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On Public-Goods Provision in the Neoclassical Growth Model: A Special Case

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Abstract

An analytical characterization for voting outcomes over the provision of public goods in the neoclassical growth model is provided. Public goods enter the utility function of households and they are financed by proportional income taxes. Households differ with respect to their initial asset holdings. It is shown that three knife-edge assumptions on the neoclassical-growth model, namely log-preferences over private and public consumption, capital depreciation equal to one, and non-tax-exempt depreciation, make the time-inconsistency problem disappear. A common closed-form solution is found for three different dynamic voting patterns: (a) voting once and for all; (b) sequential voting with perfect memory; and (c) sequential voting without remembering previous economic/political states and actions.

Key Words: public goods, dynamic Stackelberg games, information pattern, time-consistency, tax distortions, growth.

JEL classification: C73, D72, E61, E62, O23

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1. Introduction

The architecture of optimal control in problems of household optimization relies heavily on assuming an objective function with recursive household preferences, subject to constraints that are up to first-order difference equations. These modeling ingredients are crucial for generating time-invariant and time-consistent decision rules for far-sighted players with nature.

Kydland and Prescott (1977) stressed that economic policy making is not a game with nature, but a dynamic (or differential) game between policy makers and rational economic controllers in perfect-foresight models, like the neoclassical model is. They considered optimal-controller households with recursive preferences over consumption who are confronted with an exogenously given stream of future policies. They observed that, typically, *the whole future policy sequence enters the consumption decision rules of households in each period*, like a set of exogenously given parameters.

A policy maker who maximizes the lifetime utility of a single or more households, introduces the consumption decision rules of households again in the households' objective. Given the aforementioned nature of the household decision rules, at each point in time, the whole stream of future policies is contained in the households' momentary utility function in the eyes of the policy maker. In consequence, *the policy maker's preferences over policy instruments are typically not recursive* anymore. A key ingredient of optimal-control modeling therefore disappears.

At the same time, households and policy makers have a different impact on the aggregate economy: a single household's economic decisions are of negligible impact on prices, whereas policies have substantial effects on the whole macroeconomy. This discrepancy makes households and policy makers to consider different constraints and feasible sets to their problems, erasing any possibility for resolving the complexities arising from the fact that the policy maker's preferences over policies are not recursive. This additional reason makes the policy maker's problem not representable by a Bellman equation, failing the architecture of optimal control.[1] The problem of time inconsistency therefore arises: policy makers who decide today envisioning their future decisions according to their currently calculated future states, tend to make different future decisions when they reach each corresponding future time period. The latter occurs even if the future states are the same as the ones they calculated before.

This study focuses on a special case of the neoclassical growth model with endogenous public-goods provision, where the time-inconsistency problem disappears. The key technical aspect of this special case is that conditions on the structure of the neoclassical growth model are identified guaranteeing that the households' optimal decision rules depend *only on current and no future policies* when households are confronted with an exogenous policy stream. This feature is sufficient for making a policy maker's preferences over policies recursive. The latter fact facilitates finding simple closed-form solutions, enabling a close analytical examination of the policy-choice dependence on economic taste and technology parameters.

The model is a neoclassical economy where public goods enter the utility function of households and they are financed by proportional income taxes. The government budget is balanced in each period. Households differ with respect to their initial asset holdings. Three knife-edge assumptions are made: (i) preferences over private and public consumption are log; (ii) capital depreciation is equal to one; and (iii) capital depreciation is non-tax-exempt.

A common closed-form solution is found for three different dynamic voting patterns: (a) voting once and for all; (b) sequential voting with perfect memory; and (c) sequential voting

[1] The importance of this reason for the inability of optimal control to treat the problem of a policy maker is also stressed and explained in Chari et. al. (1989).

without remembering previous economic/political states and actions. The equilibrium tax rate depends positively on the weight on the momentary utility for public goods and on the rate of time preference. It also depends negatively on the share of capital in production and the share of consumption in utility (in relation to leisure).

The fact that in this version of the neoclassical model households do not respond to future policies, might be considered as a counter-intuitive or implausible feature, especially for conducting applied research. When any of the three aforementioned knife-edge assumptions is relaxed, households tend to respond to an announced changing future policy by "smoothing" their reaction to it. Quantitatively, however, the most intense reaction to a pre-announced policy shock typically takes place at the period of occurrence of the shock, giving a high quantitative weight to the dependence of economic decisions on contemporaneous policy changes. In other words, even in an applied, carefully calibrated version of the neoclassical model, the dependence of economic decision rules and the dependence of endogenous policies on state and technology parameters might not be far from what is uncovered by the version of the model used in this paper. At least the intention of this study is to provide a stimulus for further quantitative investigation in more complex numerically simulated models and possibly for further econometric study of the links between economic features and government size.

The plan of the paper is as follows. In section 2, the main economic framework is analyzed and households' decisions are uncovered. In section 3, the political framework is explained and the main result is presented and proved. In section 4 the importance of the findings is discussed, and in section 5 general conclusions are made.

