DEPARTMENT OF ECONOMICS
UNIVERSITY OF CYPRUS

PRICE DISCRIMINATION WITH DIFFERENTIATED PRODUCTS: DEFINITION, THEORETICAL FOUNDATION, IDENTIFICATION

Sofronis K. Clerides

Discussion Paper 2000-09

P.O. Box 20537, 1678 Nicosia, CYPRUS Tel.: ++357-2-892101, Fax: ++357-2-750310
Web site: http://www.econ.ucy.ac.cy

Price Discrimination with Differentiated Products: Definition, Theoretical Foundation, Identification

Sofronis K. Clerides*

University of Cyprus

October 2000

Abstract

I consider two alternative definitions of price discrimination with differentiated products and compare them in the context of two commonly used models of differentiated products, the logit model and the vertical differentiation model. Optimal pricing implications of the models are derived and related to observed behavior in markets where price discrimination is thought to occur. Although it fails to account for some types of observed behavior, the vertical model is shown to be the more appropriate tool to use in analyzing such markets. An optimizing monopolist producer of differentiated products is always price discriminating according to the model, hence the mere act of introducing differentiated products can be considered sufficient evidence of price discrimination.

Keywords: price discrimination, logit model, vertical differentiation model.

JEL Classification: D42, L11, L12.

*Correspondence: Department of Economics, University of Cyprus, P.O.Box 20537, CY-1678 Nicosia, Cyprus. Email: `sofronis@ucy.ac.cy`; phone: +357-2-892108; fax: +357-2-750310.

1 Introduction

Price discrimination (PD) is said to exist when the same product is sold by the same seller to different customers at different prices. Typical examples include practices such as student discounts, senior citizen discounts and coupons. Complications arise when we try to extend this definition beyond the case of identical products and as a result different definitions have been proposed. The purpose of this paper is twofold. First, to derive the pricing implications of the logit and vertical differentiation models, which are commonly used to model product differentiation, and evaluate their usefulness in the analysis of markets where we think price discrimination occurs. Second, to compare and contrast the proposed definitions in the context of these models and assess the recent empirical literature that identifies instances of price discrimination.

The paper is organized as follows. Section 2 discusses the problem of defining PD when products are differentiated. Section 3 introduces the general setup and derives pricing implications from the logit and vertical models. Section 4 discusses the results from the models and section 5 considers their implications for the identification of PD. Finally, section 6 concludes.

2 Defining price discrimination

Both definitions of PD with differentiated products are based on the same general idea. If differences in prices between different varieties of a commodity are primarily demand-driven and can not be justified by differences in production costs, PD is said to exist.

Frequently used examples are first-class passenger service, which is a lot more expensive than economy service, and hardcover books, which are substantially more expensive than their paperback versions. Although the costs of providing the two different products in each of these cases are not the same, the additional cost associated with the higher quality product appears to be very small compared to the premium the consumer has to pay for it. Hence these are usually thought of as examples of PD.

One issue that is left unresolved is the definition of the relevant market. When can two products be considered different varieties of the same good (in which case we can apply the definitions) and when are they two completely different goods? One might suggest using a substitutability test; but this does not get rid of the problem because we still have to decide on an appropriate threshold that determines whether the two goods are close enough substitutes. A more radical but effective solution is to limit ourselves to cases where the products in question are perfect substitutes. The products in the examples cited above can be thought of as such. One can not consume both an economy seat and a first-class seat, and a book buyer will purchase either a hardcover or a paperback, but not both. This distinction also illustrates why discrete choice models, in which consumers are assumed to choose one option from a set of alternatives, have been so extensively used to model product differentiation.[1]

Formally, suppose that there exist varieties 1 and 2 of some good and they are priced at p_1 and p_2 respectively. Marginal costs of production are assumed to be constant in quantity produced and equal to c_1 and c_2 respectively. The following two definitions of PD with differentiated products have been proposed:

Definition 1 *Price discrimination occurs whenever* price-cost differentials *(PCDs) of two (or more) varieties of the same good are not the same; that is, whenever* $p_1 - c_1 \neq p_2 - c_2$.

Definition 2 *Price discrimination occurs whenever* price-cost ratios *(PCRs) of two (or more) varieties of the same good are not the same; that is, whenever* $p_1/c_1 \neq p_2/c_2$.

Definition 1 has been advocated by Phlips (1983) and adopted by Tirole (1990). It has been used as the criterion for PD in empirical work by Borenstein (1991), Verboven (1996) and Giulietti and Waterson (1997).[2] Definition 2 is preferred by Stigler (1987) and Varian (1989) and has been used in empirical work by Borenstein and Rose (1994).[3]

The reasons for choosing one definition over the other are not always made explicitly clear. Phlips (1983) does not consider the ratio definition while Varian (1989) appears to accept Stigler's argument, which goes as follows:

> The proportionality definition has the merit of separating a monopolist's behavior into two parts: (1) the simple restriction of output such that price is greater than marginal cost; and (2) the misallocation of the two or more goods among buyers when they are charged different prices, which is zero if prices are *proportional* to marginal costs.[4]

Stigler's preference for the proportionality definition rests on its intuitive appeal. Although this is certainly an advantage, it does not appear powerful enough to settle the issue. A convincing argument would have to explicitly analyze the relationship between these two definitions in a general sense but also compare their implications in specific settings. Surprisingly, this has not been done in the literature.

A good way to start such an analysis is by comparing the two definitions in a purely algebraic sense.

Proposition 1 *The identities* $p_2 - c_2 = p_1 - c_1$ *and* $p_2/c_2 = p_1/c_1$ *will* both *hold if and only if* $p_1 = c_1$ *and* $p_2 = c_2$*.*

The proofs of this and all other propositions are relegated to the appendix. The corollary below follows immediately from the proposition.

