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ARE NASH TAX RATES TOO LOW OR TOO HIGH?
THE ROLE OF ENDOGENOUS GROWTH IN MODELS WITH PUBLIC GOODS*

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Abstract: We reconsider the conventional wisdom that, in the presence of public goods, Nash tax rates are too low. We use a general equilibrium dynamic model of a world economy, in which world-wide environmental quality has public good features. We show that the type of the spillover effect from one country to another (and hence whether we under-tax, or over-tax, in a Nash equilibrium relative to a coordinated one) can be reversed when we introduce dynamics. Specifically, the spillover effect changes from positive (which is the static, traditional case) to negative once the same model allows for long-term endogenous growth. This happens because in a growing economy, long-run capital tax bases are elastic, so that a higher tax rate leads to lower economic growth, smaller tax bases, lower tax revenues, lower clean-up policy in each country, and this eventually implies a negative spillover effect upon other countries. Then, negative spillovers imply that long-run Nash tax rates on polluting firms' output are too high.

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I. INTRODUCTION

Recall two popular results from the literature on public goods: First, when decision-making is decentralized and voluntary, Nash tax rates - and the associated private provision of public goods - are suboptimally low. Hence, when we switch from Nash to coordinated decision-making, tax rates increase and this is welfare improving.[1] Second, the size of under-provision of public goods increases with the size of population. That is, Nash tax rates decrease with the size of population.[2] This is because as the number of participants increases, the incentive to free ride on the supply provided by others becomes stronger and so the willingness to pay taxes decreases.

In terms of externalities in the game formulation, the above two results presuppose that the spillover effect from one agent to another is *positive*. That is, an increase in agent j's action leads to an external welfare benefit upon the remaining agent $i \neq j$. For instance, in the context of public goods, an increase in j's tax rate leads to higher tax revenues and hence higher public good provision for $i \neq j$. Then, in the presence of positive spillovers, players' actions increase when we switch from an uncoordinated (i.e. Nash) equilibrium to a coordinated (i.e. cooperative) equilibrium. See e.g. Cooper and John [1988].[3]

This paper shows that the type of the spillover effect (and hence whether we under-tax, or over-tax, in a Nash equilibrium relative to a coordinated one) can be reversed, when we introduce dynamics into a fairly standard model with public goods. In particular, the nature of spillover effect *changes from positive to negative*, once the same model allows for long-term endogenous growth. As a result, Nash tax rates are

[1] This is when coordinating or centralized tax authorities act as benevolent planners.
[2] For instance, Mueller [1989, chapter 2] points out that "there is an under-provision whose relative size grows with the size of the community" (see his equation (2.20) for individual voluntary contribution). Oakland [1987, p. 514] also states that "when the number of participants is large, … the outcome will diverge further from the efficient outcome". Thus, in the "basic" model of public goods, the relation between the Nash tax rate and population size is monotonically negative (for details, see our subsection V.1 below). On the other hand, in "richer" models, the relation between population size and success in providing public goods is not clear depending on the nature of the public good, the nature of payoffs, etc (for a survey, see Drazen [2000, subsection 9.4]).
[3] As Cooper and John [1988] show, when they focus on symmetric equilibria, in the presence of positive (resp. negative) spillovers, players' actions increase (resp. decrease) when we switch from Nash to cooperative equilibria. This is because coordinated decision-making enables agents to internalize the positive (resp. negative) spillover effects.

inefficiently high in the long run. They also increase with the size of population. Therefore, although decentralized voluntary decision-making leads to too low tax rates in a static economy, it can lead to too high tax rates (and hence too small tax bases) in a dynamic growing economy. In both the static and dynamic cases, there are too little tax revenues allocated to public goods provision in a Nash equilibrium.[4]

We focus on that category of public goods whose quality is damaged by economic activity, but can be improved by policy intervention.[5] That is, the government taxes the activity of agents who generate negative externalities, and then uses the collected tax revenues to maintain the public good. A classic example is the environment.[6] We therefore use an endogenous growth model where the public good is renewable natural resources. Also, since we want to compare non-cooperative and cooperative equilibria, we choose to work in the context of a world economy composed of a number of countries. In each country, the national government sets its environmental policy by either playing Nash or cooperating with other national governments. We believe that our setup is fairly general.[7]

The model is as follows. Consider a world economy composed of a number of countries. In each country, private agents consume, save in domestic capital[8] and produce

[4] As Baumol and Oates [1988, p. 99] point out, "imagine a community in which police protection against crime is provided by private policemen hired by individuals who can afford it. In that case, optimality may well require more police protection, but it need not call for the hiring of more policemen". This is what happens in our dynamic growing economy. Baumol and Oates [1988, chapter 7] also show that several conventional results can break down when one introduces more than one externality-generating private activity. Here, our results are driven by long-term endogenous growth and not by the number of such activities.

[5] For various categories of public goods, see e.g. Oakland [1987] and Cornes and Sandler [1996, chapter 3].

[6] Environmental quality (i.e. water, air and soil quality) has been one of the most popular examples of public goods, because of the obvious open-access character of natural resources (see e.g. Baumol and Oates [1988] and Cornes and Sandler [1996, chapter 3]). However, our results apply to any public good whose provision is reduced by private economic activity, but the government can contribute to its maintenance (e.g. public facilities, roadways, national parks, health).

[7] For instance, we could alternatively use a closed economy, in which individuals (rather than nations) choose their contributions to economy-wide clean-up policy in a decentralized way. By contrast, in a cooperative equilibrium, a mechanism (e.g. the government) coordinates individuals' actions. The results would be conceptually the same.

[8] For simplicity, factors of production are internationally immobile.Thus, there is only one spillover effect from one player/country to another, and this arises from the public-good character of the environment. This allows us to focus on the main issue, which is the efficient management of public good provision in static and dynamic setups. If factors were internationally mobile, players/countries would also compete with each other for mobile tax bases. This would add spillover effects, which could mitigate or exacerbate our results depending on their nature (i.e. whether the additional spillover effects are positive or negative). For instance, in the literature on international capital mobility and tax competition, the spillover effect is

goods by using a linear Ak technology.[9] Pollution occurs as a by-product of output produced. That is, private production degrades environmental resources, but clean-up policy can improve them. Clean-up policy is financed by taxes on polluting firms' output. Then, world environmental resources (which enter the private agents' utility function in each country as a pure public good) are the sum of national environmental resources. Benevolent national governments decide what pollution tax rate their citizens are going to pay for clean-up policy, and hence how much of the public good (i.e. renewable world environmental resources) will be eventually provided. In doing so, national governments either play Nash vis-à-vis each other, or they cooperate. We focus on symmetric equilibria in national policies. Within this framework, we solve for a long-run equilibrium in which the economy can grow at a constant positive rate without damaging the environment. This constant rate is a function of economic policy, as in Barro's [1990] model of endogenous growth and fiscal policy.

Our main results are as follows. In a static setup, capital tax bases are exogenously given. This implies a *positive* spillover from one country/player to another. Namely, a rise in country j's tax rate leads always to higher tax revenues and higher clean-up policy in j, and this is also good for environmental quality in country $i \neq j$. Then, a positive spillover implies that the Nash tax rate is inefficiently low. Also, this tax rate decreases with the size of population. These are the standard results.

In contrast, in a dynamic setup with long-term endogenous growth, capital tax bases are elastic. Now, although higher tax rates can lead to higher tax revenues and clean-up policy in the very short run, they discourage private capital accumulation and economic growth in the long run. In this case, the implications of lower economic activity can generate a *negative* spillover from one country to another. Namely, a rise in country j's tax rate leads to lower capital accumulation, smaller tax bases, lower tax revenues, lower clean-up policy, and eventually a lower natural resources-to-capital ratio in j. The latter (i.e. a lower natural resources-to-capital ratio in j) implies an external cost upon

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positive: an increase in country j's tax rate stimulates growth in country $i \neq j$. Then, Nash tax rates are inefficiently low.