2. Economic Framework and Competitive Equilibrium

A set $\mathcal{I}$ of individuals who differ only with respect to their initial endowment of physical-capital claims is considered. Assets are denoted as a_0^i for person $i \in \mathcal{I}$ at time 0.[2] At any point in time, the aggregate (average) capital level is given by:

$$\mathbf{k}_t = \int_{\mathcal{I}} a_t^i \mu_t (i)\, di \ , \quad t = 0, 1, ... \tag{1}$$

where $\mu_t (i)$ is the measure of individuals of type i at time t. All aggregate variables will be denoted by bold characters in order to be distinguished from ones pertaining to individual agents. It is also assumed that $\mathbf{k}_0 > 0$. All individuals are infinitely-lived. There is a single private consumable good produced by a Cobb-Douglas aggregate production function:

$$\mathbf{y}_t = A \mathbf{k}_t^{\alpha} \mathbf{l}_t^{1-\alpha} \ , \tag{2}$$

with $A > 0$ and $\alpha \in (0, 1]$.[3] The marginal returns to capital and wages are:

$$R_t = \alpha A \left(\frac{\mathbf{k}_t}{\mathbf{l}_t} \right)^{\alpha - 1} \quad \text{and} \quad w_t = (1 - \alpha) A \left(\frac{\mathbf{k}_t}{\mathbf{l}_t} \right)^{\alpha} \ . \tag{3}$$

The one-period depreciation rate of capital, δ, is assumed to be 100%. Therefore, one can think that a period in the model is a sufficiently long interval of time, to allow for such a value for δ.[4]

There is a government which is constitutionally constrained to collecting taxes by setting a common marginal tax on personal income every period, denoted by τ_t, and to keeping a

[2] The set of individuals can be either a large finite number or a continuum. The support of the personal physical-capital claims is Borel measurable.
[3] The "AK" growth model is a special case for the analysis below. In this particular case one can think of two different formulations: (i) there is no labor income and output comes solely from capital investments; and (ii) there is accumulable human capital. Both a re simple extensions of the model and are not undertaken here. For the general case of the "AK" production, long-run growth becomes very sensitive to policy differences, as it was pointed out by Rebelo (1991). Therefore, the framework of Rebelo (1991) can be extended to a politicoeconomic setup with majority voting following the analysis below.
[4] The assumption that $\delta = 1$ is also made, for example, in Stokey and Lucas (1989; p.12), again for the purpose of obtaining a closed-form solution. However, as it will be noted below, for the case where $\alpha = 1$, this assumption can be relaxed, i.e. $\delta \in [0, 1]$ will be allowed.

balanced fiscal budget every period. Therefore, government revenues (and expenditures) are given by:

$$\mathbf{g}_t = \tau_t A \mathbf{k}_t^{\alpha} \mathbf{l}_t^{1-\alpha}. \tag{4}$$

It should be noted that depreciated capital is not exempted from taxation.[5] Tax rates are restricted to taking values within the following interval:

$$\tau_t \in [0, 1) \ , \tag{5}$$

i.e. taxing with a 100% rate is not allowed.[6]

It is assumed that all public revenues become parks, hospitals, theaters, art schools, defense, etc., all comprising a single composite public good. Nothing is returned back to the individuals in the form of a direct monetary transfer. The following momentary utility function for any household i applies:

$$u(c_t, l_t, \mathbf{g}_t) = \ln\left[c_t^{\theta}(1-l_t)^{1-\theta}\right] + \chi \ln(\mathbf{g}_t) \ , \tag{6}$$

with $\chi > 0$ and $\theta \in (0, 1]$.[7] The assumption of additive separability and homotheticity of preferences with respect to private and public consumption is not extreme. Amano and Wirjanto (1998) estimate a utility function of the average American household from aggregate data with constant intertemporal and intratemporal elasticity of substitution. Their regressions are based on Euler equations that stem from a permanent-income model with lump-sum taxes.[8] They find that private and public consumption are unrelated in the

[5] Without this assumption tax revenues would be: $\mathbf{g}_t = \tau_t \left(A\mathbf{k}_t^{\alpha}\mathbf{l}_t^{1-\alpha} - \delta \mathbf{k}_t\right)$.

[6] Since the depreciation rate is 100% and there is proportional income taxation, a tax of 100% would mean to confiscate the whole wealth of the economy and make it public consumable goods, so the production possibility set becomes empty. Therefore, it is assumed that the constitution does not allow for this possibility, in order to preserve the infinite-horizon setup and focus on the recursive solution of the competitive economy.

[7] Superscripts i will be dropped throughout the text unless necessary.

[8] Even though the permanent-income Euler equations of Amano and Wirjanto (1998) differ from the politico-economic equilibrium conditions of this setup, their findings should uncover similar estimates for the intra- and inter-temporal elasticities of substitution with these implied by a model distorted by marginal taxes.

Edgeworth-Pareto sense (they are neither substitutes nor complements). They also find that the elasticities of intertemporal and intratemporal substitution are both about 1.56. The latter means that the assumption of natural-log preferences is not an extreme deviation from their findings.

