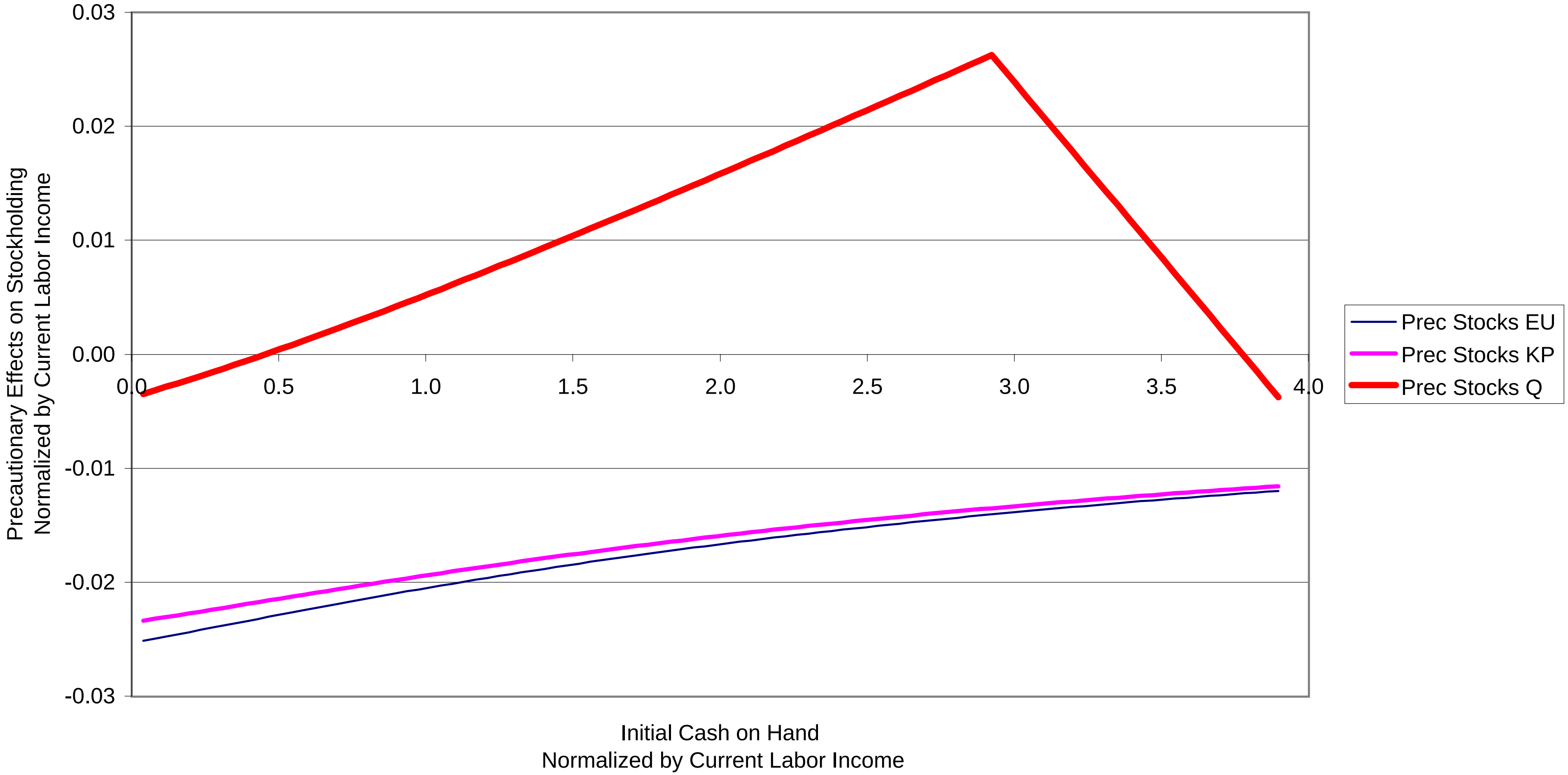


Fig.7 : Normalized Consumption (varying rho)