[9] The Ak model is the simplest and one of the most popular models of long-term endogenous growth. See e.g. Barro and Sala-i-Martin [1995, chapter 4] and Jones and Manuelli [1997].

country $i \neq j$. Thus, the cross-country spillover effect is negative. Hence, the Nash tax rate is inefficiently high. It also increases with the size of population. However, we also show that if clean-up technology is relatively good and/or economic activity shifts in the direction of less pollution-intensive goods, a rise in country j's tax rate can eventually lead to a higher natural resources-to-capital ratio in j (despite the fall in tax revenues), and hence generate a positive spillover effect. Then, the Nash tax rate is too low and decreases with the size of population, as in the conventional static case.

Consequently, in a dynamic setup, the decentralized world economy can end up with too high tax rates on polluting producers, relative to the case in which there is centralized decision-making. The intuition is as follows. Without coordination, national policymakers do not internalize the harmful effect of their own tax rates (and hence low economic growth, small tax bases, low clean-up policy and low nature-to-capital ratio at home) on the provision of world-wide environmental quality. They therefore set too high tax rates on polluting producers. A switch to coordination will lead to lower tax rates, and this is good for both economic growth and the environment.

Therefore, while in the short run higher tax rates on polluting firms' output are good for the environment but at the cost of lower consumption, in the long run higher tax rates can be bad for both economic growth and the environment. That is, economies that achieve a sustained growth path can also afford a better environmental quality (despite the adverse effect of higher growth and pollution). This is a known result (see e.g. John and Pecchenino [1994]).[10] However, here we show something more, when we introduce public good type externalities and optimal maintenance policy: the type of the spillover effect (and hence whether we under-tax, or over-tax, in a Nash equilibrium relative to a coordinated one) can be reversed once there is long-term endogenous growth.

[10] Grossman and Krueger [1995], Cavlovic et al [2000] and Bradford et al [2000] provide empirical support for this prediction, i.e. the link between income level and environmental quality is not monotonic. Also, Aghion and Howitt [1998, p. 151], although they focus on R&D activities, they make an analogous point when they say that "it is roughly consistent with historical experience in industrialized countries that the chances of achieving sustainable growth depend critically on maintaining a steady flow of technological innovations". All this is consistent with the general view that the best way to lower poverty or promote equity on an enduring basis is to enhance prospects for sustained economic growth. For the link among economic growth, natural resources and economic policy, see among others Tahvonen and Kuuluvainen [1993], Lighthart and van der Ploeg [1994], Bovenberg and Smulders [1995] and Nielsen et al [1995].

The rest of the paper is organized as follows. Section II presents the model and solves for a world competitive equilibrium. Section III solves for Nash national policies. Section IV solves for coordinated national policies. Section V presents special cases and extensions. Section VI summarizes and concludes. Mathematical proofs are gathered in an Appendix.

II. THE MODEL AND COMPETITIVE EQUILIBRIUM

Consider a world economy composed of a finite number of countries, $i = 1,2,...,I$. Each country i is populated by a representative private agent and a national government.[11] We assume continuous time, infinite horizons and perfect foresight. In each time-period, national governments move first by choosing economic policy, and then private agents make their decisions.[12] This section solves for the private agents' optimization problem and a world competitive equilibrium, given economic policy. The next two sections will endogenize economic policy.

II. 1 Private agents

The representative private agent in each country i gets utility from private consumption, c^i, and the average stock of natural resources across countries, $\overline{N}$. That is, $\overline{N} \equiv \frac{\sum_{i=1}^{I} N^i}{I}$, where N^i denotes the stock of natural resources in country i.[13] The infinite-lived private agent maximizes intertemporal utility given by:

$$\int_0^{\infty} [u(c^i, \overline{N})]e^{-\rho t} dt \qquad (1)$$

[11] Our results do not change if we use a decentralized setup with households and firms in each country.
[12] See below in footnote 17 for more details.
[13] Our results do not change if we use the aggregate stock, $\sum_{i=1}^{I} N^i$, in (1)-(2) so that there is no congestion. Results are available upon request.

where the parameter $r > 0$ is the rate of time preference. The function $u(.)$ is increasing and concave in its two arguments, and also satisfies the Inada conditions. For algebraic simplicity, we use an additively separable and logarithmic function:

$$u(c^i, \overline{N}) = \log c^i + n \log \overline{N} \tag{2}$$

where the parameter $n > 0$ is the weight given to natural resources relative to private consumption.

Private agents produce goods by using a Ak technology. That is, in each country i, output, y^i, is linear in capital, k^i:

$$y^i = Ak^i \tag{3}$$

where $A > 0$ is a parameter.

If $0 \leq q^i < 1$ is a proportional tax on output, the budget constraint of the representative private agent in each country i is:

$$\dot{k}^i + c^i = (1 - q^i)Ak^i \tag{4}$$

where a dot over a variable denotes a time derivative. Initial capital, k_0^i, is given.

Private agents act competitively by taking tax policy, q^i, and public goods provision, $\overline{N}$, as given. Then, the first-order conditions for a maximum are the budget constraint in (4) and the familiar Euler equation:

$$\dot{c}^i = [(1 - q^i)A - r]c^i \tag{5}$$

II. 2 Government budget constraint

Each national government i finances its clean-up policy, g^i, by taxes on domestic output, $q^i y^i$. Thus, at any point of time, the government's balanced budget is:

$$g^i = q^i y^i \tag{6}$$

II. 3 Natural resources and pollution

The motion of natural resources in each country i, N^i, is:

$$\dot{N}^i = dN^i - p^i + g^i \tag{7}$$

where the parameter $d > 0$ is the rate of regeneration of natural resources. That is, natural resources increase over time with natural regeneration, dN^i, and clean-up policy, g^i, but they decrease with pollution emission, p^i. Initial resources, N_0^i, are given.

Pollution, p^i, is a by-product of output produced, y^i.[14] Specifically, we assume:

$$p^i = y^i \tag{8}$$

that is, for simplicity, one unit of output generates one unit of pollution.[15] See below in subsection V.2 for a richer specification of (7)-(8).

Using (3), (6) and (8) into (7), the motion of natural resources in each i becomes:

$$\dot{N}^i = dN^i - (1-q^i)Ak^i \tag{9}$$

[14] Our results do not change if pollution is also a by-product of consumption. On the other hand, this is different from the case of resource extraction, where natural resources are extracted from preserved natural environments to be used as inputs in production. Irrespectively of whether we treat the generation of pollution as a by-product of economic activity or as resource extraction, the main issue is the public-good character of the environment and its efficient management.

[15] Equation (8) also implies that, in our model, output taxes act as pollution taxes.

II. 4 World competitive equilibrium: summary

In this section, we have solved for a World Competitive Equilibrium (WCE). In a WCE: (i) Private decisions maximize utility [this is summarized by equations (4) and (5)]. (ii) All constraints are satisfied and all markets clear [this is summarized by equation (9)]. This WCE holds for any initial conditions and any feasible economic policy, where the latter is summarized by national tax rates on polluting firms' output, θ^i .

Since private agents have not internalized the effects of their decisions on the public good, the WCE is inefficient. This provides a rationale for government intervention: namely, the government taxes the externality-generating activity and uses the collected tax revenues to finance the maintenance of the public good. Therefore, the next sections will endogenize θ^i by assuming that national governments act as benevolent Stackelberg leaders vis-à-vis private agents. Also, in section III national governments play Nash vis-à-vis each other, while in section IV they coordinate their policies.