It is assumed for the moment that a stream of taxes and aggregate capital and labor $\{(\tau_t, \mathbf{k}_t, \mathbf{l}_t)\}_{t=0}^{\infty}$ is pre-determined exogenously. Chatterjee (1994) rigorously proves that for a general class of neoclassical-growth setups, within which the model of this study falls, households need to know *only* the future stream of aggregate capital levels and not the future distributions of physical-capital claims in order to accurately calculate their optimal path of savings.[9] Therefore, the problem of an individual household i, is adequately described by the following constrained problem:

Household Problem

$$\max_{\{(c_t, l_t, a_{t+1})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left[c_t^{\theta} \left(1 - l_t\right)^{1-\theta} \right] + \chi \ln \left(\mathbf{g}_t \right) \right\}$$

subject to:

$$a_{t+1} = \left(1 - \tau_t\right) \left(R_t a_t + w_t l_t\right) - c_t \ , \tag{7}$$

equations (2), (3) and (4),

given a_0, $\{(\tau_t, \mathbf{k}_t, \mathbf{l}_t)\}_{t=0}^{\infty}$,

with $\{(\mathbf{k}_t, \mathbf{l}_t)\}_{t=0}^{\infty}$ conforming to every household's solution to the same problem and market-clearing conditions.

Individuals have negligible economic weight and they assume that their personal decisions do not have any impact on aggregate variables. For this reason, the optimality conditions

[9] Krusell and Rios-Rull (1999) show it in an economy with proportional taxation and lump-sum transfers. Interestingly, Krusell and Smith (1997) find that in neoclassical models modified in ways such that Chatterjee's (1994) theoretical argument fails, at a numerical level, households calculating distribution moments that capture future individual asset distributions, can very well approximate their optimal path by relying only on the future stream of first moments, namely on the future sequence of aggregate capital.

of a household i are given by:

$$\frac{c_{t+1}}{c_t} = (1 - \tau_{t+1})\,\beta R_{t+1}\ , \tag{8}$$

$$\frac{c_t}{1 - l_t} = \frac{\theta}{1 - \theta}(1 - \tau_t)\,w_t \tag{9}$$

equation (7), and the transversality condition $\lim_{t\to\infty} \frac{a_{t+1}}{\prod_{s=0}^{t}(1-\tau_s)\beta R_s} = 0$. One can obtain a closed-form solution for this problem, with a slight restriction on the initial conditions, as it is stated by the following Proposition.

Proposition 1 *Let the household with the maximum wealth at time* 0*, be denoted as* $a_0^{\max}$*. Then if*

$$\frac{a_0^{\max}}{\mathbf{k}_0} < \frac{\theta(1-\alpha)}{(1-\theta)\,\alpha(1-\beta)} + \frac{1-\alpha\beta}{\alpha(1-\beta)}\ , \tag{10}$$

the competitive-equilibrium laws of motion for the ***Household Problem*** *of any individual household* i*, are given by:*

$$a_{t+1} = (1-\tau_t)\,\beta R_t a_t\ , \tag{11}$$

with $\frac{a_t}{\mathbf{k}_t} = \frac{a_{t+1}}{\mathbf{k}_{t+1}}$ *for all* $t \geq 0$*, and*

$$c_t = (1-\tau_t)\,\theta\left[\alpha(1-\beta)\frac{a_0}{\mathbf{k}_0} + 1 - \alpha + \frac{1-\theta}{\theta}(1-\alpha\beta)\right]\mathbf{y}_t\ , \tag{12}$$

$$l_t = l\left(\frac{a_0}{\mathbf{k}_0}\right) = \theta\left[1 - \frac{\frac{1-\theta}{\theta}\alpha(1-\beta)\frac{a_0}{\mathbf{k}_0}}{1-\alpha+\frac{1-\theta}{\theta}(1-\alpha\beta)}\right]\ \text{for all } t \geq 0. \tag{13}$$

The aggregate-economy laws of motion for capital and labor are:

$$\mathbf{k}_{t+1} = (1-\tau_t)\,\alpha\beta\mathbf{y}_t\ , \tag{14}$$

and

$$\mathbf{l}_t = \mathbf{l} = \frac{1}{1+\frac{(1-\theta)(1-\alpha\beta)}{\theta(1-\alpha)}}\ . \tag{15}$$

Proof. See the Appendix.

The constraint on the initial wealth holdings of the richest household is placed in order to guarantee that, at all times, all households supply a positive level of labor hours. In this way it is secured that no household has a corner solution, in which case the above competitive-equilibrium equations would be disfigured. Condition (10) can be easily derived from (13).

A "back-of-the-envelope" calculation shows that a reasonable number for this model would be $\frac{a_0^{\max}}{\mathbf{k}_0} < 13.4$.[10]

An important remark about the nature of the competitive equilibrium of this version of the neoclassical model should be made. In the neoclassical model with $\alpha, \delta \in (0,1)$ and an elasticity of intertemporal substitution for consumption $\frac{1}{\sigma} \neq 1$, if one considers two future policy plans at time t, $\{\tau_s\}_{s=t}^{\infty}$ and $\left(\{\tau_s\}_{s=t}^{t+n}, \widetilde{\tau}_{t+n+1}, \{\tau_s\}_{s=t+n+2}^{\infty}\right)$, with $n \geq 0$ and $\widetilde{\tau}_{t+n+1} \neq \tau_{t+n+1}$, then typically individual policies at time t would change, i.e. $\widetilde{a}_{t+1} \neq a_{t+1}$ and $\widetilde{c}_t \neq c_t$.[11] Equations (11) through (15) indicate that this will not be the case for the particular economy studied in this paper. But it should be emphasized that it is neither that agents are not forward-looking anymore, nor that one cannot study forward-looking policy issues through this version of the model. The agents see future changes in policy but they optimally decide to follow the same path as if the future policy change was not there, until they reach the period of occurrence of the change. This is because they do not gain extra utility from "consumption smoothing."