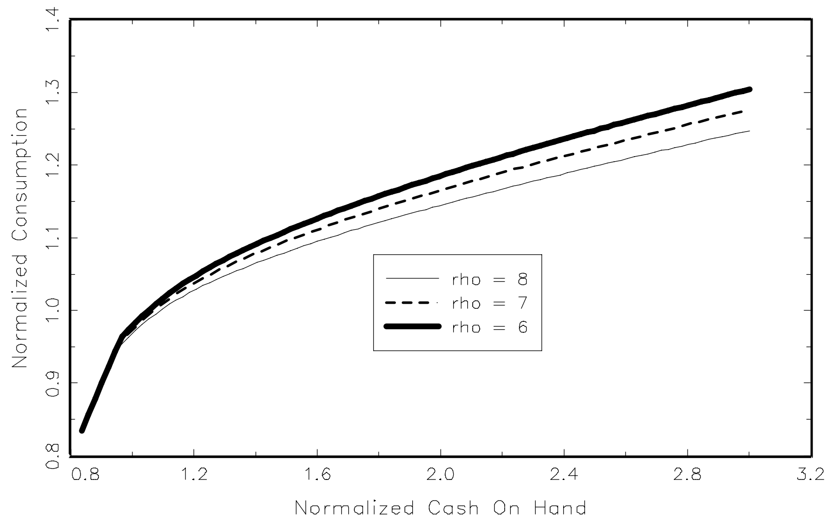


Fig.8 : Share of Wealth in Stocks (varying rho)

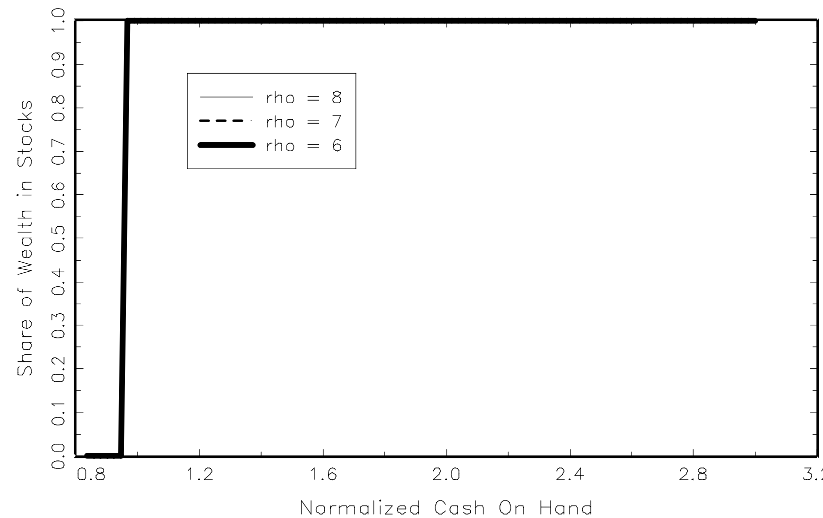


Fig.9 : Normalized Stock Holdings (varying rho)

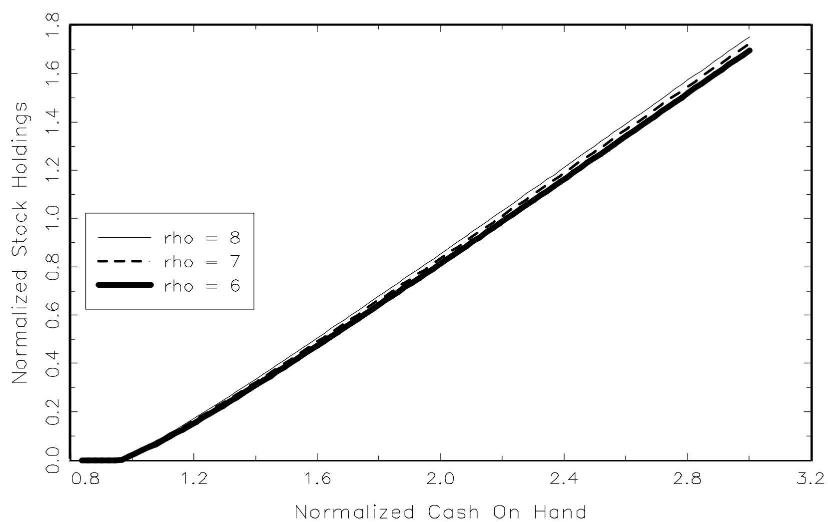


Fig.10 : Normalized Bond Holdings (varying rho)