III. UNCOORDINATED (NASH) ECONOMIC POLICIES

This section solves for a Nash game among benevolent national governments. Each national government i maximizes the utility of its own private agent by taking into account the WCE specified above and by taking as given the policies of other national governments $j \neq i$. Thus, each i chooses θ^i, c^i, k^i, N^i to maximize (1) in the form of (2), subject to equations (4), (5) and (9), and by taking as given θ^j, c^j, k^j, N^j, where $j \neq i$.[16] The current-value Hamiltonian, H^i, of government i is:

$$H^i \equiv \log c^i + \nu \log \overline{N} + \lambda^i c^i \left[(1-\theta^i)A - \rho\right] + \gamma^i \left[(1-\theta^i)Ak^i - c^i\right] + \mu^i \left[\delta N^i - (1-\theta^i)Ak^i\right] \tag{10}$$

[16] Here, we use the so-called "primal" approach to optimal policy, according to which the Stackelberg government chooses private allocations again (and of course its policy instruments). A famous example is Chamley [1986]. This is equivalent to the "dual" approach to optimal policy, according to which the

where λ^i, γ^i and μ^i are dynamic multipliers associated with the constraints (5), (4) and (9) respectively. Thus, λ^i is the social price of private consumption, γ^i is the social price of capital, and μ^i is the social price of natural resources in country i.[17]

III. 1 Symmetric Nash equilibrium

We will focus on Symmetric Nash Equilibria (SNE) in policy strategies. At a SNE, strategies are symmetric *ex-post.* Thus, ex post $x^i = x^j \equiv x$, where $i \neq j$ and $x \equiv (\theta, c, k, N, \gamma, \lambda, \mu)$.[18] Invoking symmetry into the first-order conditions for $\theta^i, c^i, \lambda^i, \gamma^i, k^i, \mu^i, N^i$, we get respectively (we now omit the country-superscript i):

$$\lambda c + \gamma k = \mu k \tag{11a}$$

$$\dot{\lambda} = \rho \lambda - \frac{1}{c} - \lambda\left[(1-\theta)A - \rho\right] + \gamma \tag{11b}$$

$$\dot{c} = \left[(1-\theta)A - \rho\right]c \tag{11c}$$

$$\dot{k} = (1-\theta)Ak - c \tag{11d}$$

$$\dot{\gamma} = \rho\gamma - (1-\theta)A\gamma + (1-\theta)A\mu \tag{11e}$$

$$\dot{N} = \delta N - (1-\theta)Ak \tag{11f}$$

$$\dot{\mu} = \rho\mu - \frac{\hat{\nu}}{N} - \delta\mu \tag{11g}$$

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government maximizes an indirect utility function with respect to its policy instruments only. See e.g. Glomm and Ravikumar [1997] for a discussion.

[17] The way we solve the government's optimization problem is as if we have assumed commitment technologies on behalf of governments. However, with an Ak technology and a log-linear utility function, the optimal tax rate is constant over time (see below) and hence there are no time inconsistency problems a la Chamley [1986]. In other words, in this setup, commitment and non-commitment equilibria coincide, as in e.g. Barro [1990]. For details, see Benhabib, Rustichini and Velasco [1996] and Benhabib and Velasco [1996].

[18] Focusing on symmetric equilibria is not too restrictive for what we want to do here (i.e. to examine how spillover effects and incentives are affected depending on whether the setup is dynamic or static). On the other hand, we cannot study e.g. the interaction between industrialized and non-industrialized countries.

where $\hat{n} \equiv \frac{n}{I}$ denotes the "effective" weight given to environmental quality relative to private consumption (recall that I is the exogenous number of countries).

These necessary conditions are completed with the addition of a transversality condition that guarantees utility is bounded. A sufficient condition for this to hold is:

$$[(1-q)A - r] + d < r \qquad (12)$$

so that the growth rate of consumption, $[(1-q)A - r]$, plus the rate of regeneration of natural resources, d, is less than the rate of time preference, r.

Following usual practice, we transform the variables to facilitate analytical tractability. Let us define $z \equiv \frac{c}{k}$, $y \equiv mk$ and $f \equiv mN$. Appendix A shows that the dynamics of (11a)-(11g) are equivalent to the dynamics of (13a)-(13d) below:

$$\dot{z} = (z - r)z \qquad (13a)$$

$$\dot{y} = \left[(1-q)A - z + r - \frac{\hat{n}}{f} - d\right]y \qquad (13b)$$

$$\dot{f} = \left[r - \frac{\hat{n}}{f} - \frac{(1-q)Ay}{f}\right]f \qquad (13c)$$

$$\left[z + d + \frac{\hat{n}}{f}\right]y = 1 \qquad (13d)$$

where (13a)-(13d) constitute a system in z, y, f, q. Note that (13d) is static. Thus, the dynamics of q follow from the dynamics of z, y, f.

To sum up, we have solved for a Symmetric Nash Equilibrium in tax rates among national policymakers. This equilibrium is summarized by equations (13a)-(13d) and the terminal condition (12).

III. 2 Long-run symmetric Nash equilibrium: solution and properties

This subsection solves for a long-run Nash equilibrium. We will study Sustainable Balanced Growth Paths (SBGPs). That is, long-run equilibria in which consumption and capital can grow at a constant positive rate without damaging the environment.[19]

Specifically, we look for a long-run equilibrium of (13a)-(13d) in which $\dot{z} = \dot{\psi} = \dot{\phi} \equiv 0$. Since $z \equiv \frac{c}{k}$, $\psi \equiv mk$ and $\phi \equiv mN$, conditions $\dot{z} = \dot{\psi} = \dot{\phi} \equiv 0$ imply that natural resources can grow at the same constant positive rate with capital and consumption.[20] This is typical of Ak-type growth models in which all per capita quantities grow at the same positive rate.[21] Notice three things: First, renewable natural resources can indeed grow, especially when there is maintenance policy and regeneration, as it is the case here (see e.g. soil, water and air quality). Second, the common rate at which c, k and N can grow in the long run (called SBGP) is a function of economic policy, θ, where the latter has been optimally chosen. This is as in Barro [1990]-type endogenous growth models. Third, the link between the Nash tax rate and the SBGP is negative in the long run (see below for intuition).[22]

Let us denote the long-run equilibrium values of (z, ψ, ϕ, θ) by $(\tilde{z}, \tilde{\psi}, \tilde{\phi}, \tilde{\theta})$. Appendix B solves for $(\tilde{z}, \tilde{\psi}, \tilde{\phi}, \tilde{\theta})$ and proves existence and uniqueness. The solution for $\tilde{\theta}$ will then give us a solution for the SBGP via e.g. equation (11c) above. The main results are as follows:

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[19] See e.g. Aghion and Howitt [1998, chapter 5] and the references cited therein for the notion of sustainable development.

[20] Since $z \equiv \frac{c}{k}$, $\dot{z} = 0$ implies that c and k grow at the same rate. Since $\psi \equiv mk$ and $\phi \equiv mN$, $\dot{\psi} = \dot{\phi} = 0$ implies that N and k also grow at the same rate. Therefore, c, k and N grow at the same rate, i.e. $\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{N}}{N}$.

[21] See e.g. Barro and Sala-i-Martin [1995, chapter 4].

[22] This can be seen, for instance, by using (11c). Thus, $\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{N}}{N} = A(1-\tilde{\theta}) - \rho$, so that the link between $\tilde{\theta}$ and the SBGP is negative.

PROPOSITION 1: Focusing on symmetric Nash equilibria in national policies, if the parameter values satisfy the conditions:

$$A > \boldsymbol{r} + \boldsymbol{d} \qquad (14a)$$

$$\boldsymbol{r} > 2\boldsymbol{d} \qquad (14b)$$

$$2\hat{\boldsymbol{n}}\boldsymbol{d} > \boldsymbol{r} - \boldsymbol{d} \qquad (14c)$$

there exists a unique long-run tax rate on polluting firms' output, denoted by $\tilde{\boldsymbol{q}}^{nash}$*. This tax rate lies in the region* $0 < 1 - \frac{\boldsymbol{r}+\boldsymbol{d}}{A} < \tilde{\boldsymbol{q}}^{nash} < 1 - \frac{\boldsymbol{r}}{A} < 1$*, and is a solution to:*

$$\hat{\boldsymbol{n}}\left[\boldsymbol{r} + \boldsymbol{d} - (1-\tilde{\boldsymbol{q}})A\right]\left[\boldsymbol{r} + (1-\tilde{\boldsymbol{q}})A\right] = (1-\tilde{\boldsymbol{q}})A\left[(1-\tilde{\boldsymbol{q}})A - \boldsymbol{d}\right] \qquad (15)$$

This tax rate supports a unique balanced growth path in which consumption, capital and renewable natural resources can grow at the same constant positive rate. Hence, this is a Sustainable Balanced Growth Path (SBGP).