The three knife-edge assumptions do not "trim" the influence of the continuation of a tax plan, $\{\tau_s\}_{s=t+1}^{\infty}$, on the decision rules. They simply "fix" this influence into some constant parameters in the decision rules, independent of the nature of the future tax plan and the levels of the state variables. These constants contain non-trivial information about the far-sightedness of individuals with respect to policies. The structure of household preferences and technology "enfold" the effects of future taxes back to current taxes. Current taxes are multiplied by geometric sums of the rate of time preference that span the infinite horizon in

[10] If one assumes that the annual depreciation rate is 10%, then a reasonable duration for each period of this model for matching $\delta = 100\%$, is $\frac{\ln(2)}{\ln(1.10)} \simeq 7$ years, so $\beta \simeq 0.96^7 \simeq 0.75$. With $\theta = 0.30$ (Chari et. al. (1994) use $\theta = 0.25$ in a similar framework with log-preferences, but the value of 0.30 is more "attuning" for matching reasonable levels of $\frac{\mathbf{c}_t}{\mathbf{y}_t}$ and $\mathbf{l}_t$ in this model), and with $\alpha = 0.33$, $\frac{\theta(1-\alpha)}{(1-\theta)\alpha(1-\beta)} + \frac{1-\alpha\beta}{\alpha(1-\beta)} \simeq 13.4$.

[11] This feature is well-known to anyone who has studied impulse-response functions in the standard neoclassical model, as in King, Plosser and Rebelo (1988).

the decision rules. These geometric sums reflect how taxation in each period has *cumulative* distortionary effects on households' optimal plans over the infinite horizon.[12]

In fact, this model is a very good vehicle for studying the most important determinants of forward-looking voting behavior, because of its simplicity. As it will be clearer below, it eliminates complexities related to envisioning all the strategic interactions among voters for solving for politicoeconomic equilibria in Markov- and non-Markov-perfect games, where the continuation or the history of actions greatly perplexes the derivation of equilibria.

How sensitive the results of the following section are to different parameter values for δ and the elasticities of intra- and inter-temporal substitution that make economic and voting decision rules perplexingly responsive to future policy paths, is an open question for quantitative investigation. It is conjectured that the most important determinants of the behavior of forward-looking voters in neoclassical-growth frameworks are captured by this version of the model and there should not be significant qualitative differences with different parametrizations. Nevertheless, it is useful to use this particular paradigm as a stimulus for interesting quantitative questions of positive explanations of the size of government.[13]

Another remark is that labor supply is not affected by taxes, and there is a wealth effect on it, as it can be seen from (13) and (15). This is because an intra-temporal substitution effect on leisure due to current taxes on labor income is eliminated one-to-one by the exact tax distortion on the concurrent equilibrium consumption, since in this framework only the current tax affects the momentary marginal utility of current consumption. The assumption of momentary log-utility and the proportionality of taxes on momentary income due to the

[12]So, even though the current-period decision rules of a household when facing two alernative plans, say $\{\tau_t, 0, 0, ...\}$ and $\{\tau_t, 99.\overline{9}\%, 99.\overline{9}\%, ...\}$, would only depend on τ_t, and current decisions would be exactly the same in both cases, when the household is called to judge upon the continuation of the policy plan from period $t+1$ and on, the current-period decision rule, the level of τ_t, the household's rate of time preference and technology parameters are critical for *how much* the household likes or dislikes each of the two plans, $\{\tau_t, 0, 0, ...\}$ and $\{\tau_t, 99.\overline{9}\%, 99.\overline{9}\%, ...\}$, and *how strongly* it prefers the one over the other.

[13]For example, a paper that studies such quantitative questions is Krusell and Rios-Rull (1999).

assumption of $\delta = 1$, fixes inter-temporal effects of future taxes on current consumption and, consequently, on leisure. On the other hand, the wealth effect on labor supply is always present, because all agents have the same time endowment and labor productivity, so the opportunity cost of leisure is affected only by how much personal wealth can support higher demands for the two normal private goods (consumption and leisure).

Finally, if one sets $\theta = 1$, equations (11) through (15) describe the competitive-equilibrium conditions for the case of exogenous labor with labor supply normalized to one for every individual. This is because in the proof of Proposition 1 in the appendix, condition (9) is not needed for proving (14), and since the equilibrium labor supply is also constant over time in the case of $\theta \in (0, 1)$, the remaining arguments of the proof for $\theta \in (0, 1)$ are the same with the case of $\theta = 1$.

3. Political Framework and Majority-Voting Equilibrium

Since the constitution restricts governments to keeping a balanced budget, the only subject of voting each period is the current level of the income tax rate. Majority voting over taxes takes place at the beginning of every period, *before* households and producers make their economic decisions. Therefore, the electoral process is a *leader* in the dynamic game of politics and economics in this framework.

All households vote (or, using game-theoretic language, "move") simultaneously, before they move again simultaneously in the markets in order to make their economic decisions given the current period's electoral outcome. So, while each household decides independently about its best voting strategy, it takes into account the full impact of its voting action on the current electoral outcome and how this electoral outcome will affect its own current and future economic decisions, its own future voting decisions, and the current and future economic and voting decisions of others in politico-economic equilibrium over the infinite horizon.