Corollary 1 *Definition 1 and Definition 2 will both reject the price discrimination hypothesis* ***only*** *when price equals marginal cost for both products.*

The corollary points out that anything that is not marginal cost pricing will be identified as price discrimination by at least one of the two definitions. The importance of selecting a definition ex ante and for the right reasons is obvious. Otherwise one could look at the data and pick the definition that delivers the "right" answer.

In order to consider cases when prices are greater than marginal costs suppose, without loss of generality, that good 2 is no cheaper than good 1; that is, $p_2 \geq p_1$. No restriction is placed on the relative level of marginal costs. We can state the following proposition.

Proposition 2 *Assume* $p_2 \geq p_1$ *and* $p_j > c_j$*,* $j = 1, 2$*. Then,*

(i) if $c_2 > c_1$ *then* $p_2/c_2 \geq p_1/c_1 \quad \Rightarrow \quad p_2 - c_2 > p_1 - c_1$*;*

(ii) if $c_2 \leq c_1$ *then* $p_2 - c_2 > p_1 - c_1$ *and* $p_2/c_2 > p_1/c_1$.

The implications of these results are summarized below.

Corollary 2 *The following statements hold if* $p_2 \geq p_1$ *and* $p_j > c_j$, $j = 1, 2$*:*

(i) If $c_2 > c_1$*, then* ΔPCD *and* ΔPCR *will not necessarily have the same sign. In particular, it is possible to observe* $\Delta PCD \geq 0$ *and* $\Delta PCR \leq 0$.

(ii) If $c_2 \leq c_1$*, then* ΔPCD *and* ΔPCR *will both be strictly positive and our two definitions will be qualitatively equivalent.*

The case $c_2 \leq c_1$ is not common but it does occur,[5] so it is reassuring that the two definitions coincide in that case. The definitions will disagree, on the other hand, in the more common case where $c_2 > c_1$. Note that the definitions do not impose any restriction on the sign of the difference. Hence if the cheaper product has a both a higher PCR and a higher PCD than the more expensive product, this would pass the test. Of course we typically think that the expensive (high quality) product is the one with the higher markup, so empirically this may not be a big problem. What is more worrisome is the possibility of observing ΔPCD > 0 and ΔPCR $= 0$, which can not be ruled out according to Corollary 2, and is an empirically relevant scenario.

The fact that the two definitions will give different answers in many situations is troubling and calls for a closer examination of the circumstances under which this will occur. In order to do that we need to appeal to theory. In the next section I consider with two models of product differentiation that are commonly used in empirical work, the simple logit model and the vertical differentiation model.

3 The model

3.1 General setup

Consider a firm that produces two different varieties of the same product. The firm faces downward sloping demand for the two varieties, generically written as $Q_1(p_1, p_2)$ and $Q_2(p_1, p_2)$ for varieties 1 and 2 respectively. Marginal costs are assumed to be constant at c_1 and c_2 and, without loss of generality, we can assume that variety 2 is the more costly one to produce: $c_1 \leq c_2$. The firm chooses prices to maximize profits given by

$$\Pi(p_1, p_2) = (p_1 - c_1)\, Q_1(p_1, p_2) + (p_2 - c_2)\, Q_2(p_1, p_2).$$

Standard profit maximization techniques yield the following pricing equations:

$$\begin{aligned} p_2 - c_2 &= (1/H)\, [+d_{12}\, Q_1 - d_{11}\, Q_2] \\ p_1 - c_1 &= (1/H)\, [-d_{22}\, Q_1 + d_{21}\, Q_2]\,, \end{aligned}$$

where $d_{ij} = \partial Q_i / \partial p_j$ and $H = d_{11}d_{22} - d_{12}d_{21}$. Note that these are not complete solutions for the prices since the Q_j's are actually functions of the prices. They will nevertheless be useful for our purposes. We will be interested in looking at the differences between PCDs and between PCRs:

$$\Delta PCD = (1/H) \Big[(d_{12} + d_{22})\; Q_1 - (d_{11} + d_{21})\; Q_2 \Big] \tag{1}$$

$$\Delta PCR = \frac{1}{H c_1 c_2} \Big[(c_1 d_{12} + c_2 d_{22})\; Q_1 - (c_1 d_{11} + c_2 d_{21})\; Q_2 \Big], \tag{2}$$

where $\Delta\text{PCD} = (p_2 - c_2) - (p_1 - c_1)$ and $\Delta\text{PCR} = (p_2/c_2) - (p_1/c_1)$.

Independent markets. The differentiated product models considered here do not, by construction, nest the case of independent product demands (that is, of zero cross-price elasticities). In order to accommodate that scenario (which will serve as a useful benchmark) I will consider the case of the markets for the two varieties being independent, that is, $Q_j(p_1, p_2) = Q^j(p_j)$, for $j = 1, 2$. Profit maximization then delivers the usual inverse elasticity rule that holds in each market separately (superscripts are used to denote the relevant market for variables that can vary across models):

$$p^j - c_j = -\frac{Q^j}{dQ^j/dp^j} \qquad \Rightarrow \qquad \frac{p^j - c_j}{p^j} = -\frac{1}{\eta^j}. \tag{3}$$

This completes the general framework. I now proceed to analyze the logit and vertical models.