Proof: See Appendix B.[23]

(15) is an equation in $\tilde{\boldsymbol{q}}$ only. Total differentiation implies that the long-run Nash tax rate on polluting firms' output, $\tilde{\boldsymbol{q}}$, is:[24]

$$\tilde{\boldsymbol{q}} = \boldsymbol{q}(\overset{-}{\boldsymbol{d}}, \overset{-}{\boldsymbol{r}}, \overset{+}{A}, \overset{-}{\hat{\boldsymbol{n}}}) \qquad (16)$$

[23] Conditions (14a)-(14c) are jointly sufficient for a well-defined and unique long-run equilibrium to exist. The details are in Appendix B. Here, we just point out that (14a) requires the productivity of private capital, A, to be higher than the rate of time preference, $\boldsymbol{r}$, plus the regeneration rate of natural resources, $\boldsymbol{d}$. This is a familiar condition for long-term growth (see Barro and Sala-i-Martin [1995, p. 142]), but here we require a stronger condition than usually because the economy must also devote resources to clean-up policy. Condition (14b) guarantees that (12) holds and so the attainable utility is bounded (see Barro and Sala-i-Martin [1995, chapter 2]). Condition (14c) implies that existence gets easier when: (i) The rate of regeneration of natural resources, $\boldsymbol{d}$, increases. (ii) Agents care about the future, i.e. $\boldsymbol{r}$ is low. (iii) Agents value environmental quality, i.e. $\hat{\boldsymbol{n}}$ is high. (iv) The size of population, I, decreases (recall that $\hat{\boldsymbol{n}} \equiv \frac{\boldsymbol{n}}{I}$). Point (iv) is a familiar result in the literature on public goods: as the size of population increases, any problems associated with decentralized (i.e. Nash) decision-making become worse (see also below).

[24] Signs above parameters give equilibrium properties.

Thus, among other comparative static results,[25] the more economic agents value environmental quality relative to private consumption (i.e. the higher is $\hat{n}$), the lower is the optimal tax rate. This seemingly counter-intuitive result is explained in detail below. The main idea is that, when agents value environmental quality, the government needs resources, or tax revenues, to finance its clean-up policy, and this can be achieved only by large tax bases.

Having solved for the long-run Nash tax rate, $\tilde{q}$, we can now solve for the Sustainable Balanced Growth Path (SBGP). As we said above, the properties of the SBGP are symmetrically opposite from those of $\tilde{q}$.[26] In other words, any changes in the exogenous factors (i.e. $d, r, A, \hat{n}$ as shown in equation (16) above), that cause a rise (resp. fall) in the Nash tax rate on polluting firms' output, lead not only to lower (resp. higher) rates of economic growth but also to deteriorating (resp. improving) environmental quality. Intuitively, in a dynamic setup, economic activity, pollution (which is a by-product of economic activity) and capital tax bases are all endogenous in the long run. Although higher tax rates can lead to higher tax revenues and higher clean-up policy in the short run, they discourage private capital accumulation and economic growth. Hence, in the long run, higher tax rates lead to smaller tax bases, which reduce the ability to engage in clean-up policy. Eventually, higher tax rates imply a deterioration in environmental quality (despite the fact that lower economic activity brings down pollution emissions).[27]

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[25] That is, we also have: (i) The easier natural resources regenerate themselves (i.e. the higher is d), the smaller the tax rate. (ii) The more we care about the future (i.e. the lower is r), the higher is the tax rate. The idea is that a low r leads to high growth, so that a higher $\tilde{q}$ is needed to slow growth down and make utility bounded (see e.g. Barro and Sala-i-Martin [1995, chapter 2]). (iii) When the productivity of private capital is high (i.e. A is high), we can afford higher tax rates and lower economic growth.

[26] Recall that $\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{N}}{N} = A(1-\tilde{q}) - r$.

[27] Using a model with richer production technology and transitional dynamics, John and Pecchenino [1994] show the possibility of multiple steady states, some of which are characterized by high growth and high environmental quality, and some of which are characterized by low growth and bad environmental quality. In our simpler Ak model, we get a unique equilibrium in which higher tax rates on polluting firms' output are bad for both economic growth and the environment in the long run.

The above are also reflected in a negative link between (any given) Nash tax rate, $\tilde{q}$, and the nature-to-capital ratio, $\frac{\tilde{N}}{\tilde{k}}$, in the long run:[28]

$$\frac{\tilde{k}}{\tilde{N}} = \frac{d + r - A(1-\tilde{q})}{A(1-\tilde{q})} \tag{17a}$$

as well as a negative link between $\tilde{q}$ and the nature-to-consumption ratio, $\frac{\tilde{N}}{\tilde{c}}$, in the long run:[29]

$$\frac{\tilde{c}}{\tilde{N}} = r\left[\frac{d + r - A(1-\tilde{q})}{A(1-\tilde{q})}\right] \tag{17b}$$

so that a high tax rate is bad for public goods provision vis-a-vis private goods, because a higher tax rate harms tax bases.[30]

The above are intuitive results. However, notice the effect of the number of countries, I, on equilibrium outcomes. Since $\hat{n} \equiv \frac{n}{I}$, (16) implies that an increase in I leads ceteris paribus to an increase in the long-run Nash tax rate. Such a positive effect from the size of population on the Nash tax rate seems to be opposite from the standard one, which is traditionally negative (see the discussion in the Introduction). As we argue below, this seemingly paradoxical result arises because the cross-country spillover effect

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[28] Since $\frac{\dot{c}}{c} = \frac{\dot{N}}{N}$ in the long run, (11c) and (11f) imply $A(1-\tilde{q}) - r = d - A(1-\tilde{q})\frac{\tilde{k}}{\tilde{N}}$, which gives (17a).

[29] Since $\frac{\tilde{k}}{\tilde{N}} = \frac{d + r}{A(1-\tilde{q})} - 1$ and $\frac{\tilde{c}}{\tilde{k}} = r$ in the long run, we get (17b).

[30] It is important that there is endogenous growth. If the production function in (3) is of Cobb-Douglas type, so that the solution does not display balanced growth in the long run i.e. $\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{N}}{N} \equiv 0$, then the ratios $\frac{\tilde{k}}{\tilde{N}}$ and $\frac{\tilde{c}}{\tilde{N}}$ in (17a)-(17b) are independent of tax policy, $\tilde{q}$, and the interaction among countries/players.

is in fact negative. Then, a negative spillover effect can justify the positive effect of I on Nash tax rates.

To confirm the above, we also have to solve for a coordinated equilibrium. Before we do so, we check whether the above long-run equilibrium is dynamically stable. Appendix C shows that this is the case indeed. In particular, there are no transitional dynamics and the economy jumps immediately to its long-run equilibrium, as in the basic Ak model.

IV. COORDINATED ECONOMIC POLICIES

We now solve for a coordinated or cooperative equilibrium defined as the solution to the problem of maximizing the sum of national utilities.[31] That is, now a hypothetical benevolent social planner chooses jointly all q^i, c^i, k^i, N^i for $i = 1,2,...,I$ to maximize the sum of national utilities (1) in the form of (2), subject to equations (4), (5) and (9) for each i. This coordinated equilibrium is Pareto-optimal. The current-value Hamiltonian, H, is:

$$H \equiv \sum_i \log c^i + \sum_i n \log \overline{N} + \sum_i l^i c^i [(1-q^i)A - r] + \sum_i g^i [(1-q^i)Ak^i - c^i]$$
$$+ \sum_i m^i [dN^i - (1-q^i)Ak^i] \qquad (18)$$

where l^i, g^i and m^i are new multipliers associated with (5), (4) and (9) respectively for each i.

