University of Cyprus
Department of Economics

Working Paper 03-2013

Piecemeal Reform of Domestic Indirect Taxes toward Uniformity in the Presence of Pollution: with and without a Revenue Constraint

Michael S. Michael, Sajal Lahiri and Panos Hatzipanayotou

Department of Economics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus
Tel.: +357-22893700, Fax: +357-22895028, Web site: www.econ.ucy.ac.cy

Piecemeal Reform of Domestic Indirect Taxes toward Uniformity in the Presence of Pollution: with and without a Revenue Constraint*

Michael S. Michael♣, Sajal Lahiri♦ and Panos Hatzipanayotou♠†

January 2013

Abstract

The literature on indirect tax reforms in pollution-ridden economies is quite limited. This paper, using a general equilibrium model of a perfectly-competitive small open economy with both production and consumption generated pollution, considers the welfare implications of tax reforms that take the structure of consumption and production taxes toward uniformity. Specifically, both in the presence and absence of a binding government revenue constraint, we derive sufficient conditions for welfare improvement in the case where we implement *(i)* reforms in either production or consumption taxes, and *(ii)* reforms in both consumption and production taxes.

Keywords: Indirect tax reforms, Production and consumption generated pollution, Welfare, Government tax revenues.

J.E.L Classification: H21: Efficiency, Optimal Taxation
H23: Production taxes and subsidies

♣ Department of Economics, University of Cyprus; P.O.Box 20537, CY 1678, Nicosia, Cyprus, and CES-ifo, Email: m.s.michael@ucy.ac.cy

♦Department of Economics, Southern Illinois University Carbondale, Carbondale, IL. 62901-4515, U.S.A., E-mail: lahiri@siu.edu

♠ Department of International and European Economic Studies, Athens University of Economics and Business; 76, Patission str., Athens 104 34, Greece, and CES-ifo, Email: hatzip@aueb.gr

† **Acknowledgements:** We thank the editor and the anonymous referee of the Journal, and participants at the 2011 12th Annual Conference of the Association of Public Economic Theory (PET 11) for very helpful comments. The usual disclaimer applies.

1. Introduction

During the past couple of decades there has been a general consensus regarding the reforms of national tax systems. International institutions, e.g., the WTO, the IMF and the World Bank, encourage governments to reform their indirect and direct tax structure in a way of reducing economic distortions, improving welfare and possibly ensuring higher levels of government tax revenues.[1] Amongst the various types of recommended reforms, two seem to stand out. First is the need of countries to reduce their reliance on discriminatory trade taxes and switch to domestic taxes such as income taxes and consumption taxes.[2] The second class of recommended reforms involves just domestic taxes such as the movement of taxes towards uniformity.

Motivated by such developments in the policy arena, a voluminous academic literature on tax reforms has been developed examining a wide range of reforms in direct and indirect taxes. This paper is not about the first type of reforms mentioned above,[3] but about the second type. Within the class of reforms of domestic taxes, there are many subclasses. A strand of this literature examines the relationship between direct and indirect taxes (see, for example, Atkinson and Stiglitz, 1976); another examines the movement from destination to origin principle of commodity taxation (see, for example, Lockwood *et al*., 1994 and Keen and Lahiri, 1998); a third examines the implications of moving domestic taxes on different goods towards a uniform rate (see, for example, Hatta, 1977, 1986).[4]

Specifically, of interest to our study is the literature that considers the implications of a move towards uniformity of domestic taxes across goods. The origins of this literature dates back to

---

[1] This latter concern becomes even more important for revenue-strained developing economies. Achieving these two goals, countries are able to attain a so-called “double-dividend”. That is, a tax system which improves welfare and does not reduce tax revenues.

[2] According to the World Bank (2002), during the 1990s in low- and medium-income countries, the share of domestic indirect taxes (i.e., taxes on goods and services) in total current government revenue rose from 26 percent in 1990 to 36 percent in 1999. During the same period the share of trade taxes fell from 17 percent to 9 percent.

[3] The literature here is quite substantial and growing. See. For example, Diewert et al., 1989; Michael et al., 1993; Hatzipanayotou et al., 1994, Abe, 1995; Neary, 1998; Keen and Ligthart, 2002; Lahiri and Nasim, 2005; Emran and Stiglitz, 2005; Boadway and Sato, 2009. All the above studies examine the welfare and revenue implications of domestic and/or trade tax reforms in the context of a static general equilibrium model of a small open economy. Among others, Majumdar (2004), Keen and Ligthart (2005) and Naito and Abe (2008) examine the welfare implications of indirect tax reforms under a revenue neutrality constraint in the context of imperfect competition. Naito (2005 and 2006) examine dynamic policy aspects, e.g., the growth rate of output, of such tax reforms.

[4] There is also a large literature on the uniformity of domestic taxes across tax jurisdictions ---the issue of tax harmonization --- starting with the seminal work by Keen (1987), and on the uniformity of domestic environmental taxes across heterogeneous firms within an industry (see, for example, Fullerton et al., 2008).

Atkinson and Stiglitz (1976) who show that, when income tax is set optimally, differential commodity taxation is inefficient. Hatta (1977) in the context of a closed economy and without considering a tax revenue constraint, examines the welfare implications of moving consumption taxes towards uniformity, while Hatta (1986), re-examines the implications of the above tax reforms under a revenue constraint. The broad argument here is that non-uniformity in commodity taxation distorts consumption choices and therefore is inefficient. A move toward this type of uniformity is also a live issue in the policy-making sphere (see, for example, The European Union, 2010).