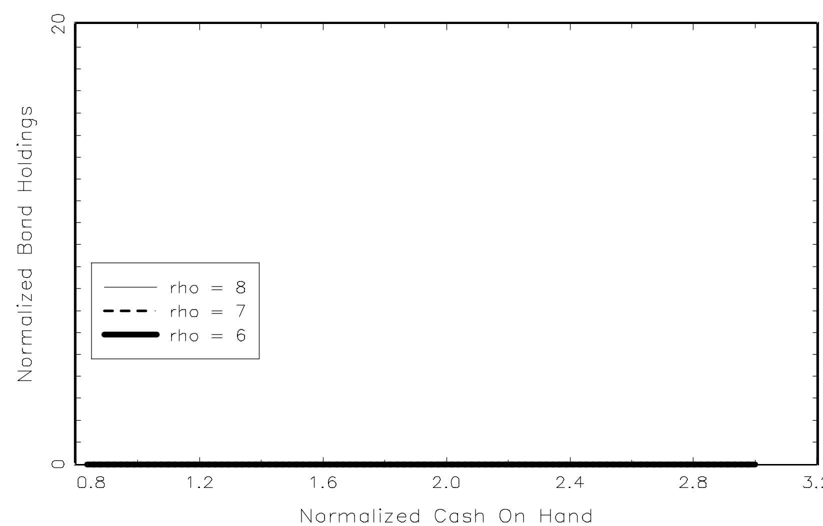


Fig.11 : Normalized Consumption

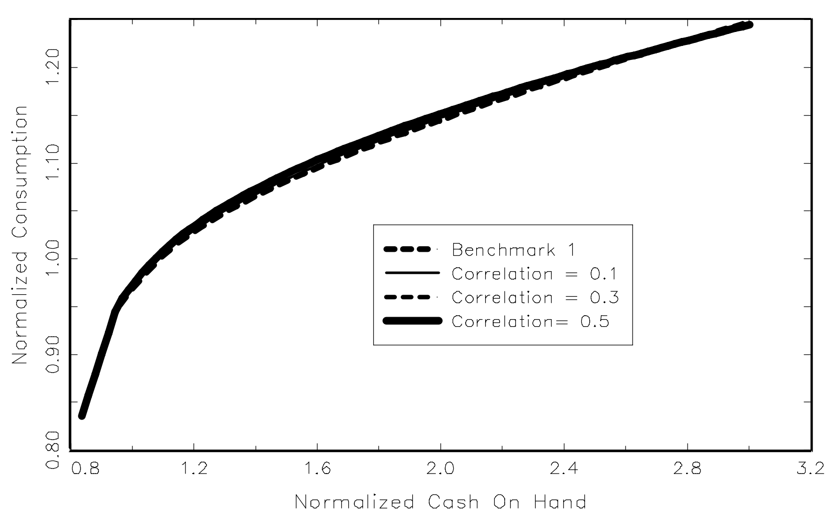


Fig.12 : Share of Wealth in Stocks

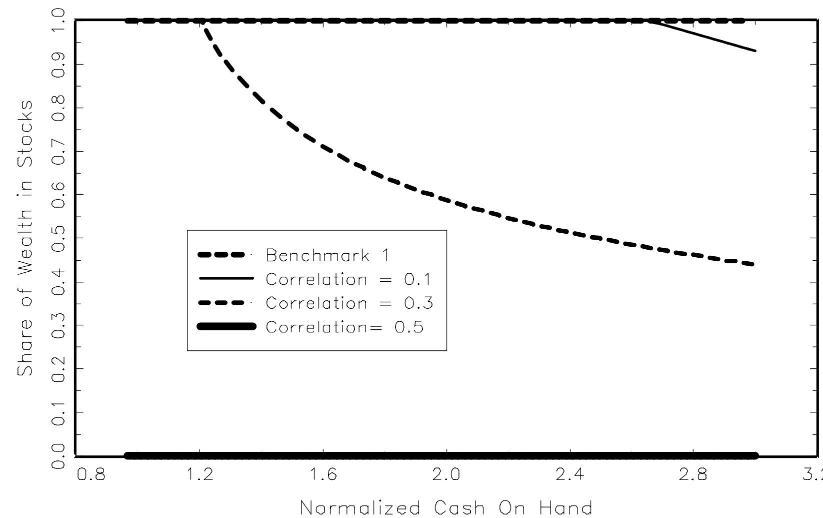


Fig.13 : Normalized Stock Holdings

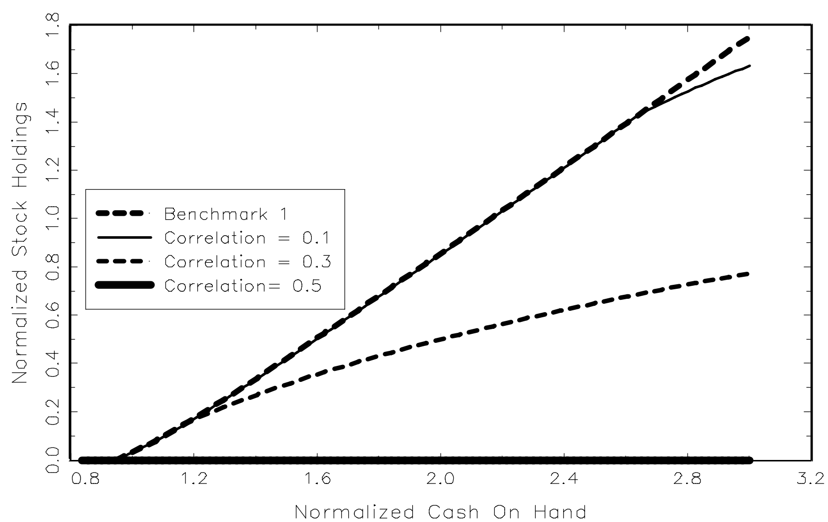
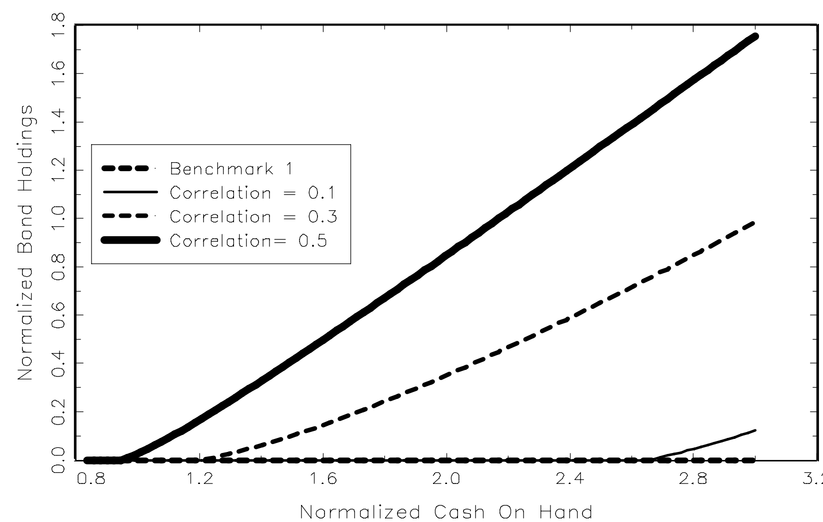


Fig.14 : Normalized Bond Holdings



Market Timing vs No Market Timing, rho=3, $\rho_{\varepsilon,z}=0$ and $\rho_{n,z}=0$