Since in the above setup the political process leads the simultaneous making of economic decisions in each period, the dynamic game between polity and economy falls in the class of dynamic Stackelberg games of a leader and a follower. There are many variations of dynamic Stackelberg games, depending on what information is utilized by the players each time. In the game-theoretic literature, any possible information that is utilized each period is formalized by the definition of an *information pattern*.

Following definitions 5.1 and 5.2 by Basar and Olsder (1982; p.205-207), an *information pattern at time* t, η_t, is a subcollection of all possible actions of players and some observation function of the state variables. It will be assumed that actions of other players and state variables are perfectly observed ("perfect-state observation"), except from when actions are taken simultaneously, like in the case of voting in a certain period and making economic decisions, again in a certain period. The state variable in any period $t \geq 0$, is $\mathcal{A}_t \equiv \{a_t^i, \mu_t(i) | i \in \mathcal{I}\}$, i.e. the personal asset-distribution among households in period t. One can list three different cases of information patterns for all $t \geq 0$:

(i) Open-loop: $\eta_t = \{\mathcal{A}_0\}$,

(ii) Closed-loop: $\eta_t = \{\mathcal{A}_0, ..., \mathcal{A}_t\}$,

(iii) Feedback: $\eta_t = \{\mathcal{A}_t\}$.

The open-loop information pattern pertains voting "once and for all" in period 0, so it is irrelevant to sequential voting. It is, however, related to the Ramsey problem of optimal fiscal policy, if the only fiscal-policy decision maker is a benevolent social planner who aims at maximizing a utilitarian social-welfare function.[14] Therefore, it is a useful benchmark

[14]

for comparing the socially optimal open-loop policy with the voting outcomes found below.

The closed-loop information pattern implies that players are able to remember the whole history of economic and political actions of all other players and their own. Therefore, in a sequential-voting setup, the calculation of optimal strategies becomes very complex if one takes into account these histories together with the far-sightedness of households. Such an information pattern can generally give rise to trigger and punishment strategies that are elements of strategic voting. A paper examining closed-loop equilibria, for analyzing the issue of optimal policies is Chari and Kehoe (1990). They examine the actions of a benevolent social planner and the private sector in a setup where the history of actions is taken into account by both players. They characterize a "feedback Stackelberg equilibrium with a closed-loop information pattern" (see definition 3.29 in Basar and Olsder (1982; p. 131) for a formal general statement of the concept in dynamic games) in the model of Fischer (1980), and they call the equilibrium policies "sustainable plans." Even though this concept is not directly related to sequential voting, one can think of voting under a closed-loop information pattern, where every one decides remembering all the histories of actions by all players. Nevertheless, identification of sustainable plans is again a good benchmark for comparing outcomes from voting and what a social planner would do if the information pattern is closed-loop.

Finally, the feedback information pattern is a special case of the closed-loop information pattern without memory. In this case, the game of voting is always Markov perfect and all strategies are subgame perfect.[15] Such an equilibrium for voting is computed in Krusell

Some papers in the literature dealing with this problem are, for example, Kydland and Prescott (1977), Lucas and Stokey (1983), Chamley (1986), Aiyagari (1995), and Chari and Kehoe (1998).

[15]There is an extensive literature studying feedback Stackelberg equilibria with a feedback information pattern, when the objectives of each player are quadratic and the constraints on their action sets linear. Some examples are, Basar and Olsder (1982), Cohen and Michel (1988), Miller and Salmon (1985). In these frameworks, global equilibrium existence and stability results are obtained by standard linear-algebra arguments, based on the identification of non-explosive eigenvalues. On the other hand, Basar (1988) and

and Rios-Rull (1994) and (1999). Also, in Klein and Rios-Rull (1999) the issue of best time-consistent policies in a representative-agent economy is examined, where they compute the "best time-consistent policy" when there is only instantaneous pre-commitment on fiscal policies from the side of the policy maker who decides first and her actions are observable by the economic agents. The latter setup is also studied by Cohen and Michel (1988).[16]

It turns out that in the setup examined here, none of the aforementioned complexities or distinctions arise for the particular setup of this section, because under any of the three information patterns, no matter if one considers voting or policies set by planners, the game drops to the same Markov-perfect structure. This is revealed by the following proposition.

Proposition 2 *The voting outcome, independently of whether the information pattern is open-loop, closed-loop or feedback, is given by:*

$$\tau_t^* = \tau^* = \frac{\frac{\chi}{\theta+\frac{\alpha\beta(\theta+\chi)}{1-\beta}}}{1+\frac{\chi}{\theta+\frac{\alpha\beta(\theta+\chi)}{1-\beta}}} \quad \text{for all } t \geq 0 \ . \tag{16}$$

All voters unanimously agree upon this policy. This policy also coincides with the Ramsey solution by a benevolent social planner who maximizes a utilitarian social welfare function at time 0*, the "sustainable plan," and the "best time-consistent policy."*