During the past few decades most countries including many developing ones -- e.g., the so-called BRICS countries Brazil, Russia, India, China and South Africa -- have enjoyed strong growth. For theorists and policy makers, however, this process of economic growth has raised a number of serious concerns. Among those foremost is the threat to the quality of the environment due to the intensification of economic activity. To deal with these concerns, a new strand of the tax reform literature has been developed, which examines the implications of changes in the structure of indirect taxes in the context of pollution ridden open economies. Among others, Copeland (1994), Beghin et al. (1997), Turunen-Red and Woodland (2004), Kayalica and Kayalica (2005) consider the welfare and environmental implications of reforms in trade and domestic taxes in economies where pollution is a by-product of production and/or consumption. This literature however does not account for a binding government revenues constraint, nor does it consider the reforms that take environmental taxes toward uniformity.[5]

This paper considers reforms of indirect taxes along the lines of the literature on tax reforms in pollution ridden economies. The present study, however, extends the above literature in two ways. First, although our analytical framework is one of an open economy, we depart from the standard paradigm of reforms in domestic vs. trade taxes by considering reforms of only domestic taxes and consider reforms that move production and consumption taxes toward uniformity across goods. Such a framework could be more relevant since trade barriers have

[5] A different literature examines the so-called double-dividend hypothesis of green tax reforms, whereby pollution taxes simultaneously corrects for the pollution externality and raises government revenues, e.g., Bovenberg and van der Ploeg (1994), Bovenberg and De Mooij (1994) Goulder (1995) and Bovenberg (1999) provide various meanings of the term “double-dividend” and extensive surveys of this literature. Finally, a different framework of pollution tax reforms is developed by Hatzipanayotou *et al.* (2005) who in a two-country model with cross-border pollution and public pollution abatement examine the welfare implications of selected multilateral environmental policy reforms.

been rapidly going down. Second, and in contrast to the bulk of the relevant literature, the proposed tax reforms also account for a binding government revenue constraint.

To this end, we consider a small open economy where pollution is generated either by production or by consumption, and where the government raises revenue by imposing production and/or consumption taxes. We consider the cases where government revenue constraint is binding as well as when it is not binding. Under these different scenarios, we derive sufficient conditions for welfare improvement in the specific types of reforms mentioned above; we consider reforms of consumption taxes and production taxes on their own and also the case when both types of taxes are reformed at the same time. [6]

2. The General Model

We consider a small open, perfectly competitive economy which produces and consumes K internationally traded goods. Pollution is modeled as a by-product of both production and consumption. For, analytical convenience, we assume that the production of any good generates the same type of pollutant (z) but at different units per unit of output produced. Similarly, consumption pollution of any good is the same (r), but at different units per unit of output consumed.[7] Pollution generated by the production or consumption of each commodity affects negatively the households' utility. The government imposes consumption and production taxes and all tax revenues are lump-sum distributed. The country is endowed with an inelastic supply of primary factors. The international prices of all goods are fixed and assumed to equal unity.[8]

The economy's production side is represented by the revenue function $R(q)$ which captures the economy's maximum revenue from production of the goods with the vector of producer prices q, where $q_j = 1 - t_j$ is the domestic producer price of the j^{th} good and t_j is the specific production tax levied on it**.** The $R(q)$ function is assumed convex and homogeneous of

[6] We do not consider the distributional implications of the reforms. See, for example, Saez (2002) for an analysis of the Atkinson-Stiglitz (1976) results in the presence of heterogeneous consumers.

[7] A more general specification would allow for different types of pollutants, at different rates, for every unit of the output produced and consumed. Such a specification results to cumbersome algebraic calculations without adding substantively to the generality of the results.

[8] We follow a standard practice of the literature of indirect tax reforms, which, by and large, for analytical convenience confines the analysis of such tax reforms in the context of small open economies, i.e., terms of trade considerations, are unaccounted for.

degree one in producer prices. By the envelop theorem $R_{q_j} (= \partial R / \partial q_j)$ is the supply function of the j^{th} good.

Turning to the demand side of this economy, it comprises identical households who consume the K commodities, and whose utility is adversely affected by production and consumption generated pollution. A representative household's preferences are captured by the expenditure function $E(p,z,r,u)$ denoting the minimum expenditure on goods achieving a certain level of utility (u), at consumer price vector p and vectors of production pollutants z and consumption pollutants r. The domestic consumer price for the j^{th} commodity is $p_j = 1 + \tau_j$, where τ_j denotes the specific consumption tax levied on it. The $E(p,z,r,u)$ function is increasing in u, in levels of pollution z and r, and non-decreasing and concave in p.[9] The derivative $E_{p_j} = \partial E / \partial p_j$ is the compensated demand for the j^{th} good and E_{pp} is a $(K \times K)$ negative semi-definite matrix. The derivative E_u is the inverse of the marginal utility of income. The derivatives E_z and E_r, respectively, denote the marginal damage caused by the pollutant z or r, and thus they represents the household's *marginal willingness to pay* for its reduction (e.g., see Copeland, 1994).