IV.1 Symmetric cooperative equilibrium

We will work exactly as in the previous section. That is, we will first derive the first-order conditions for $q^i, c^i, l^i, g^i, k^i, m^i, N^i$ and then focus on Symmetric

[31] Alternatively, this equilibrium could be defined as the solution to the problem of maximizing the welfare of a representative country; this would be equivalent. Also, note that we take this structure as given, i.e. we do not worry about coalition formation or stability.

Cooperative Equilibria (SCE) in national policies. Then, instead of equations (11a)-(11g), we get:

$$lc + gk = mk \tag{19a}$$

$$\dot{l} = rl - \frac{1}{c} - l\left[(1-q)A - r\right] + g \tag{19b}$$

$$\dot{c} = \left[(1-q)A - r\right]c \tag{19c}$$

$$\dot{k} = (1-q)Ak - c \tag{19d}$$

$$\dot{g} = rg - (1-q)Ag + (1-q)Am \tag{19e}$$

$$\dot{N} = dN - (1-q)Ak \tag{19f}$$

$$\dot{m} = rm - \frac{n}{N} - dm \tag{19g}$$

Observe that (19a)-(19g) differ from (11a)-(11g) only in that we now have n instead of $\hat{n} \equiv \frac{n}{I}$. To understand this, recall that the only externality present is due to public good provision, i.e. $\overline{N} \equiv \frac{\sum_{i=1}^{I} N^i}{I}$. In a cooperative equilibrium (in which all actions are coordinated and all externalities are internalized), equilibrium outcomes are not affected by such public-good type problems.

In turn, we have in place of equations (13a)-(13d):

$$\dot{z} = (z - r)z \tag{20a}$$

$$\dot{y} = \left[(1-q)A - z + r - \frac{n}{f} - d\right]y \tag{20b}$$

$$\dot{f} = \left[r - \frac{n}{f} - \frac{(1-q)Ay}{f}\right]f \tag{20c}$$

$$\left[z + d + \frac{n}{f}\right] y = 1 \tag{20d}$$

Therefore, (20a)-(20d), together with the terminal condition (12), summarize a Symmetric Cooperative Equilibrium in tax rates among national policymakers.

IV. 2 Long-run symmetric cooperative equilibrium: solution and properties

We again solve for a long-run equilibrium in which $\dot{z} = \dot{y} = \dot{f} \equiv 0$. Let $\left(\tilde{z},\ \tilde{y},\ \tilde{f},\ \tilde{q}\right)$ denote the new long-run values in (20a)-(20d). Since the only difference between (20a)-(20d) and (13a)-(13d) is that we now have n instead of $\hat{n} \equiv \frac{n}{I}$, Proposition 1 above still holds if we simply replace $\hat{n}$ with n. In other words, the cooperative long-run tax rate denoted by $\tilde{q}^{coop}$ lies in the region $0 < 1 - \frac{r+d}{A} < \tilde{q}^{coop} < 1 - \frac{r}{A} < 1$ and is a solution to:

$$n\left[r + d - (1-\tilde{q})A\right]\left[r + (1-\tilde{q})A\right] = (1-\tilde{q})A\left[(1-\tilde{q})A - d\right] \tag{21}$$

The functional forms in (15) and (21) are exactly the same with n instead of $\hat{n}$ in the latter. Since $n > \hat{n} \equiv \frac{n}{I}$, the comparative static results in (16) imply that the optimal tax rate decreases when we switch from non-cooperation to cooperation, i.e. $\tilde{q}^{nash} > \tilde{q}^{coop}$. As we said above this is different from the standard result, and can happen only when the spillover effect (arising from the presence of public goods in the form of world-wide natural resources) from one country/player to another is negative.

IV. 3 Interpretation of results

To understand our results, it is convenient to discuss first the static, special case (the formal proof of this case will be given in subsection V.1 below). In a static setup, capital tax bases are exogenously given. Then, an increase in tax rates leads always to an increase in tax revenues and hence an increase in resources available for clean-up policy. This direct revenue effect implies a positive spillover effect from one country/player to another. That is, an increase in country j's tax rate leads to higher tax revenues and higher clean-up policy in j, and this is good for world-wide environmental quality. In other words, there is an external welfare benefit upon country $i \neq j$. In turn, a positive spillover implies that there are inefficiently low tax rates in a Nash equilibrium.

In contrast, in a setup with endogenous growth, economic activity, pollution and capital tax bases are all endogenous in the long run. As a result, as shown by equations (17a)-(17b) above, there is a negative link between the tax rate on polluting firms' output ($\tilde{q}$) on the one hand, and the nature-to-capital and the nature-to-consumption ratios ($\frac{\tilde{N}}{\tilde{k}}$ and $\frac{\tilde{N}}{\tilde{c}}$) on the other hand. This is what happens in each country, given the rest of the world. In turn, in terms of cross-country externalities, national tax policy generates a negative spillover from one country to another. An increase in country j's tax rate leads to lower capital accumulation, smaller tax bases, lower tax revenues, lower clean-up policy and lower nature-to-capital ratio in j; this is bad for world-wide environmental quality. In other words, there is an external welfare cost upon country $i \neq j$. Since the spillover effect is negative, the Nash tax rate is inefficiently high, and this inefficiency increases with the size of population.

V. SPECIAL CASES AND EXTENSIONS

To support our results, we will now study a special case and an extension.

V. 1 Standard results in a static special case of our model

We first show that a static version of our model can produce the standard results.

If we set $\dot{k} \equiv 0$, equation (4) gives for consumption in each i:

$$c^i = (1-q^i)Ak_0^i \tag{22}$$

while equation (9) gives for the end-of-period natural resources in each i:

$$N^{i} = (1+\boldsymbol{d})N_{0}^{i} - (1-\boldsymbol{q}^{i})Ak_{0}^{i} \tag{23}$$

where k_0^i and N_0^i are given initial values.

In a Nash game, each national government i chooses $\boldsymbol{q}^i$ to maximize:

$$\log c^{i} + \boldsymbol{n} \log \overline{N} \tag{24}$$

where c^i and N^i are given by (22) and (23) respectively, and also $\overline{N} \equiv \dfrac{\sum_{i=1}^{I} N^{i}}{I}$. Then, in a symmetric Nash equilibrium, the first-order condition is:

$$\frac{N}{c} = \hat{\boldsymbol{n}} \tag{25}$$

which is a well-known condition (see e.g. Mueller [1989, chapter 2] and Mas-Colell et al. [1995, chapter 11]).

Using (22) and (23) for c^i and N^i into (25) and totally differentiating, we get:

$$\boldsymbol{q} = \boldsymbol{q}(\overset{-}{\boldsymbol{d}}, \overset{+}{A}, \overset{+}{\hat{\boldsymbol{n}}}, \overset{+}{k_0}, \overset{-}{N_0}) \tag{26}$$

so that, since $\hat{\boldsymbol{n}} \equiv \dfrac{\boldsymbol{n}}{I}$, the Nash tax rate decreases with group size, I, in the static case (to further confirm this result and our claims in the Introduction, we also present a textbook model with public goods in Appendix D).

In a coordinated game, the hypothetical social planner chooses $\boldsymbol{q}^i$ to maximize:

$$\sum_i \log c^i + \sum_i \nu \log \overline{N} \qquad (27)$$

so that, in a symmetric cooperative equilibrium, the first-order condition is:

$$\frac{N}{c} = \nu \qquad (28)$$

which is the Samuelson condition for Pareto optimality. Therefore, using the comparative static properties in (26) and since $\nu > \hat{\nu} \equiv \frac{\nu}{I}$, inspection of (28) and (25) reveals that $\tilde{\theta}^{nash} < \tilde{\theta}^{coop}$. That is, in the static case, a switch to coordination will lead to a higher tax rate, higher tax revenues, more clean-up policy and better environment.