Proof. With substitution of the decision rules (11) through (15) into an individual household's objective, a household i's value function at any time $t \geq 0$, given its information set η_t, is given by:

$$\begin{aligned} V^i\left(\eta_t, \{\tau_s\}_{s=t}^{\infty}\right) &= \xi + \frac{\ln\left[\alpha\left(1-\beta\right)\frac{a_0}{\mathbf{k}_0} + 1 - \alpha + \frac{1-\theta}{\theta}\left(1-\alpha\beta\right)\right]}{1-\beta} + \\ &\quad + \Lambda\left(\{\tau_s\}_{s=t}^{\infty}\right) + \frac{\alpha\left(\theta+\chi\right)}{1-\alpha\beta}\ln\left(\mathbf{k}_t\right) \ , \end{aligned} \tag{17}$$

where ξ is a constant and $\Lambda\left(\{\tau_s\}_{s=t}^{\infty}\right)$ satisfies:[17]

$$\Lambda\left(\{\tau_s\}_{s=t}^{\infty}\right) = \varphi\left(\tau_t\right) + \beta\Lambda\left(\{\tau_s\}_{s=t+1}^{\infty}\right) \ , \tag{18}$$

Cruz (1975) are excellent introductions to this concept.

[16] It is what the name "TC_1" equilibrium.

[17]

with

$$\varphi(\tau_t) = \left[\theta + \frac{\alpha\beta(\theta+\chi)}{1-\beta}\right] \ln(1-\tau_t) + \chi \ln(\tau_t) \ . \tag{19}$$

So, when the information pattern is open-loop, equation (17) should be considered only at time $t = 0$. When the information pattern is closed-loop, at any time $t \geq 0$, all past histories and continuations of the game are "enfolded" into equation (17). Finally, when the information pattern is feedback, past histories do not matter and only the continuation of the game is considered, however, equation (17) still describes the value function of an arbitrary voter i.

Equation (18) reveals that tax policies in a household's value function are additively separable over time. This means that in the open-loop case there are no conflicts between current and future policy choices, in other words, policy choice is time-consistent. In the closed-loop case, equation (18) means that there are no incentives to follow any trigger or punishment strategies, since again inter-temporal policy-choice conflicts do not arise in equation (17). It is therefore clear, that under any information pattern, there is a unique strategy by all voters, namely to vote setting $\varphi'(\tau_t) = 0$ at all $t \geq 0$, agreeing unanimously with all the other voters in every period upon the tax rate given by (16), which is the obvious solution to their problem. It should be noted that, as it turns out, voters do not need to

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It is:

$$\xi = \tfrac{1}{1-\beta} \Big\{ \tfrac{1}{1-\alpha\beta} \big[(\theta+\chi)(\alpha\beta \ln(\alpha\beta) + \ln(A)) + \\ + (1+\chi-\alpha(\chi+\theta+\beta(1-\theta))) \ln(1) \big] + \\ + \theta \ln(1-\theta) + (1-\theta)\ln(1-\alpha) \Big\} \ ,$$

and

$$\Lambda\left(\{\tau_s\}_{s=t}^{\infty}\right) = \sum_{s=0}^{\infty} \beta^s \left[\theta \ln(1-\tau_{t+s}) + \chi \ln(\tau_{t+s})\right] + \\ + \alpha(\theta+\chi) \sum_{s=1}^{\infty} \beta^s \sum_{j=1}^{s} \alpha^{s-j} \ln(1-\tau_{t+j-1}) \ .$$

know the whole distribution of personal assets in order to calculate their value functions and the voting outcome, but only the aggregate level of capital at any time $t \geq 0$.

Finally, when a utilitarian social planner is the single controller of fiscal policy, (16) again gives the solution under any of the three aforementioned information patterns, since linear aggregation of the personal value functions, $\int_{\mathcal{I}} V^i \left(\eta_t, \{\tau_s\}_{s=t}^{\infty}\right) \omega_i \mu_t (i)\, di$ for any set of weights $\omega_i \geq 0$ on household i's utility, leaves the condition $\varphi' (\tau_t) = 0$ at all $t \geq 0$ intact. **Q.E.D.**

An immediate extension to Proposition 2 is the characterization of the politicoeconomic equilibrium with respect to key parameters of tastes and technology, given by the following Corollary:

Corollary 1. *The equilibrium tax rate depends positively on the weight χ on the momentary utility for public goods and on the rate of time preference $\frac{1-\beta}{\beta}$. It also depends negatively on the share of capital, α, in production and the share of consumption, θ, in utility. The politicoeconomic equilibrium is characterized by balanced growth with all households' assets and consumption growing at the rate:*

$$\gamma_t = \frac{\alpha \beta A \mathbf{k}_t^{\alpha-1}}{\left(1 + \frac{\chi}{\theta + \frac{\alpha\beta(\theta+\chi)}{1-\beta}}\right)\left(1 + \frac{(1-\theta)(1-\alpha\beta)}{\theta(1-\alpha)}\right)^{1-\alpha}}. \tag{20}$$

Proof. These relationships can be directly verified from (16), and (20) can be derived by combining (16), (15) and (14). **Q.E.D.**

It should also be noted that when $\theta = 1$ both the politicoeconomic equilibrium tax rate given by (16) and the growth rate given by (20) correspond to the solution for the case of exogenous labor that is normalized to one for every household. This is because of the reasons explained in the last paragraph of the previous subsection.