Fig.15 : Normalized Consumption

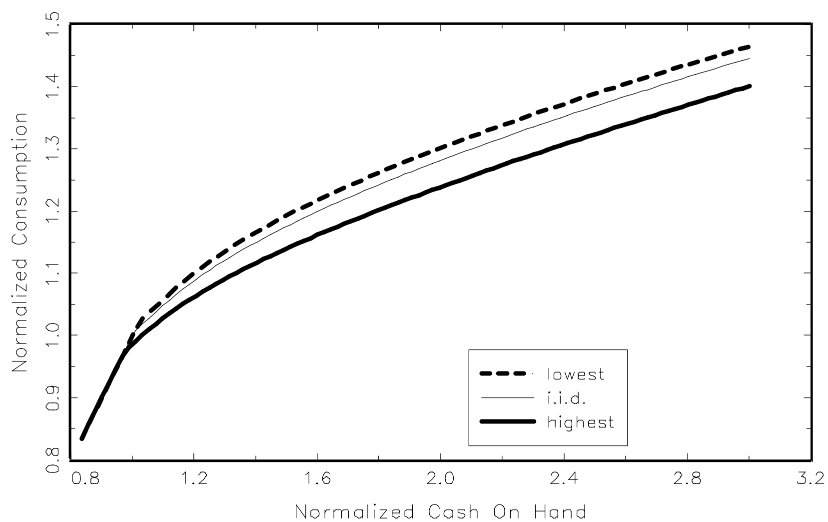


Fig.16 : Share of Wealth in Stocks

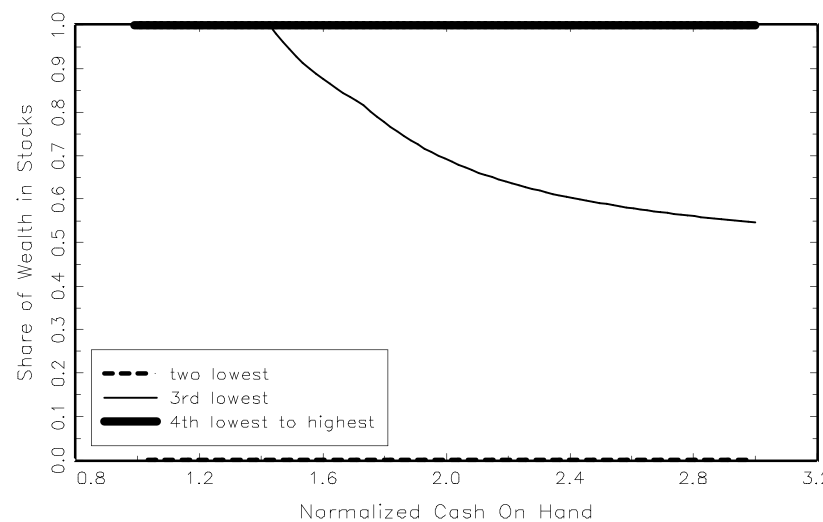


Fig.17 : Normalized Stock Holdings

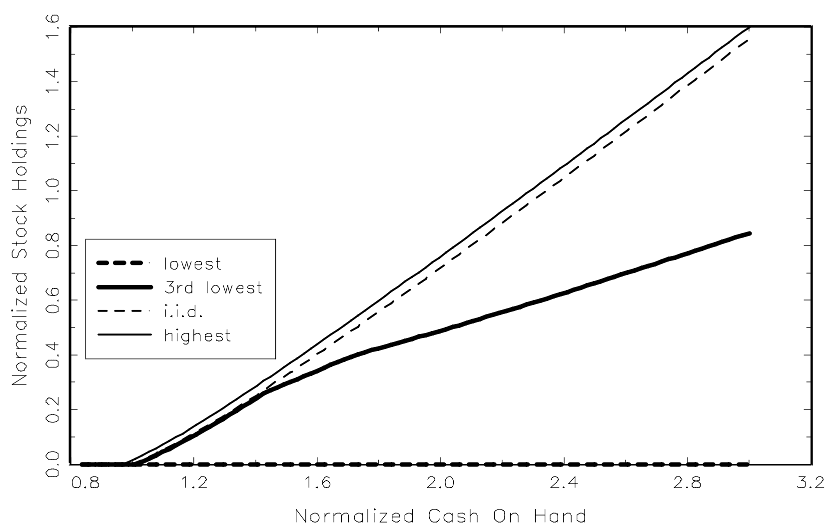


Fig.18 : Normalized Bond Holdings