3.2 The logit model

The logit model is often used to model demand for differentiated products. There are M consumers and individual i's utility for product j is given by

$$u_{ij} = \delta_j - \alpha p_j + \varepsilon_{ij}, \qquad j = 1, 2. \tag{4}$$

The outside good gives utility $u_{i0} = \varepsilon_{i0}$. The sum $\delta_j + \varepsilon_{ij}$ is individual i's assessment of the quality of product j; δ_j is the mean quality over all consumers and ε_{ij} is individual i's deviation from this mean. If ε_{ij} has the extreme value distribution then the well-known market shares for product j are given by

$$s_j = \frac{e^{\delta_j - \alpha p_j}}{1 + e^{\delta_1 - \alpha p_1} + e^{\delta_2 - \alpha p_2}}, \qquad j = 1, 2.$$

The derivatives with respect to price can be written in terms of the market shares as $\partial s_1/\partial p_1 = -\alpha s_1(1 - s_1)$, $\partial s_2/\partial p_2 = -\alpha s_2(1 - s_2)$, and $\partial s_1/\partial p_2 = \partial s_2/\partial p_1 = \alpha s_1 s_2$.[6] Using this information we can derive the following pricing equations for the logit model:

$$p_j - c_j = \frac{1}{\alpha s_0}, \tag{5}$$

where s_0 is the market share of the outside good. This is the standard form of the pricing equation in logit models: price equals marginal cost plus a markup. The associated differences in PCDs and PCRs are easy to calculate:

$$\Delta(p - c) = 0 \tag{6}$$

$$\Delta\text{PCR} = \frac{1}{\alpha s_0} \frac{c_1 - c_2}{c_1 c_2}. \tag{7}$$

The implications of these results are best summarized in the following corollary.

Corollary 3 *Joint profit maximization in the logit model implies identical price-cost differentials for the two products, and thus* ***no price discrimination*** *according to Definition 1. Moreover, the same conclusion is reached by Definition 2 in the case of equal marginal costs.*

The implication that joint profit maximizing prices will not be considered discriminatory according to Definition 1 does not bode well for the logit model. Also problematic is the fact that the logit model rules out the possibility that two differentiated products with the same marginal cost can have different prices. Moreover, equation (7) suggests that the higher cost product should always have a *lower* PCR. This runs counter to many empirical examples. For example, Clerides (1999a) reports that PCRs of (higher cost) hardcover books are higher than those of paperbacks for the great majority of his sample of 66 titles. Also, Deneckere and McAfee (1996) report examples of firms that produce high and low quality versions of a product and the low quality version (dubbed a "damaged good") actually has a higher marginal cost and thus, since its price is lower, a lower PCR.

Independent markets. For the independent market scenario suppose that consumers are split in two groups (markets 1 and 2) and only one product is available to each group. Let s^j denote the market share of product j when it is the only one available in market j. The inverse elasticity rule yields the pricing equation $p^j - c_j = 1/[\alpha(1-s^j)] = -p^j/\eta^j$. Thus PCDs will be the same only when elasticities are equal or, equivalently, when market shares are the same. Thus within the framework of the logit model we can not observe PD according to Definition 1 unless we assume that the markets for the two products are completely independent. This, of course, defeats the purpose of using a model of differentiated products in the first place.

Furthermore, note also that with independent markets we have

$$\Delta PCD = \frac{s^2 - s^1}{\alpha(1-s^2)(1-s^1)} \tag{8}$$

$$\Delta PCR = \frac{c_1(1-s^2) - c_2(1-s^2)}{\alpha c_1 c_2 (1-s^2)(1-s^1)}. \tag{9}$$

These pricing rules are much more flexible than the ones derived from the joint profit maximization case: they can accommodate both different pricing of same-cost products and a higher PCR for the high cost product.

3.3 The vertical differentiation model

The vertical differentiation model was introduced by Shaked and Sutton (1982) and was featured in Bresnahan's (1987) seminal study of the automobile industry. It is called the vertical model because it incorporates an explicit ranking of products in order of quality. All consumers agree on this ranking, but they differ in their willingness to pay for quality. This is in contrast to the simple logit model where preference for quality is common among all consumers and what differs is their assessment of product quality.

In the vertical model individual i's utility for product j is given by

$$u_{ij} = v_i \delta_j - p_j,$$

where δ_j denotes the quality of the product and v_i represents the individual's preference for quality. The outside good is normalized to give utility $u_{i0} = 0$. Quality levels are given at δ_1 and δ_2 and I assume, without loss of generality, that $\delta_1 < \delta_2$. The firm has the option of selling both qualities, or just one of the two. I leave the derivation of demand and profit maximization for the appendix. Suffice it to say that, unlike the logit model, the vertical model delivers analytic solutions if consumer preferences v_i are uniformly distributed on the interval $[0, V]$. This enables us to delve further into the specifics of the model and draw concrete conclusions.

Proposition 3 *The firm's optimal policy in the model outlined above is:*

(i) If $\frac{c_2-c_1}{V\delta_2-V\delta_1} \geq 1$*, sell only the low quality product at a price* $p_1 = (V\delta_1 + c_1)/2$.

(ii) If $\frac{c_2-c_1}{V\delta_2-V\delta_1} \leq \frac{c_1}{V\delta_1}$*, sell only the high quality product at a price* $p_2 = (V\delta_2 + c_2)/2$.

(iii) If $\frac{c_1}{V\delta_1} < \frac{c_2-c_1}{V\delta_2-V\delta_1} < 1$*, sell both qualities at prices* $p_1 = (V\delta_1 + c_1)/2$ *and* $p_2 = (V\delta_2 + c_2)/2$.

The above also require that $V\delta_j \geq c_j$ *for* $j = 1, 2$.