4. What can(not) be learned from Proposition 2

One should be careful on how to interpret the results of the previous subsection. Equation (16) is the common solution to six different but interrelated problems for the specific version of the neoclassical model considered here.[18] How different these problems can be from each other is well surveyed in the literature, and some of the differences in the nature of their setup were summarized in the previous subsection.

The coincidence of solutions comes from the fact that only contemporaneous policies enter the decision rules and preferences are "log," so (18) holds. It was also argued that these decision rules contain non-trivial information about the far-sightedness of households. Nevertheless they are a special case, relying on the following five assumptions: (i) the depreciation rate of capital is one; (ii) the elasticity of intertemporal substitution is one; (iii) capital depreciation is not tax-exempt; (iv) the tax rates on capital and labor are restricted to be the same; and (v) the fiscal budget is restricted to be balanced.

Relaxing gradually these five assumptions would involve examining $2^5 - 1 = 31$ more cases (the case of having all five assumptions there is already examined here, so it is subtracted). Examining both the cases of voting and optimal policy for each of the three information patterns would make $6 \times 31 = 186$ cases. Little is general in the literature about studying the issues of existence, computability and the nature of existing politicoeconomic equilibria when such departures from the framework examined here are made.[19] For any of the 186

[18]In non-game-theoretic terms these problems are: (i) sequential voting with no memory; (ii) sequential voting with memory; (iii) voting once and for all in period 0; (iv) the best time-consistent policy with instantaneous pre-commitment; (v) the Chari-Kehoe (1990) problem of sustainable plans; and (vi) the Ramsey problem of optimal fiscal policy.

[19]It should be noted, however, that two seminal relevant studies in the theoretical literature of optimal taxation with an open-loop information pattern are these of Chamley (1986) and Aiyagari (1995). Chamley (1986) characterizes the open-loop equilibrium for the social planner's problem of financing an exogenous stream of government expenditures when the five assumptions made in this paper are relaxed alltogether. Aiyagari (1995) goes even further by relaxing a sixth extra assumption, this of complete capital markets. Finally, Marcet and Marimon (1999) provide tools for computing recursively a large class of such complex

cases of relaxing the above five assumptions the answer can be quite different, or, for some cases, an equilibrium may not exist.

The conjecture that can be made here is that Proposition 2 and Corollary 1 might be useful for understanding key dependencies of politico-equilibrium tax rates on parameters of tastes and technology for the cases of voting and optimal policies *mainly for the case of a feedback information pattern (sequential playing and no memory).* This finding should not be considered as a significant insight for the cases of open- and closed-loop equilibria.

The reason for making this conjecture is the following. In the feedback information pattern it is more likely that political choice and preference coincide. Therefore the dependencies of the politicoeconomic equilibrium on taste and technology parameters that are revealed by corollary 1, should generally prevail. Krusell and Rios-Rull (1999) observe such dependencies in their numerical experiments in a setup with lump-sum transfers.

In the closed-loop case, strategic voting of trigger and punishment policies can generate multiple equilibria, in many of which political preference and choice may diverge significantly.

In the open-loop case, there are very-well known corner solutions, especially at period 0 and close to a steady state that could still disfigure the result in (16) and the conclusions by Corollary 1.

Finally, it is useful to observe that unanimity comes from the fact that public and private consumption neither complements nor substitutes in the Edgeworth-Pareto sense when momentary preferences over these two goods are additively separable and "log." Unanimity would also be the case in a static world with the same preferences.

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problems, based on the philosophy suggested by Kydland and Prescott (1980). Klein and Rios-Rull (1999) apply such methods for computing the optimal open-loop fiscal policy in a stochastic neoclassical framework.

5. Conclusion

A special case of the neoclassical model with endogenous public-goods provision was studied here. Knife-edge assumptions were identified enabling a common closed-form solution for sequential voting over public goods with and without memory of previous states and actions. The solution is time-consistent and it also coincides with the case of voting once and for all.

The main virtue of this finding is that it uncovers the dependence of endogenous tax rates on economic preference and technology parameters. Distortionary income tax rates depend negatively on taste and technology parameters that boost growth in the undistorted version of the neoclassical model (except from total factor productivity that does not affect taxes at all). Therefore, the politicoeconomic channel reinforces, ceteris-paribus, the positive role of these parameters for growth, since it additionally implies less savings distortions.

This model can serve as a guide for quantitative/empirical research related to the relationship between economic parameters, the size of government and growth.

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Appendix

Proof of proposition 1.

From (8) one can see that this optimality condition admits linear aggregation, so:

$$\mathbf{c}_{t+1} = \beta \frac{p_t}{p_{t+1}} \mathbf{c}_t \quad \text{for all } t \geq 0, \tag{A1}$$

where p_t is the shadow price of capital with $p_0 = 1$ (the numeraire) and:

$$\frac{p_{t-1}}{p_t} = (1 - \tau_t) R_t \quad \text{for all } t \geq 0, \tag{A2}$$

with $p_{-1} = (1 - \tau_0) R_0$. From the aggregate-resource constraint it is:

$$\mathbf{c}_t + \mathbf{k}_{t+1} = (1 - \tau_t) \mathbf{y}_t \ ,$$

since $\delta = 1$. Also since the production function is Cobb-Douglas, $\mathbf{y}_t = \frac{1}{\alpha} R_t \mathbf{k}_t$, so using (A2), the aggregate-resource constraint becomes:

$$\mathbf{c}_t + \mathbf{k}_{t+1} = \frac{1}{\alpha} \frac{p_{t-1}}{p_t} \mathbf{k}_t \Rightarrow p_t \mathbf{c}_t = \frac{1}{\alpha} p_{t-1} \mathbf{k}_t - p_t \mathbf{k}_{t+1} \ .$$