Let, $z = \sum_{j=1}^{K} \alpha_j R_{q_j}(q) = \alpha' R_q(q)$ and $r = \sum_{j=1}^{K} \beta_j E_{p_j}(p,z,r,u) = \beta' E_p(p,z,r,u)$, respectively denote the levels of production and consumption pollution. The scalars $\alpha_j > 0$ and $\beta_j > 0$ denote, respectively, the units of production and consumption pollution per unit of the j^{th} commodity. $\alpha' = [\alpha_1, \alpha_2, ..., \alpha_K]$ and $\beta' = [\beta_1, \beta_2, ..., \beta_K]$.[10]

The government's tax revenue, (T), which is lump-sum distributed to households, equals the sum of consumption and production tax revenues, i.e.,

$$T = \tau' E_p(p,z,r,u) + t' R_q(q) = \sum_{j=1}^{K} \tau_j E_{p_j}(p,z,r,u) + \sum_{j=1}^{K} t_j R_{q_j}(q), \tag{1}$$

[9] The $E(.)$ function is increasing in z or in r since an increase in any type of pollutant is assumed to harm the households' utility. Therefore, to attain a given level of utility, u, private spending on consumption must rise.

[10] A prime $(')$ denotes a transposed vector or matrix.

where E_p and R_q, respectively, are the vectors of compensated demand and supply of goods. The country's income-expenditure identity requires that private spending on goods must equal income from production plus income from government taxes. Thus, the country's budget constraint is given as follows:

$$E(p,z,r,u) = R(q) + \tau' E_p(p,z,r,u) + t' R_q(q)\,. \tag{2}$$

Differentiating equation (2), we obtain:

$$E_u du = \left(\tau - \beta E_r\right)' dE_p + \left(t - \alpha E_z\right)' dR_q\,, \tag{3}$$

where $dE_p = E_{pp} d\tau + E_{pr} dr + E_{pz} dz + E_{pu} du$, $dR_q = -R_{qq} dt$, $dz = -\alpha' R_{qq} dt$, and

$dr = \delta^{-1}\beta'\left(E_{pp} d\tau + E_{pz} dz + E_{pr} dr + E_{pu} du\right)$. E_{pu} is a vector whose elements are all positive assuming that goods are normal in consumption. The vectors E_{pz} and E_{pr} respectively capture the changes in compensated demand for goods due to changes in production and consumption generated pollution. In case that compensated demands for good and clean environment are independent, all elements of the aforementioned vectors are zeroes. $\delta = \left(1 - \beta' E_{pr}\right)$ is a positive scalar indicating that an increase in income and consumption result to higher level of consumption generated pollution.

Equation (3) can be rewritten so as to capture the welfare effect of changes in a single consumption tax, say that on the i^{th} good, and of changes in a single production tax, say on the n^{th} good. That is:

$$\Omega du = \sum_{j=1}^{K} \sigma_j p_j E_{p_j p_i} d\tau_i - \sum_{j=1}^{K} s_j q_j R_{q_j q_n} dt_n\,, \tag{4}$$

where $\Omega = E_u - \tau' E_{pu} + \left(E_r - \tau' E_{pr}\right)\delta^{-1}\beta' E_{pu}$, and it is assumed positive.[11] Ω^{-1} represents an augmented income tax multiplier adjusted for consumption taxes and consumption generated pollution. Equation (4) can be further elaborated on by using the properties of the expenditure and revenue functions that compensated demand and supply functions are homogeneous of

[11] The positive sign of Ω is justified in various ways. Here Ω^{-1} the tax multiplier, is equivalent to the so-called tariff multiplier, for example in Copeland (1994), Neary and Ruane (1988). A negative Ω^{-1} would imply that an increase in lump-sum taxes on consumers would raise utility.

degree zero in prices. Specifically, $\sum_{j=1}^{K} p_j E_{p_j p_i} = 0$ and $\sum_{j=1}^{K} q_j R_{q_j q_n} = 0$, respectively, yield $E_{p_i p_i} = -\sum_{j \neq i}^{K} \left(p_j / p_i \right) E_{p_j p_i}$ and $R_{q_n q_n} = -\sum_{j \neq n}^{K} \left(q_j / q_n \right) R_{q_j q_n}$. Note that $p_k = 1 + \tau_k$, $q_k = 1 - t_k$, $k = j, i, n$, and by the reciprocity conditions $E_{p_k p_j} = E_{p_j p_k}$ and $R_{q_k q_j} = R_{q_j q_k}$. Using the above properties, we obtain:

$$\Omega du = \sum_{j \neq i}^{K} (\sigma_j - \sigma_i) p_j E_{p_j p_i} d\tau_i - \sum_{j \neq n}^{K} (s_j - s_n) q_j R_{q_j q_n} dt_n \, , \tag{5}$$

where $\sigma_k = \mu_k - \delta^{-1} E_r (\beta_k / p_k)$, $s_k = \varphi_k - \left(E_z + E_r \delta^{-1} \sum_{j=1}^{K} \beta_j E_{p_j z} \right) \left(\alpha_k / q_k \right)$, and $\mu_k = (\tau_k + \delta^{-1} \tau' E_{pr} \beta_k) / p_k$, $\phi_k = \left[t_k + \tau' \left(E_{pz} + E_{pr} \delta^{-1} \sum_{j=1}^{K} \beta_j E_{p_j z} \right) \alpha_k \right] / q_k$ for k= i, j, n.