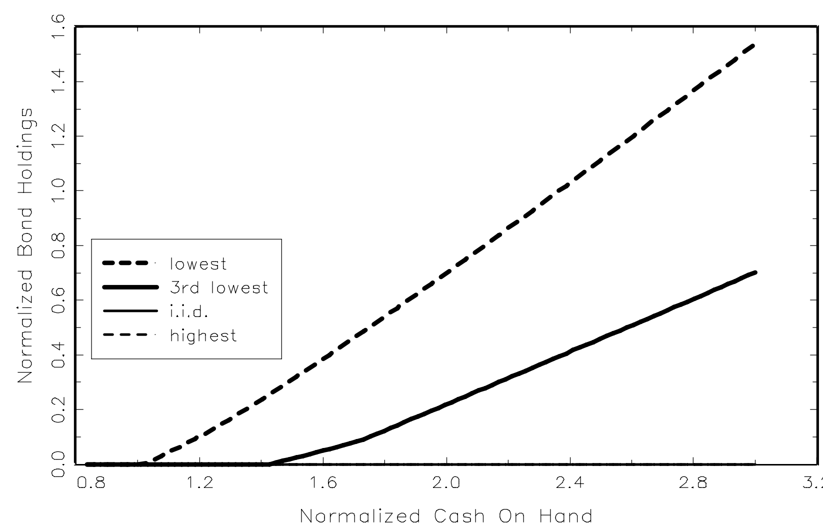


Fig.19 : Normalized Consumption (Retirement)

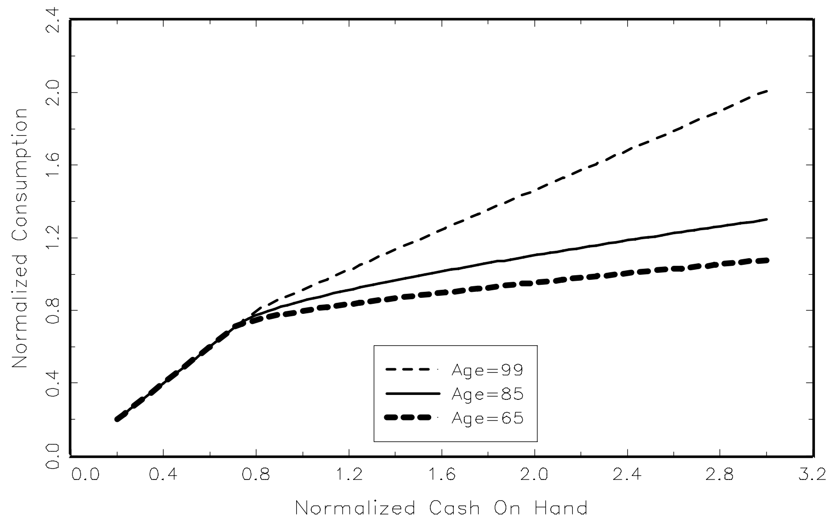


Fig.20 : Share of Wealth in Stocks (Retirement)

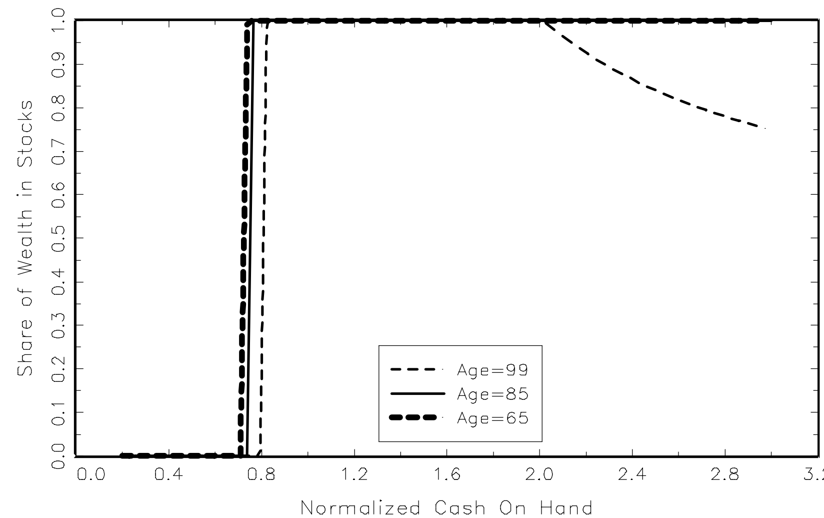


Fig.21 : Normalized Consumption (Working Life)