These results are intuitive. Note that $V\delta_j$ is the highest willingness to pay for δ_j by any consumer, or the maximum value of quality. If the cost of the additional quality is higher than the additional value it creates for the top consumer, then the high quality product is not worth producing (statement (i)). If, on the other hand, the cost is very small relative to the value it creates, then it is optimal to sell only the high quality product (statement (ii)). Perhaps the most interesting result is that the price of each product is independent of whether the other one is produced. This means that the addition of a new variety of a product has no effect on the price of the existing variety; in other words, discriminatory prices are the same as profit maximizing prices from each variety individually.[7] This result may not be surprising given that optimal prices depend only on the cost and quality of the product itself, and not on the cost or quality of the competing product. This is in contrast to the logit model where the optimal price of each version depends on the quality of the other.[8]

The differences between PCDs and PCRs when both qualities are produced are:

$$\Delta\text{PCD} = (1/2)\,[(V\delta_2 - c_2) - (V\delta_1 - c_1)] \tag{10}$$

$$\Delta\text{PCR} = (V/2)\,[\delta_2/c_2 - \delta_1/c_1] \tag{11}$$

Note the correspondence between the PD definitions and the results of the vertical model. The following corollary is of interest.

Corollary 4

(i) When both qualities are produced, the high quality product will have a higher PCD and a lower PCR than the low quality product; that is, $\Delta PCD > 0$ and $\Delta PCR < 0$.

(ii) If $c_2 \leq c_1$ only the high quality product will be produced.

Thus the vertical model gives us a definitive answer as to the signs of the differences in PCDs and PCRs. Most importantly, it tells us that the differences cannot be zero; in other words, a profit maximizing monopolist in this model is *always* price discriminating. Statement (ii) of the corollary indicates that the model is not rich enough to cover all scenarios of interest. Specifically, in the vertical model it is never optimal to market a damaged good, and we can never have both ΔPCD and ΔPCR being positive.

Independent markets. In order to accommodate damaged goods we need to go to the independent market scenario. This requires some further elaboration of the model. Suppose that consumers are divided such that a proportion λ, $0 < \lambda < 1$, of them belong to market 1, the market for the low quality product, and the rest to market 2. Therefore the distribution of consumer preferences for quality in each market is $v_i \sim U[0, \lambda V]$ and $v_i \sim U[\lambda V, V]$ in markets 1 and 2 respectively. One can then show that optimal prices are $p_1 = (\lambda V \delta_1 + c_1)/2$ and $p_2 = (V\delta_2 + c_2)/2$. That is, the price of the high quality product remains the same, while that of the low quality product is lower than in the general case. Thus both the PCD and PCR of the low quality product will decrease, meaning that

ΔPCD will remain positive, while ΔPCR could also become positive (though the exact sign is indeterminate). Hence the independent market scenario can accommodate the case of both ΔPCD and ΔPCR being positive.

Overall, the vertical model appears to be a suitable framework for the analysis of situations where PD occurs. One problem that remains is its inability to handle cases where $\Delta\text{PCD} > 0$ and $\Delta\text{PCR} < 0$ are both positive and the need to assume independent markets. In the next section I take a closer look at this issue.

4 Comparison of the logit and vertical models

The above results suggest that the vertical model is more suited to the analysis of price discrimination than the logit model. A natural question is, why does the latter do so badly? A possible explanation is the inability of the model to deal with situations where cross-price elasticities are very small. As we noted already, the cross-price derivative in the logit model is given by $\partial Q_i/\partial p_j = \alpha M s_i s_j$. This derivative can only approach zero if α approaches zero. But this would also drive the *own* price derivative to zero. So the model cannot accommodate situations where there is price sensitivity for a given product but not across products.

In the vertical model the cross-price derivative is given by $\partial Q_i/\partial p_j = 1/(\delta_2 - \delta_1)$. This derivative can get close to zero if the quality difference is large. This will also have the effect of driving the own-price derivative of the high quality product to zero since $\partial Q_2/\partial p_2 = -1/(\delta_2 - \delta_1)$. The own-price derivative of the low quality product, on the other hand, will *not* be driven to zero because it includes two terms: $\partial Q_1/\partial p_1 = -1/(\delta_2 - \delta_1) - 1/\delta_1$. Thus the vertical model can handle situations where consumer demand for the high quality product is fairly inelastic, while demand for the low quality product is more elastic. This is probably a good description of what happens in many markets.

Intuitively it seems reasonable that price discrimination using product differentiation will only be profitable (when compared to a single-product monopoly) when the resulting market segmentation is fairly robust; that is, when shifting of consumers between segments is limited and the firm can essentially treat each segment as a distinct market. Thus it is not surprising that the logit model does not fit these situations well.

The implication here is that it may be better to treat the two markets as independent. Although this may sound far-fetched, there is some recent empirical evidence that this may be what is going on in many markets. For example, both Berry, Carnall, and Spiller (1996) and Clerides (1999b) have been successful in estimating models of demand in the air travel and book markets respectively where different types of consumers have very different preferences and there is very little cannibalization of one product's sales by the introduction of the other.

5 Identifying price discrimination

What are the implications of all this for the empirical literature that aims to identify instances of price discrimination? Two major issues arise. First, the criteria used to identify PD suffer from a robustness problem. There is not (and there can not be) any statistical theory that tells us the extent to which PCDs or PCRS must differ for PD to exist. Therefore, if we were to follow the criteria strictly, we would have to proclaim PD even when the differentials or ratios differ by very small amounts. In other words, the data will almost never reject the PD hypothesis. This problem is a general one and is not restricted to any particular model or mode of behavior.

In addition, the results from the vertical model suggest that an optimizing monopolist would always price in a discriminatory way. These results are, of course, drawn from a stylized model with restrictive assumptions. Nonetheless, it is a model that seems to reflect the behavior of price discriminating firms quite well and delivers reasonable and intuitive conduct implications. But if a profit maximizing monopolist is always price discriminating, the requirement for the identification of PD is substantially reduced. It is no longer necessary to look at prices in order to assess the practice of PD. Instead, the mere act of introducing multiple versions of the same product should be sufficient evidence for it.

Do these results render the literature redundant? Not necessarily. The identification of instances of PD remains an interesting and important task. Estimated PCDs and PCRs can serve as indicators of the nature of demand while they can also be thought of as measures of the degree of PD. If anything, the results from this paper suggest that a lot more work is needed in order to understand pricing.