We call the ratio μ_k the *consumption-tax impact factor*; where, τ_k captures the direct effect on tax revenues due to a unit changes in the consumption of the k^{th} good. Changing, however, the consumption of the k^{th} good changes also consumption generated pollution (r), which in turn induces a change in consumption of all other goods, thus, entailing a further, indirect change in consumption tax revenues captured by the term $\delta^{-1} \tau' E_{pr} \beta_k$. Therefore, μ_k is the total, direct and indirect, tax revenue effect of a unit change in the consumption of the k^{th} good as a fraction of the consumer price p_k. Next, we can interpret $\left(\beta_k / p_k \right)$ as the rate of pollution per unit value of consumption of the k^{th} good, and $\delta^{-1} E_r \left(\beta_k / p_k \right)$ the total negative effect on welfare when pollution increases due to the increase in the consumption of a unit of the k^{th} commodity, taking into account that when pollution increases, consumption of all goods changes. It is also called, the total willingness to pay for reducing pollution created from the consumption of a unit of the k^{th} good. The ratio σ_k we call the *rate of excess taxation of consumption-pollution*. It can be positive or negative depending on whether the total tax revenue effect of a unit change in the consumption of the k^{th} good exceeds the welfare damage caused from pollution of consuming the extra unit of the k^{th} good. Similarly, we call the ratio φ_k the *production-tax impact factor*. It consists of the direct and indirect, tax revenue effect of a unit change in the production of the k^{th} commodity as a fraction of the producer price q_k. The term $\left(E_z + E_r\delta^{-1}\sum_{j=1}^{K}\beta_j E_{p_j z}\right)(\alpha_k / q_k)$ is the total welfare damage due to pollution per unit value of production of the k^{th} commodity. It consists of the direct pollution welfare damage per unit value of production, i.e., $(E_z\alpha_k)/q_k$, and the indirect pollution welfare damage per unit of value of production, i.e., $\left(E_r\delta^{-1}\sum_{j=1}^{K}\beta_j E_{p_j z}\right)\alpha_k / q_k$, due to the induced change in the consumption of all goods. Alternatively, this term captures the total willingness to pay for reducing pollution generated from an increase in the production of a unit of the k^{th} good. Then, the ratio s_k we call the *rate of excess taxation of production-pollution*, and it is positive (negative) if the total tax revenue effect of a unit change in the production of the k^{th} good exceeds (falls short of) the total welfare damage from pollution caused from producing the extra unit of the k^{th} good.

2.1 Optimality of consumption and production taxes in the presence of pollution

It is of importance to note on the choice of optimal, first best, consumption and production taxes in the present context of production and consumption pollution. In standard competitive models without pollution, it is well-known that any uniform consumption/production tax gives the first best as the relative post-tax prices are the same as the relative pre-tax prices. In this case, with uniform *rates of excess taxation of consumption-pollution* σ_i s, i.e., $\sigma_i = \sigma, \forall i$, and *rates of excess taxation of production-pollution* s_i s, i.e., $s_i = s, \forall i$,we get,

$$\Omega du = \sum_{i=1}^{K}\left[\sum_{j=1}^{K}\sigma_j p_j E_{p_i p_j}\right]d\tau_i - \sum_{i=1}^{K}\left[\sum_{j=1}^{K}s_j q_j R_{q_i q_j}\right]dt_i = \sigma\sum_{i=1}^{K}\left[\sum_{j=1}^{K}p_j E_{p_i p_j}\right]d\tau_i - s\sum_{i=1}^{K}\left[\sum_{j=1}^{K}q_j R_{q_i q_j}\right]dt_i = 0. \quad (6)$$

Note that the compensated demand and supply functions are homogeneous of degree zero in consumer and producer prices, respectively. Thus, as it can be seeing from equation (6), the uniform *rates of excess taxation of consumption-pollution* and uniform *rates of excess taxation of production-pollution* satisfy the first-order necessary conditions for optimality.[12]

---

[12] Appendix I uses the two-goods model as an illustrative example to characterize optimal taxes.

3. Reforms without a binding revenue constraint

In this section we assume a non-binding revenue constraint, and we examine the welfare implications of (i) reforms in consumption taxes when only consumption pollution exists, and (ii) reforms in production taxes when only production pollution exists.

3.1 Reforms in consumption taxes with only consumption pollution

We first consider the case where production is "clean" and only consumption generates pollution. The aggregate level of pollution in the economy is $r = \beta' E_p(p, r, u)$. We derive the conditions under which welfare improves by changing the consumption tax on a certain good, holding consumption taxes on all other goods and all production taxes constant. Then, equation (5) reduces to:

$$\Omega \frac{du}{d\tau_i} = \sum_{j \neq i}^{K} (\sigma_j - \sigma_i) p_j E_{p_j p_i} \ . \tag{7}$$

From equation (7) it follows that an increase in the consumption tax rate for the i^{th} good increases welfare if the deviation in the *rate of excess taxation of consumption pollution* of any j^{th} good from the *rate of excess taxation of consumption-pollution* of the i^{th} good is positively correlated with the value of the consumption substitutability between the i^{th} *and* the j^{th} good. Formally,

Theorem 1: *Suppose only consumption pollution exists. Then, an increase in the consumption tax rate of the* i^{th} *good increases welfare if and only if* $\sum_{j \neq i}^{K} (\sigma_j - \sigma_i) p_j E_{p_j p_i} > 0$.

From the above general results, we can derive two specific results under more restrictive assumptions. First, when $\sigma_j = \sigma$ for all $j \neq i$**,** then increasing τ_i increases welfare if and only if $\sigma > \sigma_i$ for all $j \neq i$. Formally,

Proposition 1: *Suppose only consumption pollution exists and let* $\sigma_j = \sigma$ *for all* $j \neq i$. *Then an increase in the consumption tax rate of the* i^{th} *good increases welfare if and only if* $\sigma > \sigma_i$.