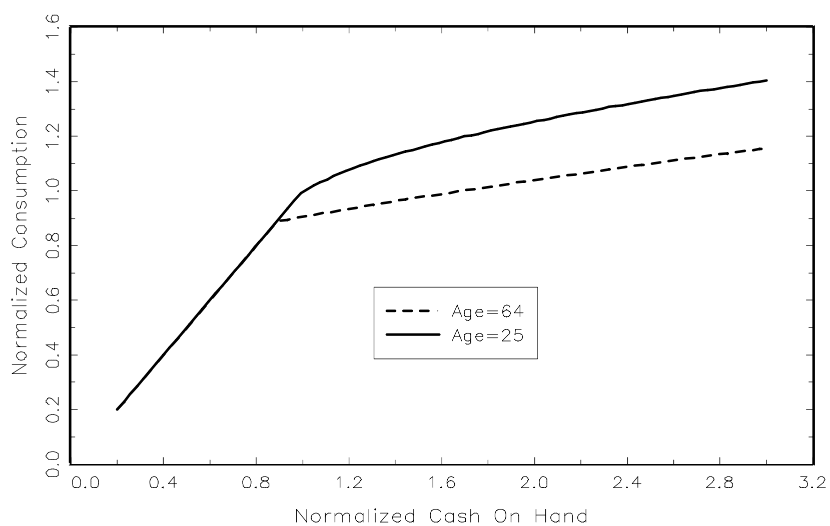


Fig.22 : Share of Wealth in Stocks (Working Life)