In summary, when the model is static, we get the standard results: Nash tax rates on public goods provision are too low and decrease with the size of population. By contrast, when there is long-term endogenous growth, these results are reversed. In particular, in the Ak model of endogenous growth presented in sections II-IV above, Nash tax rates on public good provision are too high and increase with the size of population.

Note that we have either studied “the very short run” where capital tax bases are fully inelastic (see the static case in this subsection), or “the very long run” where capital tax bases are fully elastic (see the growing economy in sections II-IV above). In other words, here we cannot get an inverse U-shaped *dynamic* Laffer curve, where a higher tax rate on polluting firms’ activity is bad for growth and good for the environment in the short run, but it starts getting bad for both growth and the environment as the time horizon becomes longer and tax bases become more elastic.[32] This is because we have used the Ak model of endogenous growth, in which the return to capital and the rate of growth are independent of the initial capital stock, and there are no transitional dynamics.

[32] We wish to emphasize that we are talking about a *dynamic* Laffer curve. That is, we are not referring to the relationship between income level and environmental quality as in e.g. Grossman and Krueger [1995]. Also, note that Agell and Persson [2000] use a Ak model similar to ours to study the implications of tax policy for long-term public expenditures. Their main results are not different from ours (see below for

In a more general model, where the state of the economy would matter more, we could possibly get a really dynamic Laffer curve.[33] Nevertheless, we wish to point out that: (i) The Ak model has enabled us to get analytical results that show clearly our main point; namely, the type of externalities present can be reversed once we introduce long-term endogenous growth. (ii) The Ak model is one of the most popular models of endogenous growth. (iii) This model is widely used in the literature on growth, natural resources and policy (see e.g. Lighthart and van der Ploeg [1994] and Nielsen et al [1995]).

V. 2 More general clean-up technologies

So far we have assumed one-for-one relationships between economic activity and pollution (see equation (8) above) and between resources used for clean-up policy and environmental improvement (see equations (6)-(7) above). We now generalize the model by assuming that the motion of natural resources in each country i changes from (9) to:

$$\dot{N}^i = dN^i - (e - pq^i)Ak^i \tag{29}$$

where the parameter $e > 0$ measures the degree of degradation of the environment as a result of production, while the parameter $p > 0$ measures the degree of environmental improvement when resources are used for clean-up policy (see also John and Pecchenino [1994, p. 1395]). In the previous sections II-IV, we had $e = p \equiv 1$.

Working exactly as above, (17a) and (17b) become respectively:

$$\frac{\tilde{k}}{\tilde{N}} = \frac{d + r - A(1-\tilde{q})}{A(e - p\tilde{q})} \tag{30a}$$

$$\frac{\tilde{c}}{\tilde{N}} = r\left[\frac{d + r - A(1-\tilde{q})}{A(e - p\tilde{q})}\right] \tag{30b}$$

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details). However, they use the term “dynamic Laffer curve”, even if they focus only on the long run, like us. We feel that a really dynamic analysis of the Laffer curve requires a model with transitional dynamics.

[33] We believe that more general results can follow if we modify the production function in (3) to take e.g. the form $y = Ak + Bk^b$, where $A > 0$, $B > 0$ and $0 < b < 1$ (see Barro and Sala-i-Martin [1995, chapter 4.5]). This function is a combination of the Ak and Cobb-Douglas functions; hence it still displays endogenous long-term growth but it also exhibits transitional dynamics as in the Ramsey model.

Equations (30a)-(30b) imply that the link between $\frac{\tilde{N}}{\tilde{k}}$ and $\tilde{q}$, and the link between $\frac{\tilde{N}}{\tilde{c}}$ and $\tilde{q}$, depend on the sign of $(e - p\tilde{q})$.[34] Note that it is this link that drives our results (see also below). That is, for any given Nash tax rate $\tilde{q}$, if $(e - p\tilde{q}) > 0$, $\frac{\tilde{N}}{\tilde{k}}$ and $\frac{\tilde{N}}{\tilde{c}}$ decrease with $\tilde{q}$; on the other hand, if $(e - p\tilde{q}) < 0$, $\frac{\tilde{N}}{\tilde{k}}$ and $\frac{\tilde{N}}{\tilde{c}}$ increase with $\tilde{q}$.

The intuition is as follows. When clean-up technologies are good and/or economic activity shifts in the direction of less pollution-intensive goods (i.e. $(e - p\tilde{q}) < 0$), then an increase in the tax rate on polluting firms' output may lead to lower growth and tax revenues, but can nevertheless improve the nature-to-capital and the nature-to-consumption ratios. By contrast, when clean-up technology cannot counter-balance the adverse effect of higher economic activity on the environment (i.e. $(e - p\tilde{q}) > 0$), then an increase in the tax rate on polluting firms' output leads unavoidably to a deterioration in the nature-to-capital and the nature-to-consumption ratios.[35]

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[34] (30a)-(30b) imply that the sign of $[d + r - (1 - \tilde{q})A]$ is the sign of $(e - p\tilde{q})$. That is, for any given tax rate $\tilde{q}$, if $(e - p\tilde{q}) > 0$, $[d + r - (1 - \tilde{q})A] > 0$; on the other hand, if $(e - p\tilde{q}) < 0$, $[d + r - (1 - \tilde{q})A] < 0$. The special case studied above ($e = p \equiv 1$) belongs to the former case, i.e. $(e - p\tilde{q}) > 0$. These results are useful for the comparative static exercises below.

[35] Agell and Persson [2000] also use an Ak model to show that the public consumption-to-output ratio in the long run (something like the $\frac{\tilde{N}}{\tilde{k}}$ ratio here) cannot be decreasing in the tax rate (thus, it is not possible a decrease in the tax rate to lead to an increase in the relative provision of public goods). Here, we get the same qualitative result with Agell and Persson when $(e - p\tilde{q}) < 0$ (this is consistent with Agell and Persson who set $e = 0$ and $p = 1$). However, when $(e - p\tilde{q}) > 0$, $\frac{\tilde{N}}{\tilde{k}}$ can be decreasing in $\tilde{q}$. The intuition behind this case (i.e. when $(e - p\tilde{q}) > 0$) is as follows: When the tax rate increases, lower economic activity has two opposite effects on the environment: on the one hand, it leads to smaller tax bases and lower revenues available for clean-up policy and public good provision in general; on the other hand, it brings down pollution emissions and this takes the pressure off the environment. The latter (i.e. low activity) gives an incentive for a further increase in tax rates as a way of fighting pollution. Higher tax rates lead to a new round of low activity, small tax bases, low clean-up policy, high tax rates, and so on. Eventually, in combination with "poor" clean-up technologies, the excessive rise in tax rates implies a negative link between the tax rate and the nature-to-capital ratio.

In turn, following exactly the same steps as above, equation (15) for the Nash tax rate changes to:

$$\hat{n}\left[r+d-(1-\tilde{q})A\right].\left[pr+(e-p\tilde{q})A\right]=(e-p\tilde{q})A[(1-\tilde{q})A-d] \qquad (31)$$

Working as in (14)-(16) above, comparative static exercises can show that if the parameter values ($d, r, A, \hat{n}, e, p$) are such that $(e-p\tilde{q})>0$, then the Nash tax rate, $\tilde{q}$, increases with I, while if $(e-p\tilde{q})<0$, $\tilde{q}$ decreases with I.[36] This is because the former case implies a negative externality across countries (i.e. there is a negative link between $\frac{\tilde{N}}{\tilde{k}}$ and $\tilde{q}$ in each country), while the latter case implies a standard positive externality across countries (i.e. there is a positive link between $\frac{\tilde{N}}{\tilde{k}}$ and $\tilde{q}$ in each country).