Proof: From equation (7) we get:

$$\Omega \frac{du}{d\tau_i} = (\sigma - \sigma_i) \sum_{j \neq i}^{K} p_j E_{p_j p_i} \ . \tag{8}$$

Since E_{p_i} is homogeneous of degree zero in prices, we have

$$\sum_{j \neq i}^{K} p_j E_{p_j p_i} = -p_i E_{p_i p_i} > 0 \,. \tag{9}$$

Proposition 1 follows from equations (8) and (9). □

Note that when $\sigma_j = \sigma$ for all $j \neq i$, no assumption is required on the substitutability of the i^{th} good with respect to all other goods in consumption. Moreover, in the case of two goods, $\sigma_j = \sigma$ $(j \neq i)$ is trivially true, thus Proposition 1 follows immediately.

Second, when the σ_j's$(j \neq i)$ are not the same, we can still get a result similar to that of Proposition 1, but under the assumption of substitutability of the i^{th} good with respect to all other goods in consumption. The following proposition states the relevant result.

Proposition 2: *Assume the existence of only consumption pollution. Suppose, without any loss of generality, that the i^{th} good carries the lowest (highest) rate of excess taxation of consumption-pollution, i.e.,* $(\sigma_j - \sigma_i) > 0 (< 0)$, *for all* $j \neq i$. *Then, a small increase (decrease) of the consumption tax of this good improves welfare if this good is a substitute in consumption to all other goods.*

Intuitively, consider the case where the i^{th} good carries the lowest *rate of excess taxation of consumption pollution*. This emerges, for example, when the i^{th} good has the lowest *consumption-tax impact factor* and the highest rate of pollution per unit value of consumption. In this case, by increasing its consumption tax rate reduces its consumption and increases the consumption of all other goods. The reduction in the consumption of the i^{th} good has the lowest effect on tax revenue and the highest effect on the level of pollution relative to the effect of the increase in the consumption of all other goods. The results of the above proposition do not depend on whether the *rate of excess taxation of consumption-pollution* is positive or negative.

Under the conditions of the Proposition 2, the proposed consumption tax reform aims at small increases or decreases of consumption tax rates so that the *rates of excess taxation of consumption-pollution* move towards uniformity. For example, in the case of the i^{th} good carrying the lowest *rate of excess taxation of consumption-pollution* relative to all other goods, we propose successive small increases of the consumption tax on this good, so that its *rate of excess taxation of consumption-pollution* does not increase beyond the level of the second lowest *rate of excess taxation of consumption-pollution*. However, as in the case without pollution, this result depends on the relationship in consumption between the good with the lowest *rate of excess taxation of consumption-pollution*, and all other goods. This is because, assuming substitutability in consumption between the i^{th} good and the other goods, an increase in the consumption tax on the i^{th} good reduces its consumption and pollution distortion and raises the consumption and pollution distortion generated by all other goods. An analogous argument holds when the i^{th} good exhibits the highest *rate of excess taxation of consumption-pollution*, and the consumption tax on this good is reduced in such a way that, its *rate of excess taxation of consumption-pollution* does not fall below the level the second highest rate.

Comparing the above results to standard results of the literature on reforms of tariffs and consumption taxes we note the following. Michael *et al*. (1993) conclude that if, for example, the i^{th} good is burdened with the highest (lowest) consumption tax rate, then, reducing (increasing) this tax rate to the level of the next highest (lowest) consumption tax rate, improves the country's welfare if the i^{th} good is a substitute to all other goods in consumption (Proposition 1, p. 421). This result ceases to hold when introducing consumption generated pollution. As shown above, a welfare improving reform of consumption taxes requires increasing (decreasing) the consumption tax on the commodity exhibiting the lowest (highest) *rate of excess taxation of consumption-pollution*, without inferring that this commodity is the one that is also burdened with the lowest (highest) consumption tax rate. If, however, the variations in pollution generated per unit value of consumption, i.e., variations in $[\beta_j / p_j]'s$, are limited, then the rank order of the consumption tax rates is the same as that of the *rates of excess taxation of consumption-pollution.* In this case, the welfare effects of a consumption tax reform go through as originally stated by Michael *et al*. (1993) and others. In this case, the commodity carrying the highest (lowest) consumption tax rate is also the one burdened with the highest (lowest) *rate of excess taxation of consumption-pollution*. The same holds true in the presence of only production generated pollution.

3.2 Reforms in production taxes with only production pollution

Now we consider the case where consumption is "clean" and only production generates pollution. The aggregate level of pollution in the economy is $z = \alpha' R_q(q)$. We derive the conditions under which welfare improves by changing the production tax on a certain good, holding production taxes on all other goods and all consumption taxes constant. Then, equation (5) reduces to:

$$\Omega_1 \frac{du}{dt_n} = -\sum_{j \neq n}^{K} (s_j - s_n) q_j R_{q_j q_n} , \tag{10}$$

$\Omega_1 = E_u - \tau' E_{pu}$, $s_k = (t_k + \tau' E_{pz} \alpha_k - E_z \alpha_k) / q_k$.[13] Equation (10) indicates that an increase in the production tax rate for the n^{th} good increases welfare if the deviation in the *rate of excess taxation of production pollution* of any j^{th} good from the *rate of excess taxation of production pollution* of the n^{th} good is negatively correlated with the value of the production substitutability between the n^{th} *and* the j^{th} good. Formally,

Theorem 2: *Suppose only production pollution exists. Then, an increase in the production tax rate of the n^{th} good increases welfare if and only if* $\sum_{j \neq n}^{K} (s_j - s_n) q_j R_{q_j q_n} < 0$.