Considering the latter equation one period ahead, using (A1), and defining,

$$x_t \equiv p_{t-1} \mathbf{k}_t \quad \text{for all } t \geq 0 \ ,$$

it is:

$$x_{t+2} - \left(\beta + \frac{1}{\alpha} \right) x_{t+1} + \frac{\beta}{\alpha} x_t = 0 \ , \tag{A3}$$

Equation (A3) is an elementary second-order linear difference equation with two obvious solutions, $x_{t+1} = \beta x_t$ and $x_{t+1} = \frac{1}{\alpha} x_t$. The second solution is ruled out since it implies that $p_t \mathbf{k}_{t+1} = \left(\frac{1}{\alpha} \right)^{t+1} p_{-1} \mathbf{k}_0$, which would apparently violate the transversality condition for $\alpha \in (0, 1]$. Therefore, only the first solution is acceptable, proving equation (14). Note that (A3) would not be linear if preferences were not "log."

The rest of the proof follows easily. Equation (15) follows if one combines (14), the aggregate-resource constraint and (9) in its aggregated form. So, the aggregate laws of motion are in place.

For the individual laws of motion, in a similar fashion to Krusell and Rios-Rull (1999), one can substitute (9) into (7) and solve forward, imposing the transversality condition, to get:

$$\sum_{s=0}^{\infty}\frac{p_{t+s}}{p_t}c_{t+s}=\theta\left\{(1-\tau_t)\,R_t a_t+\sum_{s=0}^{\infty}\frac{p_{t+s}}{p_t}(1-\tau_{t+s})\,w_{t+s}\right\}\quad \text{for all } t\geq 0. \tag{A4}$$

Observe that by iterating (8) forward, one can get the linear characterization (again, due to the log −preferences): $\frac{p_{t+s}}{p_t}c_{t+s}=\beta^s c_t$, so:

$$\sum_{s=0}^{\infty}\frac{p_{t+s}}{p_t}c_{t+s}=\frac{c_t}{1-\beta}\quad \text{for all } t\geq 0\ . \tag{A5}$$

Also, using (A2), $\frac{p_{t+s}}{p_t}(1-\tau_{t+s})\,w_{t+s}=\frac{p_{t+s-1}}{p_t}\frac{p_{t+s}}{p_{t+s-1}}(1-\tau_{t+s})\,w_{t+s}=\frac{p_{t+s-1}}{p_t}\frac{w_{t+s}}{R_{t+s}}=\frac{1-\alpha}{\alpha}\frac{p_{t+s-1}}{p_t}\frac{\mathbf{k}_{t+s}}{\mathbf{l}_{t+s}}$. Since $\mathbf{l}_{t+s}=\mathbf{l}$, which is given from (15) and it is already proved, and since from (14) and (A2) it is $\frac{p_{t+s-1}}{p_{t-1}}\mathbf{k}_{t+s}=\beta^s\mathbf{k}_t$, for all $t\geq 0$, it follows that:

$$\sum_{s=0}^{\infty}\frac{p_{t+s}}{p_t}(1-\tau_{t+s})\,w_{t+s}=\frac{1-\alpha}{\alpha\,(1-\beta)}\frac{p_{t-1}}{p_t}\frac{\mathbf{k}_t}{\mathbf{l}}\quad \text{for all } t\geq 0,$$

so, using (A2) again,

$$\sum_{s=0}^{\infty}\frac{p_{t+s}}{p_t}(1-\tau_{t+s})\,w_{t+s}=\frac{1-\alpha}{(1-\beta)}(1-\tau_t)\,\frac{\mathbf{y}_t}{\mathbf{l}}\quad \text{for all } t\geq 0. \tag{A6}$$

So, with substitution of (A6) and (A5) into (A4), it is:

$$c_t=(1-\tau_t)\,\theta\left[\alpha\,(1-\beta)\,\frac{a_t}{\mathbf{k}_t}+\frac{1-\alpha}{\mathbf{l}}\right]\mathbf{y}_t\ . \tag{A7}$$

Combining (7) and (9) it is $a_{t+1}=(1-\tau_t)\left[\alpha\frac{a_t}{\mathbf{k}_t}+\frac{1-\alpha}{\mathbf{l}}\right]\mathbf{y}_t-\frac{1}{\theta}c_t$, so after substituting (A7) in this last equation, (11) follows. With (11) in place, it is:

$$\frac{a_t^i}{\mathbf{k}_t}=\frac{a_0^i}{\mathbf{k}_0}\quad \text{for all } t\geq 0, \text{ and } i\in\mathcal{I}. \tag{A8}$$

Using (A8) and (15), (A7) gives (12). Equation (13) comes from combining (9), (12) and (15). Condition (10) is placed in order to guarantee that all agents, at all times optimally supply a positive level of labor hours. **Q.E.D.**

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