VI. RELATED RESULTS AND CONCLUSIONS

This paper has developed a model in which the environment has public good features, and then examined whether the conventional view that Nash tax rates are too low goes through in a model with long-term endogenous growth. We showed that the type of spillover effect from one player to another arising from the public good character of the environment (and hence whether we under-tax or over-tax in a Nash equilibrium relative to a cooperative one) can be reversed when the model allows for long-term endogenous growth. We used a simple Ak model of endogenous growth to make the logic of our results as clear as possible.

As far as we know, this result has not been shown before. However, there are some papers that are related to ours. For instance, Keen and Kotsogiannis [2000] use a model of a federal economy to show that "vertical externalities" are likely to give Nash tax rates that are inefficiently high. This is because national policymakers do not internalize the pressure on federal expenditures they create, when they raise their own tax rates. However, their results are driven by a priori different types of externalities (horizontal and vertical). In our paper, by contrast, the type of the same, single externality is reversed when the same model becomes dynamic.

Glomm and Lagunoff [1999] show that concerning the provision of public goods, whether voluntary (i.e. decentralized) or coercive (i.e. centralized) mechanisms prevail, depends crucially on whether the game is static or dynamic. This is because, in a dynamic setup, the accumulation process mitigates the problem of conflicting interests occurring in coercive communities and hence such communities become more attractive.

Cornes and Sandler [1996, pp. 166-170] also study the consequences of economic growth for public good provision. They show that technical progress in private goods production will encourage further substitution out of public goods into private goods and this can lead to immiserization (since public good provision is initially too low). However, their focus is on exogenous technical progress and not on dynamics.

John and Pecchenino [1994] show (among other things) that short-lived agents may over-invest in environmental quality in a decentralized equilibrium. However, this is a natural consequence of their OLG model. As is known, OLG models lead to inefficiently low consumption and hence inefficiently low pollution (in their model, pollution is a by-product of consumption).

Our work, as well as the above papers, are consistent with the general result that many properties, concerning the comparison between non-cooperative and cooperative outcomes, may change once the model allows for dynamics. Here, we showed that the form of short-run non-cooperative behavior can be reversed in the long-run, at which point capital tax bases become elastic. We feel that we have provided a non-trivial example which shows the importance of endogenous growth in particular, and growth dynamics in general, for the type of spillover effect from one economic agent to another.

[36] All results are available upon request.

APPENDICES

APPENDIX A: From equations (11a)-(11g) to equations (13a)-(13d)

Equations (11a)-(11g) constitute a seven-equation dynamic system in q, l, c, k, g, N, m. Taking logarithms on both sides of (11a) and differentiating with respect to time, we get:

$$\frac{\dot{l}c + l\dot{c} + \dot{g}k + g\dot{k}}{lc + gk} = \frac{\dot{m}k + m\dot{k}}{mk} \tag{A.1}$$

Using (11a), (11b), (11c), (11d), (11e) and (11g) into (A.1), we get:

$$1 = mc + dmk + \frac{\hat{n}k}{N} \tag{A.2}$$

If we define $z \equiv \frac{c}{k}$, (11c) and (11d) give (13a). If we also define $y \equiv mk$ and $f \equiv mN$, (11d), (11g) and (11f) give (13b) and (13c). Using these definitions into (A.2), we get (13d) in the main text.

APPENDIX B: Proof of Proposition 1

Setting $\dot{z} = 0$, equation (13a) implies:

$$\tilde{z} = r \tag{B.1}$$

Setting $\dot{y} = 0$ and using (B.1), equation (13b) implies:

$$\tilde{f} = \frac{\hat{n}}{[(1-\tilde{q})A - d]} \tag{B.2}$$

Setting $\dot{f} = 0$ and using (B.1)-(B.2), equation (13c) implies:

$$\tilde{y} = \frac{\hat{n}[r + d - (1-\tilde{q})A]}{(1-\tilde{q})A[(1-\tilde{q})A - d]} \tag{B.3}$$

Then, using (B.1)-(B.3) into (13d), we get:

$$\hat{n}\left[r + d - (1-\tilde{q})A\right]\left[r + (1-\tilde{q})A\right] = (1-\tilde{q})A\left[(1-\tilde{q})A - d\right] \tag{B.4}$$

which is equation (15) in the text. Note that (B.4) is a quadratic equation in $\tilde{q}$ only. If we solve (B.4) for $\tilde{q}$, (B.2) and (B.3) respectively will give $\tilde{f}$ and $\tilde{y}$. So the main task is to solve (B.4). We work as follows: In the first step, we specify the region in which a well-defined solution (if any) should lie. In the second step, we establish the existence and uniqueness of such a solution.

Consider the first step. A well-defined solution requires: (i) $(1-\tilde{q})A-r>0$, i.e. $\tilde{q}<1-\frac{r}{A}$. This is required for the economy to grow in the long-run. (ii) $(1-\tilde{q})A+d<2r$, i.e. $1-\frac{2r-d}{A}<\tilde{q}$. This is required for the transversality condition (12) to hold. (iii) $r+d-(1-\tilde{q})A>0$, i.e. $1-\frac{r+d}{A}<\tilde{q}$. This follows from inspection of (B.2)-(B.4) above. (iv) $(1-\tilde{q})A-d>0$, i.e. $\tilde{q}<1-\frac{d}{A}$. Again, this follows from inspection of (B.2)-(B.4). (iv) $2(1-\tilde{q})A-d>0$, i.e. $\tilde{q}<1-\frac{d}{2A}$. This is required for the left-hand side of (B.4) to be monotonically increasing in $\tilde{q}$ (see below why we need this). Now, if we combine (i)-(iv), and given the parameter restrictions in (14a) and (14b) in Proposition 1, the "binding" lower boundary for $\tilde{q}$ is $0<1-\frac{r+d}{A}$,[37] and the "binding" upper boundary for $\tilde{q}$ is $1-\frac{r}{A}<1$.[38] Thus,

$$0<1-\frac{r+d}{A}<\tilde{q}<1-\frac{r}{A}<1 \tag{B.5}$$

which gives the region in which a well-defined solution (if any) for $\tilde{q}$ should lie.

Consider now the second step. We will study whether such a solution for $\tilde{q}$ actually exists and whether it is unique. Recall that $\tilde{q}$ solves (B.4). Define the left-hand

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[37] In particular, if we assume $r>2d$ [which is condition (14b) in the text], it follows $1-\frac{2r-d}{A}<1-\frac{r+d}{A}<\tilde{q}$. That is, when $1-\frac{r+d}{A}<\tilde{q}$, we also have $1-\frac{2r-d}{A}<\tilde{q}$, so that the terminal condition (12) is satisfied. In turn, we assume $A>r+d$ [which is condition (14a) in the text] so that $1-\frac{r+d}{A}>0$, i.e. the tax rate is positive. Therefore, the binding lower boundary is $1-\frac{r+d}{A}$ which is positive.

[38] In particular, $r>2d$ implies $1-\frac{r}{A}<1-\frac{d}{A}<1-\frac{d}{2A}$. Thus, the binding upper boundary is $1-\frac{r}{A}$.

side of (B.4) by $L(\tilde{q})$ and the right-hand side by $R(\tilde{q})$. Then, $L_q(\tilde{q}) > 0$ (see condition (iv) above) and $R_q(\tilde{q}) < 0$. Concerning the lower boundary, i.e. $1 - \frac{r+d}{A}$, we have $L\left(1 - \frac{r+d}{A}\right) = 0$ which is always smaller than $R\left(1 - \frac{r+d}{A}\right) > 0$. Concerning the upper boundary, i.e. $1 - \frac{r}{A}$, we have $L\left(1 - \frac{r}{A}\right) > R\left(1 - \frac{r}{A}\right) > 0$, if the parameter values satisfy condition (14c) in the text. Since $L_q(\tilde{q}) > 0$ and $R_q(\tilde{q}) < 0$ monotonically, these values of $L(\tilde{q})$ and $R(\tilde{q})$ at the lower and upper boundaries mean that $L(\tilde{q})$ and $R(\tilde{q})$ intersect once, as it is shown in Figure 1 below.