6 Concluding remarks

In this paper I examine two alternative definitions of price discrimination when products are not identical. I show that the two definitions are not equivalent algebraically; it is possible for one to accept price discrimination and for the other to reject it. Moreover, this scenario is likely to be relevant empirically.

In order to assess the two definitions I analyze the pricing implications of two commonly used models of product differentiation, the logit and the vertical model. The pricing implications of the logit model are problematic in many cases where we think price discrimination is practiced. For example, the model predicts identical PCDs (and also PCRs if marginal costs are the same) for all products. This runs counter to observed behavior in many situations, while it also implies that joint profit maximizing behavior is not price discrimination according to the definition based on PCDs. Moreover, the logit model predicts that the less costly product should exhibit a higher PCR, a result that is also inconsistent with empirical observation. The failure of the model seems to stem from its inability to handle situations where the market is fairly robustly divided into different segments.

The vertical model, on the other hand, appears well-suited for this kind of analysis and it delivers reasonable pricing implications. One remaining problem is its inability to handle the empirically observed case of both $\Delta\text{PCD} > 0$ and $\Delta\text{PCR} < 0$ being positive. This suggests that alternative approaches should also be considered, perhaps using demand systems defined over the product space instead of the characteristics space.

The results of the vertical model suggest that an optimizing monopolist will always price discriminate, hence the mere act of introducing multiple versions of the same product can be sufficient evidence of PD. There results are not by any means conclusive, as they come from a stylized model, considering only the case of a monopolist. More work – both theoretical and empirical – is certainly needed to understand the complex issue of pricing.

A Appendix

A.1 Proof of Proposition 1

Suppose that $p_2 - c_2 = p_1 - c_1$ and $p_2/c_2 = p_1/c_1$. From the first identity,

$$\begin{aligned} p_2 &= c_2 + p_1 - c_1 \\ &= c_2 + (c_1/c_2)p_2 - c_1 \\ &\quad \Rightarrow [1 - c_1/c_2]p_2 = c_2 - c_1 \\ &\quad \Rightarrow [(c_2 - c_1)/c_2]p_2 = c_2 - c_1 \\ &\quad \Rightarrow p_2 = c_2, \end{aligned}$$

where the second line uses the second identity and the rest is just rearranging terms. Given that $p_2 = c_2$ it follows immediately from either of the two identities above that we must also have $p_1 = c_1$; in other words, marginal cost pricing. □

A.2 Proof of Proposition 2

We have assumed that $p_2 \geq p_1$ and $p_j > c_j$ for $j = 1, 2$. There are two cases.

Case $c_2 > c_1$:

Suppose $p_2/c_2 \geq p_1/c_1$, that is, $p_2 \geq c_2 p_1/c_1$. Then,

$$\begin{aligned}
(p_2 - c_2) - (p_1 - c_1) &\geq (c_2/c_1)p_1 - c_2 - P_1 + c_1 \\
&= \frac{c_2 - c_1}{c_1} p_1 - (c_2 - c_1) \\
&= (c_2 - c_1)\left(\frac{p_1}{c_1} - 1\right) \\
&> 0.
\end{aligned}$$

Case $c_2 < c_1$:

Suppose $p_2 - c_2 > p_1 - c_1$, that is, $p_2 > c_2 + p_1 - c_1$. Then,

$$\begin{aligned}
\frac{p_2}{c_2} - \frac{p_1}{c_1} &> \frac{c_2 + p_1 - c_1}{c_2} - \frac{p_1}{c_1} \\
&= \frac{c_2 - c_1}{c_2} + \frac{p_1}{c_2} - \frac{p_1}{c_1} \\
&= \frac{c_2 - c_1}{c_2} - \frac{p_1}{c_1}\frac{c_2 - c_1}{c_2} \\
&= \frac{c_2 - c_1}{c_2}\left(1 - \frac{p_1}{c_1}\right) \\
&> 0.
\end{aligned}$$

□

A.3 Derivation of PCDs and PCRs for the general model

We are maximizing the function

$$\Pi(p_1,p_2) = (p_1 - c_1)\,Q_1(p_1,p_2) + (p_2 - c_2)\,Q_2(p_1,p_2).$$

The first order conditions for this problem are:

$$\begin{aligned} Q_1(p_1,p_2) + (p_1 - c_1)\frac{\partial Q_1}{\partial p_1} + (p_2 - c_2)\frac{\partial Q_2}{\partial p_1} &= 0 \\ Q_2(p_1,p_2) + (p_2 - c_2)\frac{\partial Q_2}{\partial p_2} + (p_1 - c_1)\frac{\partial Q_1}{\partial p_2} &= 0. \end{aligned}$$

It is convenient to write the FOCs in matrix form. Let $P = [p_1\ p_2]'$, $Q = [Q_1\ Q_2]'$, $C = [c_1\ c_2]'$, and D be the derivative matrix whose element d_{ij} is the derivative $\partial Q_i/\partial p_j$. Then the FOCs can be written as $Q + D'(P - C) = 0$ or in terms of the mark-up $P - C$ as

$$P - C = -[D']^{-1}Q$$

This expression is easy to represent in equation by equation form in the two-product case. The inverse of the matrix D' is

$$[D']^{-1} = \frac{1}{H}\begin{bmatrix} d_{22} & -d_{21} \\ -d_{12} & d_{11} \end{bmatrix},$$

where $H = d_{11}d_{22} - d_{12}d_{21}$ denotes the determinant of the matrix D. Thus we have the following pricing equations:

$$\begin{aligned} p_1 - c_1 &= (1/H)\,[-d_{22}\,Q_1 + d_{21}\,Q_2] \\ p_2 - c_2 &= (1/H)\,[+d_{12}\,Q_1 - d_{11}\,Q_2]. \end{aligned}$$