As in the case of Theorem 1, from the above general result we can derive two specific results under more restrictive assumptions. First, let $s_j = s$ for all $j \neq n$, then increasing t_n increases welfare if and only if $s > s_n$ for all $j \neq n$. Formally,

Proposition 3: *Suppose there exists only production pollution, and let $s_j = s$ for all $j \neq n$. Then an increase in the production tax rate of the n^{th} good increases welfare if and only if $s > s_n$.*

Proof: From equation (10) we get:

$$\Omega_1 \frac{du}{dt_n} = -(s - s_n) \sum_{j \neq n}^{K} q_j R_{q_j q_n} . \tag{11}$$

Since R_{q_n} is homogeneous of degree zero in prices, we have

[13] Note that assuming that consumption is clean activity, the total pollution welfare damage per unit value of production reduces to $-E_z\left(\alpha_k / q_k\right)$.

$$\sum_{j \neq n}^{K} q_j R_{q_j q_n} = -q_n R_{q_n q_n} < 0 \,. \tag{12}$$

Proposition 3 follows from equations (11) and (12). □

Observations similar to those of Proposition 1 hold here too. Namely, when $s_j = s$ for all $j \neq i$, no assumption is required on the substitutability of the n^{th} good with respect to all other goods in production; in the case of two goods, $s_j = s \; (j \neq i)$ is trivially true, thus Proposition 3 follows immediately.

Second, when the s_j '$s (j \neq n)$ are not the same, we can still get a result similar to that of Proposition 3, but under the assumption of substitutability of the n^{th} good with respect to all other goods in production.

Proposition 4: *Assume the existence of only production pollution. Suppose, without any loss of generality, that the n^{th} good carries the lowest (highest) rate of excess taxation of production pollution, i.e., $(s_j - s_n) > 0 (< 0)$, for all $j \neq n$. Then, a small increase (decrease) of the production tax of this good improves welfare if this good is a substitute in production to all other goods.*

Again, the results of the above proposition do not depend on whether *the rate of excess taxation of production-pollution* is positive or negative. The welfare improving reforms described by the above proposition aim at small increases or decreases of production tax rates so that the *rates of excess taxation of production-pollution* move towards uniformity. The intuition and the implication of these results follow the previous discussion of small reforms in the consumption tax rates.

4. Reforms under a binding revenue constraint

In this section we consider reforms in consumption and production taxes under the restriction that government revenue cannot change because of the reforms. Thus, we need to consider changes in at least two of these taxes in order to keep government revenue unchanged. Under this assumption we examine the welfare implications of consumption and production tax reforms in the following cases. First, with only consumption pollution, we examine the welfare implications of reforms in consumption taxes alone, and then in consumption and production taxes combined. Next, with only production pollution, we examine the welfare implications of reforms in production taxes.

When the government revenue constraint is binding ($dT = 0$), differentiating equation (1), using the homogeneity properties of the expenditure and revenue functions, we obtain:

$$\lambda du + \left[E_{p_i} + \sum_{j \neq i}^{K} \left(\mu_j - \mu_i \right) p_j E_{p_j p_i} \right] d\tau_i + \left[R_{q_n} + \sum_{j \neq n}^{K} \left(\varphi_n - \varphi_j \right) q_j R_{q_j q_n} \right] dt_n = 0 , \tag{13}$$

where $\lambda = \tau' \left(E_{pu} + E_{pr} \delta^{-1} \beta' E_{pu} \right)$. Now, equations (5) and (13) constitute the relevant equations system. To facilitate the analysis, we write the system of these equations for the case where the consumption and production taxes of the i[th] and n[th] good change as:

$$\Omega du = F_i d\tau_i - B_n dt_n + F_n d\tau_n - B_i dt_i , \tag{14}$$

$$\lambda du + G_i d\tau_i + D_n dt_n + G_n d\tau_n + D_i dt_i = 0 , \tag{15}$$

where $F_i = \sum_{j \neq i}^{K} (\sigma_j - \sigma_i) p_j E_{p_j p_i}$, $B_n = \sum_{j \neq n}^{K} (s_j - s_n) q_j R_{q_j q_n}$, $G_i = \left[E_{p_i} + \sum_{j \neq i}^{K} \left(\mu_j - \mu_i \right) p_j E_{p_j p_i} \right]$, and

$D_n = \left[R_{q_n} + \sum_{j \neq n}^{K} \left(\varphi_n - \varphi_j \right) q_j R_{q_j q_n} \right]$. Similarly we define F_n, B_i, G_n and D_i.

4.1 Tax reforms in the presence of consumption pollution

Using equations (14) and (15) we examine, first, the welfare effects and the required adjustments in order to maintain government revenue constant, of changes only in consumption taxes; second, of changes in consumption and production taxes. In the case of changes in consumption taxes only, using equations (14)-(15) we can write the following matrix system:

$$\begin{bmatrix} \Omega & -F_i \\ \lambda & G_i \end{bmatrix} \begin{bmatrix} du \\ d\tau_i \end{bmatrix} = \begin{bmatrix} F_n \\ -G_n \end{bmatrix} d\tau_n , \tag{16}$$

Equations (A.4) in Appendix (II. (ii)) provide the analytical equations of the above system. Then,

$$\Delta_1 \left(\frac{d\tau_i}{d\tau_n} \right) = -(\Omega G_n + \lambda F_n) , \tag{17}$$

$$\Delta_1\left(\frac{du}{d\tau_n}\right) = F_n G_i - F_i G_n , \tag{18}$$

where $\Delta_1 = (\Omega G_i + \lambda F_i)$. Equations (17) and (18), respectively, give the required adjustments in the consumption tax rate τ_i to keep tax revenues constant and the welfare effects of a change in the consumption tax τ_n. Hereon, we adopt the extensively used assumption in the tax reform literature, e.g., see Emran and Stiglitz (2005), that tax rates τ_i and τ_n are revenue increasing, i.e., the tax rates are on the "right" side of the Laffer curve. The following Lemma emerges:

Lemma 1: *When both τ_i and τ_n are revenue increasing taxes, we have $\Delta_1 > 0$ and $d\tau_i / d\tau_n < 0$.*

Proof: We write equation (15) as $-dT + \lambda du + G_i d\tau_i + G_n d\tau_n = 0$. Treating du and dT as endogenous variables and $d\tau_i$ and $d\tau_n$ as exogenous in the equation just described and equation (14), we solve $(dT / d\tau_i) = \Omega^{-1}(\Omega G_i + \lambda F_i) = \Omega^{-1}\Delta_1$ and that $(dT / d\tau_n) = \Omega^{-1}(\Omega G_n + \lambda F_n)$. Since we assume $(dT / d\tau_i) > 0$, and $(dT / d\tau_n) > 0$, then we must have $\Delta_1 > 0$ and $(\Omega G_n + \lambda F_n) > 0$, and then from equation (17) $d\tau_i / d\tau_n < 0$. □

From Lemma 1, the following Lemma follows:

Lemma 2: *Suppose, without loss of generality, that the i^{th} good has the highest rate of excess taxation of consumption-pollution and is a substitute in consumption to all other goods. Then, when both τ_i and τ_n are revenue increasing taxes, we must have $G_i > 0$.*

Proof: It is easily verifiable that under the hypothesis of the Lemma, we have $F_i < 0$. From Lemma 1 we also know that $(\Omega G_i + \lambda F_i) > 0$.Therefore, G_i must be positive. Recalling from equation (14) the expression for G_i we note that the first term $\left(E_{p_i}\right)$is always positive thus G_i can be positive when the consumption level of the i^{th} commodity is very high, regardless of the sign of the second term. □

Lemmas 1 and 2 lead to the following Proposition:

Proposition 5: *Suppose there is only consumption pollution and, without loss of generality, the i^{th} (n^{th}) good exhibits the highest (lowest) rate of excess taxation of consumption-pollution. Then, when both τ_i and τ_n are revenue increasing and both goods are substitutes to all other goods in consumption, an increase in τ_n with an accompanying reduction in τ_i, to keep tax revenue unchanged, is welfare improving if the n^{th} good carries the lowest consumption-tax impact factor*.

Proof: From Lemmas 1 and 2 and the hypothesis of the Proposition, we have $\Delta_1 > 0, F_n > 0, G_i > 0$ and $F_i < 0$. Thus, a sufficient condition for $du / d\tau_n$ to be positive is that $G_n > 0$. It follows from the expression of G_n that it is indeed positive if the n^{th} good has the lowest *consumption-tax impact factor* and is a substitute to all other goods in consumption. □

Recall that from the definitions of (σ_k) and (μ_k) we have $\sigma_k = \mu_k - \delta^{-1} E_r(\beta_k / p_k)$. Thus, if the k^{th} good has the lowest *consumption-tax impact factor* and the highest rate of pollution per unit value of consumption, then it also carries the lowest *rate of excess taxation of consumption-pollution*. That is, the assumptions and the sufficient condition in the proposition need not contradict each other. In fact, the lack of contradiction can be established under much weaker conditions. For example, if the rate of pollution per unit value of consumption for all goods is constant, then the ranking of σ_j 's and μ_j 's, for all j's are perfectly correlated. All is needed for the above rank correlation to be perfect is that the variations in (β_j / p_j)'s are limited.

Intuitively, when n^{th} good exhibits the lowest *consumption-tax impact factor*, then by increasing its consumption tax rate, decreases its consumption and thus the effect on tax revenue is the lowest. When the n^{th} good has the highest rate of pollution per unit value of consumption, then the increase in its consumption tax rate reduces its consumption and thus it reduces pollution considerably. The reduction of the tax rate on the i^{th} good which has the highest *tax impact factor* and the lowest rate of pollution per unit value of consumption causes its consumption to increase which in turn leads to a sizable increase of the consumption tax revenue and to a small effect on pollution.

Next, with reforms in production and consumption taxes, equations (14) and (15) give the welfare effect of changing the production tax t_n, and adjusting the consumption tax τ_i so that tax revenues remain constant. That is,

$$\Delta_1\left(\frac{d\tau_i}{dt_n}\right)=-\Omega D_n+\lambda B_n \tag{19}$$

$$\Delta_1\left(\frac{du}{dt_n}\right)=-(B_n G_i+F_i D_n)\,. \tag{20}$$

Equations (A.5) in the Appendix provide details for the above derivations.

Proposition 6: *Suppose there is only consumption pollution and, without loss of generality, the i^{th} (n^{th}) good exhibits the highest (lowest) rate of excess taxation of consumption-pollution (production tax). Then, when τ_i and t_n are revenue increasing and the i^{th} (n^{th}) good is substitutes to all other goods in consumption (production), an increase in t_n with an accompanying reduction in τ_i, to keep tax revenue unchanged, is welfare improving.*

Lemmas 1 and 2, respectively, ensure, that $\Delta_1>0$ and $G_i>0$. The hypotheses of the proposition ensure that $F_i<0$, and $B_n<0$ since in the absence of production pollution the rate of excess taxation of any good, its production-tax impact factor and its production tax per its producer price are all equal, e.g., $s_n=\varphi_n=t_n/q_n$. Then, a sufficient condition for du/dt_n to be positive is that $D_n>0$. It follows from the expression of D_n that it is indeed positive if the n^{th} good is substitute to all other goods in production and it carries the lowest production tax.[14]