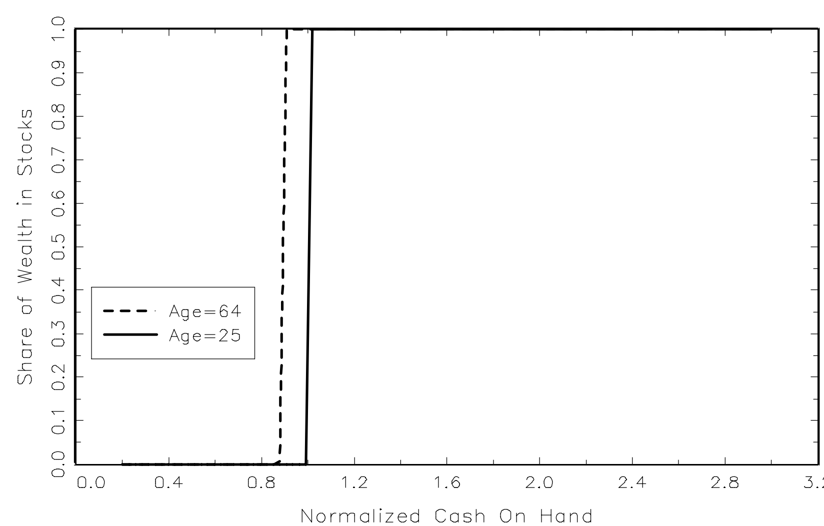


Fig.23 : Stocks, Bonds and Consumption: g=.03, delta=.1, rho=3

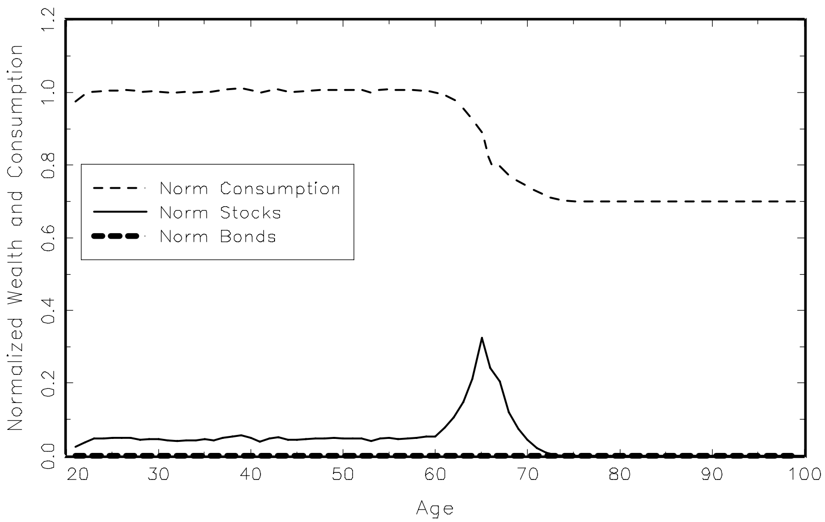


Fig.24 : Stocks, Bonds and Consumption: g=.03, delta=.1, rho=5

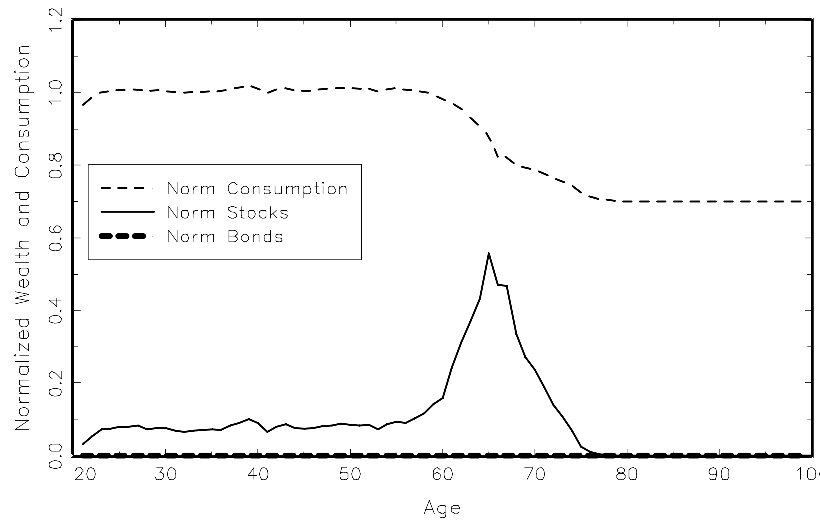