A.4 Derivatives of logit market shares

One example is sufficient to illustrate the derivation.

$$\begin{aligned}
\frac{\partial s_1}{\partial p_1} &= \frac{\partial}{\partial p_1}\left(\frac{e^{\delta_j-\alpha p_j}}{1+e^{\delta_1-\alpha p_1}+e^{\delta_2-\alpha p_2}}\right) \\
&= \frac{-\alpha e^{\delta_j-\alpha p_j}(1+e^{\delta_1-\alpha p_1}+e^{\delta_2-\alpha p_2})-(-\alpha)(e^{\delta_j-\alpha p_j})^2}{(1+e^{\delta_1-\alpha p_1}+e^{\delta_2-\alpha p_2})^2} \\
&= -\alpha(s_1-s_1^2) \\
&= -\alpha s_1(1-s_1)
\end{aligned}$$

The cross-derivatives can be derived in the same way.

A.5 Derivation of the vertical model

In order to derive demand in the vertical model one proceeds as follows. Recall that utility is given by $u_{ij} = v_i\delta_j - p_j$, where the δ_j's are the qualities. Taking quality as given, we find the threshold values of v_i that determine the indifferent consumers between each pair of products. We then integrate over the distribution of consumers between each set of cutoff points in order to get demand.

Consider the two product case with qualities δ_1 and δ_2 and $\delta_1 < \delta_2$. There will be two cutoff points, $v_{12} = (p_2 - p_1)/(\delta_2 - \delta_1)$ and $v_{01} = p_1/\delta_1$. The point v_{12} determines the marginal consumer who is indifferent between products 1 and 2; that is, $v_{12} = \{v_i \mid v_i\delta_2 - p_2 = v_i\delta_1 - p_1\}$. Similarly, point v_{01} determines the consumer who is indifferent between product 1 and the outside good ($v_{01} = \{v_i \mid v_i\delta_1 - p_1 = 0\}$). When consumer preferences v_i are uniformly distributed on the interval $[0, V]$ consumer demands are simply the distance between the cutoff points divided by the length of the interval, V. Thus, $Q_1 = (V - v_{12})/V$ and $Q_2 = (v_{12} - v_{01})/V$ or,

$$Q_2 = \frac{1}{V}\left[V - \frac{p_2 - p_1}{\delta_2 - \delta_1}\right] \tag{12}$$

$$Q_1 = \frac{1}{V}\left[\frac{p_2 - p_1}{\delta_2 - \delta_1} - \frac{p_1}{\delta_1}\right]. \tag{13}$$

It is now straightforward to calculate the required derivatives:

$$d_{11} = -\tfrac{1}{V(\delta_2 - \delta_1)} \quad d_{12} = \tfrac{1}{V(\delta_2 - \delta_1)}$$

$$d_{21} = \tfrac{1}{V(\delta_2 - \delta_1)} \quad d_{12} = -\tfrac{1}{V(\delta_2 - \delta_1)} - \tfrac{1}{V\delta_1}$$

Plugging these into equations (1) and (2) obtains the differences between price-cost differentials and price-cost ratios.

Note that the above demand functions will hold only as long as it is optimal to sell both varieties.

A.6 Proof of Proposition 3

It suffices to plug the solutions for prices into the demand equations derived above. Equation (13) becomes

$$\begin{aligned}
Q_1 &= \frac{1}{V}\left[\frac{p_2 - p_1}{\delta_2 - \delta_1} - \frac{p_1}{\delta_1}\right]. \\
&= \frac{(V\delta_2 + c_2) - (V\delta_1 + c_1)}{2(V\delta_2 - V\delta_1)} - \frac{V\delta_1 + c_1}{2V\delta_1} \\
&= \frac{V\delta_2 - V\delta_1 + c_2 - c_1}{2(V\delta_2 - V\delta_1)} - \frac{V\delta_1 + c_1}{2V\delta_1} \\
&= \frac{1}{2}\left[\frac{c_2 - c_1}{V\delta_2 - V\delta_1} - \frac{c_1}{V\delta_1}\right]
\end{aligned}$$

Thus $Q_1 > 0$ only if $\frac{c_2 - c_1}{V\delta_2 - V\delta_1} > \frac{c_1}{V\delta_1}$ or, equivalently, if $\delta_2/c_2 < \delta_1/c_1$. This will never occur when $c_2 \leq c_1$, hence in that case only the high quality product is produced.

Similarly, equation (12) becomes

$$\begin{aligned} Q_2 &= \frac{1}{V}\left[V - \frac{p_2 - p_1}{\delta_2 - \delta_1}\right] \\ &= 1 - \frac{(V\delta_2 + c_2) - (V\delta_1 + c_1)}{2(V\delta_2 - V\delta_1)} \\ &= 1 - \frac{V\delta_2 - V\delta_1 + c_2 - c_1}{2(V\delta_2 - V\delta_1)} \\ &= \frac{1}{2}\left[1 - \frac{c_2 - c_1}{V\delta_2 - V\delta_1}\right] \end{aligned}$$

This will be positive if $\frac{c_2 - c_1}{V\delta_2 - V\delta_1} < 1$ or, equivalently, if $V\delta_2 - c_2 > V\delta_1 - c_1$. □

Notes

[1]Anderson, DePalma, and Thisse (1992) provide a lucid and comprehensive discussion of those models.

[2]The latter two papers actually compare the values of $(p-c)/p$, which is qualitatively equivalent.

[3]Lott and Roberts (1991) have criticized the identification of "price anomalies" as price discrimination. They argue that there are usually cost-based explanations for these phenomena which are overlooked by most economists. Nonetheless, even their critique does not question the basic premise that price-cost differentials constitute price discrimination.

[4]Stigler (1987, page 210).