Figure 1 here

Therefore, a unique, well-defined solution for $\tilde{q}$ exists. This in turn supports - via (B.2) and (B.3) - a unique well-defined solution for $\tilde{f}$ and $\tilde{y}$.

APPENDIX C: Dynamic Stability

We study stability properties around steady state. Linearizing (13a), (13b) and (13c) around the unique steady state implies that the local dynamics are approximated by the linear system:

$$\begin{bmatrix} \dot{z} \\ \dot{y} \\ \dot{f} \end{bmatrix} = \begin{bmatrix} J_{zz} & J_{zy} & J_{zf} \\ J_{yz} & J_{yy} & J_{yf} \\ J_{fz} & J_{fy} & J_{ff} \end{bmatrix} \begin{bmatrix} z \\ y \\ f \end{bmatrix} \tag{C.1}$$

where, the elements of the Jacobian matrix evaluated at the steady state are:

$$J_{zz} \equiv \frac{\P \dot{z}}{\P z} = r > 0, \quad J_{zy} \equiv \frac{\P \dot{z}}{\P y} = 0, \quad J_{zf} \equiv \frac{\P \dot{z}}{\P f} = 0,$$

$$J_{yz} \equiv \frac{\P \dot{y}}{\P z} = -\tilde{y} < 0, \quad J_{yy} \equiv \frac{\P \dot{y}}{\P y} = 0, \quad J_{yf} \equiv \frac{\P \dot{y}}{\P f} = \frac{\hat{n}\tilde{y}}{\tilde{f}^2} > 0,$$

$$J_{fz} = \frac{\P \dot{f}}{\P z} = 0, \quad J_{fy} = \frac{\P \dot{f}}{\P y} = -(1-\tilde{q})A < 0, \quad J_{ff} = \frac{\P \dot{f}}{\P f} = r > 0.$$

The determinant of the Jacobian matrix, denoted by $\det(J)$, is $\det(J) = r\hat{n}(1-\tilde{q})A\frac{\tilde{y}}{\tilde{f}^2}$, which is positive. Hence, there are two possibilities: Either there are three positive eigenvalues, or one positive and two negative eigenvalues. Since all three variables (z, y, f) are forward-looking ones, the former possibility (i.e. three positive roots) will give local determinacy, while the latter possibility (i.e. one positive and two negative roots) will give local indeterminacy (i.e. multiple paths, each of which is consistent with the same initial condition and with convergence to the same unique steady state).[39] We therefore examine the characteristic equation of the Jacobian matrix being evaluated at the steady state. This is:

$$e^3 - 2re^2 + \left[r^2 + \frac{(1-\tilde{q})A\hat{n}\tilde{y}}{\tilde{f}^2} \right] e - \frac{r(1-\tilde{q})A\hat{n}\tilde{y}}{\tilde{f}^2} = 0 \qquad \text{(C.2)}$$

where e is an eigenvalue. Note that the coefficient on e^2 is negative, the coefficient on e is positive and the constant term is negative. That is, there are three sign alterations in (C.2). We now use Descartes' Theorem (see Azariadis [1993]), which states that the number of positive roots cannot be higher than the number of sign alterations. Thus, we cannot have more than three positive roots. Next, define $e' \equiv -e$. In this case, there are no sign alterations in (C.2). Thus, we cannot have a negative root. Combining results, there are three positive roots. Therefore, we have local determinacy.

What does it mean? Without predetermined variables, determinacy means that the forward-looking variables jump immediately to take their long-run values and stay there (until the system is disturbed in some way). Thus, there are no transitional dynamics and the saddle-path solution is equivalent to the steady state. This is as in the basic Ak model (see e.g. Barro and Sala-i-Martin [1995, chapter 4]). For a richer Ak-type model with transitional dynamics and indeterminacy but in the context of a closed economy, see Park and Philippopoulos [2001].

[39] See e.g. Blanchard and Fischer [1989, p. 225].

APPENDIX D: Nash Tax Rates and Population Size in a Textbook Static Model

To further confirm our results, we also present a textbook, static model with public goods. Assume that e^i is the exogenous endowment of agent $i = 1, 2, ..., I$, and q^i is the tax rate paid by the same agent, so that private consumption is $c^i = e^i - q^i e^i$. The utility function of agent i is $u^i(c^i, G)$, where $G = \frac{\sum_{i=1}^{I} q^i e^i}{I}$ is the public good. Assume for expositional simplicity (as in the paper above), that $u^i(c^i, G) = \log c^i + n \log G$, where $n > 0$.

In a Nash equilibrium, agent i chooses q^i to maximize his/her own utility function by taking q^j, where $j \neq i$, as given. Then, in a symmetric Nash equilibrium, the first-order condition implies $\frac{q}{1-q} = \frac{n}{I}$. Therefore, the Nash tax rate decreases monotonically with the group size, I.

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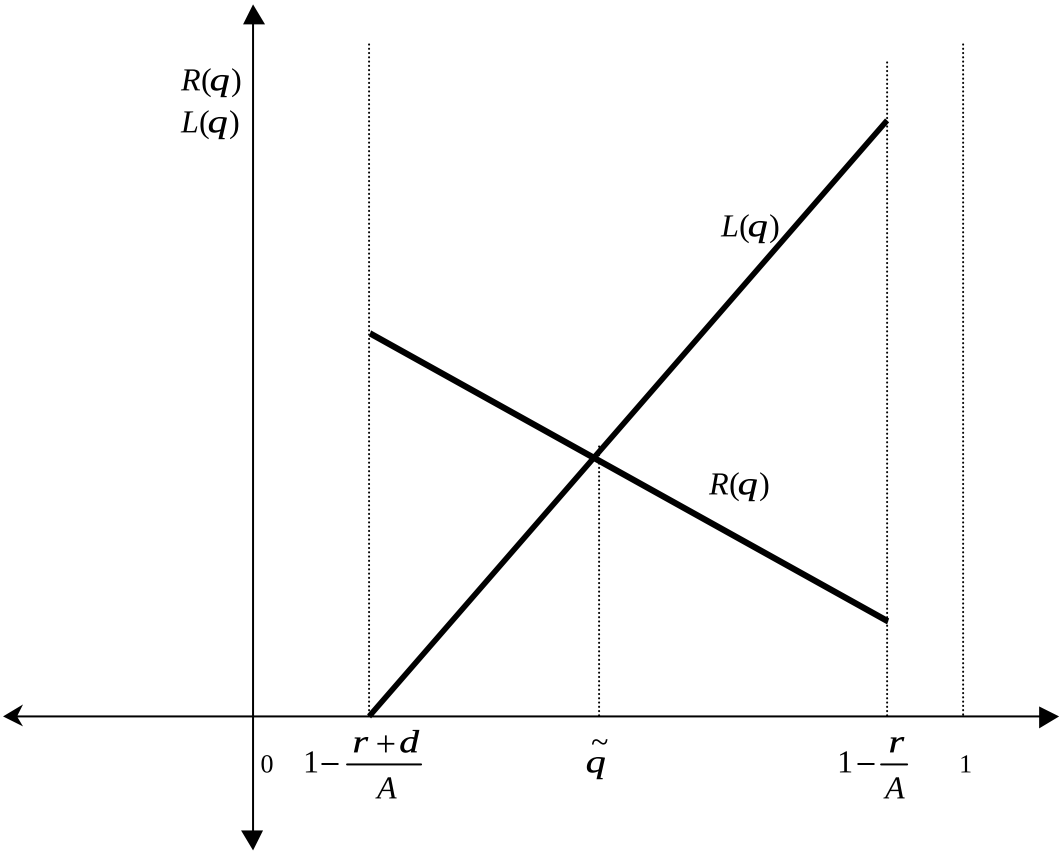
FIGURE 1



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