Fig.25 : Stocks, Bonds and Consumption: g=.01, delta=.1, rho=3

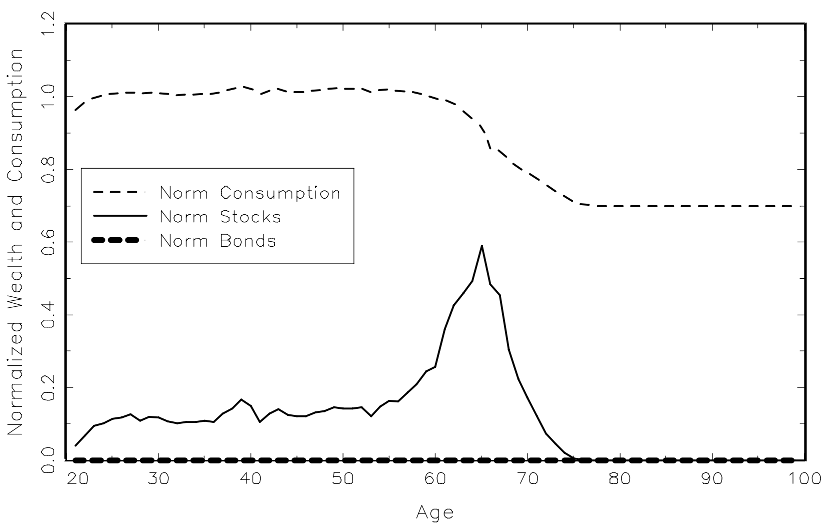
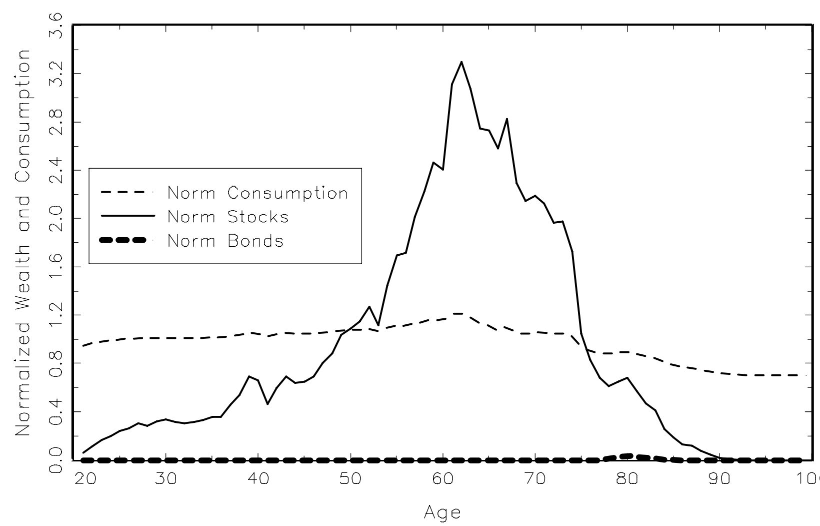


Fig.26 : Stocks, Bonds and Consumption: g=.01, delta=.1, rho=5



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