[5]See Deneckere and McAfee (1996) and also the discussion in section 4.

[6]See the appendix for the proof.

[7]This result appeared in a more general form in Itoh (1983) and is related to the fact that the uniform distribution has a linear inverse hazard rate.

[8]Even though analytic solutions for prices are not possible, this result can be easily obtained with the help of the implicit function theorem.

References

Anderson, S. P., A. DePalma, and J.-F. Thisse, 1992, *Discrete Choice Theory of Product Differentiation*, MIT Press, Cambridge, MA.

Berry, S., M. Carnall, and P. T. Spiller, 1996, "Airline Hubs: Costs, Markups, and the Implications of Customer Heterogeneity," *NBER Working Paper 5561.*

Borenstein, S., 1991, "Selling Costs and Switching Costs: Explaining Retail Gasoline Margins," *Rand Journal of Economics*, 22, 354–369.

Borenstein, S., and N. L. Rose, 1994, "Competition and Price Dispersion in the U.S. Airline Industry," *Journal of Political Economy*, 102, 653–683.

Bresnahan, T., 1987, "Competition and Collusion in the American Automobile Oligopoly: The 1955 Price War," *Journal of Industrial Economics*, 35, 457–482.

Clerides, S. K., 1999a, "Book Value: Intertemporal Pricing and Quality Discrimination in the U.S. Book Market," University of Cyprus Discussion Paper 99-15.

Clerides, S. K., 1999b, "Product Selection as Price Discrimination in the Market for Books," University of Cyprus Discussion Paper 99-08.

Deneckere, R., and R. P. McAfee, 1996, "Damaged Goods," *Journal of Economics and Management Strategy*, 5, 149–174.

Giulietti, M., and M. Waterson, 1997, "Multiproduct Firms' Pricing Behaviour in the Italian Grocery Trade," *Review of Industrial Organization*, 12, 817–832.

Itoh, M., 1983, "Monopoly, Product Differentiation and Economic Welfare," *Journal of Economic Theory*, 31, 88–104.

Lott, J. R., and R. D. Roberts, 1991, "A Guide to the Pitfalls of Identifying Price Discrimination," *Economic Inquiry*, XXIX, 14–23.

Phlips, L., 1983, *The Economics of Price Discrimination*, Cambridge University Press.

Shaked, A., and J. Sutton, 1982, "Relaxing Price Competition Through Product Differentiation," *Review of Economic Studies*, 49, 3–13.

Stigler, G. J., 1987, *The Theory of Price*, Macmillan, New York.

Tirole, J., 1990, *The Theory of Industrial Organization*, MIT Press, Cambridge, MA.

Varian, H. R., 1989, "Price Discrimination," in Richard Schmalensee, and R. D. Willig (ed.), *Handbook of Industrial Organization, Volume I*, Elsevier Science Publishers B.V, Cambridge, MA.

Verboven, F., 1996, "International Price Discrimination in the European Car Market," *Rand Journal of Economics*, 27, 240–268.

SELECTED RECENT PUBLICATIONS

Anil K. Bera and Yannis Bilias, Rao´s Score, Neyman´s C (α) and Silvey´s LM Tests: An Essay on Historical Developments and Some New Results, *Journal of Statistical Planning and Inference,* forthcoming.

Bertaut C. and M. Haliassos, Precautionary Portfolio Behavior from a Life - Cycle Perspective, *Journal of Economic Dynamics and Control*, 21, 1511-1542, 1997.

Bilias Y., Minggao Gu and Zhiliang Ying, Towards a General Asymptotic Theory for the Cox model with Staggered Entry, *The Annals of Statistics*, 25, 662-682, 1997.

Blundell R., P. Pashardes and G. Weber, What Do We Learn About Consumer Demand Patterns From Micro-Data?, *American Economic Revie*w, 83, 570-597, 1993.

Bougheas S., P. Demetriades and T. P. Mamouneas, Infrastructure, Specialization and Economic Growth, *Canadian Journal of Economics,* forthcoming.

Caporale W., C. Hassapis and N. Pittis, Unit Roots and Long Run Causality: Investigating the Relationship between Output, Money and Interest Rates, *Economic Modeling*, 15(1), 91-112, January 1998.

Caporale G. and N. Pittis, Efficient estimation of cointegrated vectors and testing for causality in vector autoregressions: A survey of the theoretical literature, *Journal of Economic Surveys*, forthcoming.

Caporale G. and N. Pittis, Unit root testing using covariates: Some theory and evidence, *Oxford Bulletin of Economics and Statistics*, forthcoming.

Caporale G. and N. Pittis, Causality and Forecasting in Incomplete Systems, *Journal of Forecasting*, 16, 6, 425-437, 1997.

Clerides K. S., Lach S. and J.R. Tybout, Is Learning-by-Exporting Important? Micro-Dynamic Evidence from Colombia, Morocco, and Mexico, *Quarterly Journal of Economics* 113(3), 903- 947, August 1998.

Cukierman A., P. Kalaitzidakis, L. Summers and S. Webb, Central Bank Independence, Growth, Investment, and Real Rates", Reprinted in Sylvester Eijffinger (ed), Independent Central Banks and Economic Performance, Edward Elgar, 416-461, 1997.

Dickens R., V. Fry and P. Pashardes, Non-Linearities and Equivalence Scales, *The Economic Journa*l, 103, 359-368, 1993.

Demetriades P. and T. P. Mamuneas, Intertemporal Output and Employment Effects of Public Infrastructure Capital: Evidence from 12 OECD Economies, *Economic Journal*, July 2000.

Eicher Th. and P. Kalaitzidakis, The Human Capital Dimension to Foreign Direct Investment: Training, Adverse Selection and Firm Location". In Bjarne Jensen and Kar-yiu

Wong (eds), Dynamics,Economic Growth, and International Trade, The University of Michigan Press, 337-364, 1997.

Fry V. and P. Pashardes, Abstention and Aggregation in Consumer Demand, *Oxford Economic Paper*s, 46, 502-518, 1994.

Gatsios K., P. Hatzipanayotou and M. S. Michael, International Migration, the Provision of Public Good and Welfare, *Journal of Development Economics*, 60/2, 561-577, 1999.

Haliassos M. and C. Hassapis, Non-expected Utility, saving, and Portfolios, *The Economic Journal*, 110, 1-35, January 2000.

Haliassos M. and J. Tobin, The Macroeconomics of Government Finance, reprinted in J.Tobin, Essays in Economics, vol. 4, Cambridge: MIT Press, 1996.

Haliassos M. and C. Bertaut, Why Do So Few Hold Stocks?, *The Economic Journal*, 105, 1110- 1129, 1995.

Haliassos M., On Perfect Foresight Models of a Stochastic World, *Economic Journal*, 104, 477-491, 1994.

Hassapis C., N. Pittis and K. Prodromidis, Unit Roots and Granger Causality in the EMS Interest Rates: The German Dominance Hypothesis Revisited, *Journal of International Money and Finance*, 18(1), 47-73, 1999.

Hassapis C., S. Kalyvitis and N. Pittis, Cointegration and Joint Efficiency of International Commodity Markets", *The Quarterly Review of Economics and Finance,* 39, 213-231, 1999.

Hassapis C., N. Pittis and K. Prodromides, EMS Interest Rates: The German Dominance Hypothesis or Else?" in European Union at the Crossroads: A Critical Analysis of Monetary Union and Enlargement, Aldershot, UK., Chapter 3, 32-54, 1998. Edward Elgar Publishing Limited.

Hatzipanayotou P., and M. S. Michael, General Equilibrium Effects of Import Constraints Under Variable Labor Supply, Public Goods and Income Taxes, *Economica*, 66, 389-401, 1999.

Hatzipanayotou, P. and M.S. Michael, Public Good Production, Nontraded Goods and Trade Restriction, *Southern Economic Journal,* 63, 4, 1100-1107, 1997.

Hatzipanayotou, P. and M. S. Michael, Real Exchange Rate Effects of Fiscal Expansion Under Trade Restrictions, *Canadian Journal of Economics,* 30-1, 42-56, 1997.

Kalaitzidakis P., T. P. Mamuneas and Th. Stengos, A Nonlinear Sensitivity Analysis of Cross-Country Growth Regressions, *Canadian Journal of Economics,* forthcoming.

Kalaitzidakis P., T. P. Mamuneas and Th. Stengos, European Economics: An Analysis Based on Publications in Core Journals, *European Economic Revie*w, 1999.

Kalaitzidakis P., On-the-job Training Under Firm-Specific Innovations and Worker Heterogeneity, *Industrial Relations*, 36, 371-390, July 1997.

Ludvigson S. and A. Michaelides, Does Buffer Stock Saving Explain the Smoothness and Excess Sensitivity of Consumption?, *American Economic Review*, forthcoming.

Lyssiotou Panayiota, Dynamic Analysis of British Demand for Tourism Abroad, *Empirical Economics*, forthcoming, 2000.

Lyssiotou P., P. Pashardes and Th. Stengos, Testing the Rank of Engel Curves with Endogenous Expenditure, *Economics Letters*, 64, 61-65, 1999.

Lyssiotou P., P. Pashardes and Th. Stengos, Preference Heterogeneity and the Rank of Demand Systems, *Journal of Business and Economic Statistics*, 17 (2), 248-252, April 1999.

Lyssiotou Panayiota, Comparison of Alternative Tax and Transfer Treatment of Children using Adult Equivalence Scales, *Review of Income and Wealth*, 43 (1), 105-117, March 1997.

Mamuneas, Theofanis P., Spillovers from Publicly – Financed R&D Capital in High-Tech Industries, *International Journal of Industrial Organization*, 17(2), 215-239, 1999.

Mamuneas, T. P. and Nadiri M. I., R&D Tax Incentives and Manufacturing-Sector R&D Expenditures, in *Borderline Case: International Tax Policy, Corporate Research and Development, and Investment*, James Poterba (ed.), National Academy Press, Washington D.C., 1997. Reprinted in *Chemtech*, 28(9), 1998.

Mamuneas, T. P. and Nadiri M. I., Public R&D Policies and Cost Behavior of the US Manufacturing Industries, *Journal of Public Economics*, 63, 57-81, 1996.

Michaelides A. and Ng, S., Estimating the Rational Expectations Model of Speculative Storage: A Monte Carlo Comparison of three Simulation Estimators, *Journal of Econometrics*, forthcoming.

Pashardes Panos, Equivalence Scales in a Rank-3 Demand System, *Journal of Public Economics*, 58, 143-158, 1995.

Pashardes Panos, Bias in Estimating Equivalence Scales from Grouped Data, *Journal of Income Distribution*, Special Issue: Symposium on Equivalence Scales, 4, 253-264,1995.

Pashardes Panos., Bias in Estimation of the Almost Ideal Demand System with the Stone Index Approximation, *Economic Journal*, 103, 908-916, 1993.

Spanos Aris, Revisiting Date Mining: ´Hunting´ With or Without a License, *Journal of Methodology*, July 2000.

Spanos Aris, On Normality and the Linear Regression Model, *Econometric Reviews*, 14,195-203, 1995.

Spanos Aris, On Theory Testing in Econometrics: Modeling with nonexperimental Data, *Journal of Econometrics*, 67, 189-226, 1995.

Spanos Aris, On Modeling Heteroscedasticity: The Student's *t* and Elliptical Linear Regression Models, *Econometric Theory*, 10, 286-315, 1994.