Note that in the case where we have only consumption pollution and adjust the consumption tax of one good and simultaneously appropriately adjust the production tax of another good so as to keep revenue constant, the sufficient conditions in Proposition 6 for welfare improvement are minimum. That is, it is only required that the two taxes are revenue increasing, and the two goods are respectively substitutes in consumption and production to all

[14] Note that $D_n=\left[R_{q_n}+\sum_{j\neq n}^{K}\left(\varphi_n-\varphi_j\right)q_j R_{q_j q_n}\right]$. Since $s_k=\varphi_k=t_k/q_k$, for k = n, j, D_n is positive if $\varphi_n-\varphi_j=(t_n/q_n)-(t_j/q_j)=(\frac{t_n}{1-t_n}-\frac{t_j}{1-t_j})=(t_n-t_j)/(1-t_n)(1-t_j)$ is negative for every j, given that goods are substitutes in production. This is true if t_n is the lowest.

other goods. As it can be easily inferred from the previous paragraph, these assumptions are similar to those invoked in the standard tax reform literature without pollution, for improving welfare.

Finally, by the same procedure, one can easily examine the welfare implications of consumption tax reforms (i.e., changes in τ_i) while appropriately adjusting the production tax t_n so as to maintain constant government tax revenue.

4.2 Tax reforms in the presence of production pollution

With only production pollution, we now use equations (14) and (15) to examine, the welfare implications and the required adjustments in order to maintain government revenue constant, of changes in production taxes.

In the case of changes only in production taxes, equations (14)-(15) produce the following matrix system:

$$\begin{bmatrix} \Omega_1 & B_i \\ \tau' E_{pu} & D_i \end{bmatrix} \begin{bmatrix} du \\ dt_i \end{bmatrix} = \begin{bmatrix} -B_n \\ -D_n \end{bmatrix} dt_n , \tag{21}$$

where, $\Omega_1 = E_u - \tau' E_{pu}$, and $\Delta_3 = (\Omega_1 D_i - \tau' E_{pu} B_i)$. Equations (A.6) in Appendix (III) provide the analytical equations of the above system. From (21), we obtain:

$$\Delta_3 \left(\frac{dt_i}{dt_n} \right) = -(\Omega_1 D_n - \tau' E_{pu} B_n) , \tag{22}$$

$$\Delta_3 \left(\frac{du}{dt_n} \right) = B_i D_n - B_n D_i . \tag{23}$$

Lemma 3: *When both* t_n *and* t_i *are revenue increasing taxes, we have* $\Delta_3 > 0$ *and* $dt_i / dt_n < 0$.

Proof: We write equation (15) as $-dT + \tau' E_{pu} du + D_i dt_i + D_n dt_n = 0.$ Treating du and dT as endogenous variables and dt_i and dt_n as exogenous in the equation just described and (14), we solve $(dT / dt_i) = \Omega_1^{-1}(\Omega_1 D_i - \tau' E_{pu} B_i) = \Omega_1^{-1}\Delta_3$ and that $(dT / dt_n) = \Omega_1^{-1}(\Omega_1 D_n - \tau' E_{pu} B_n)$. Since we assume $(dT/dt_i)>0$, and $(dT/dt_n)>0$, then we must have $\Delta_3>0$ and $(\Omega_1 D_n-\tau' E_{pu}B_n)>0$. These imply from equation (22) that $dt_i/dt_n<0$. □

Lemma 4 follows from Lemma 3.

Lemma 4: *Suppose, without loss of generality, that the i^{th} good exhibits the highest rate of excess taxation of production pollution, and it is a substitute in production to all other goods. Then when both t_n and t_i are revenue increasing taxes, we must have $D_i>0$.*

Proof: The assumption that the i^{th} good exhibits the highest rate of excess taxation of production pollution, and it is a substitute in production to all other goods implies that $B_i>0$. When both t_n and t_i are revenue increasing taxes, Lemma 5 Shows that $\Delta_3>0$, which implies that in order for $(dT/dt_i)>0$, we must have $D_i>0$. □

We can now state and prove the main result of this sub-section:

Proposition 7: *Suppose there is only production pollution and, without loss of generality, the i^{th} (n^{th}) good exhibits the highest (lowest) rate of excess taxation of production-pollution and both goods are substitutes to all other goods in production. Then, when both t_i and t_n are revenue increasing taxes, an increase in t_n with an accompanying reduction in t_i (to keep tax revenue unchanged), is welfare improving if the n^{th} good carries the lowest production-tax impact factor.*

Proof: Lemmas 3 and 4, respectively, ensure that $\Delta_3>0$ and $D_i>0$. The hypotheses of the proposition ensure that $B_i>0, D_n>0$ and $B_n<0$. Then, from equation (23) $\Delta_3\left(du/dt_n\right)>0$. □

Given the definitions of (s_n) and of (φ_n), we note that $s_n=\varphi_n-E_z(\alpha_n/q_n)$. Thus, if the n^{th} good has the lowest *production-tax impact factor* and the highest rate of pollution per unit value of production, then it also carries the lowest *rate of excess taxation of production-pollution*. That is, the assumptions and the sufficient condition in the proposition need not contradict each other. In fact, the lack of contradiction can be established under much weaker conditions. For example, if the rate of pollution per unit value of production for all goods is constant, then the ranking of s_j's and φ_j's, for all j's are perfectly correlated. All is needed for the above rank correlation to be perfect is that the variations in (α_j/q_j)'s are limited. Consider the special case where the rate of pollution per unit of value of production of all goods are the same, i.e., $[\alpha_j / q_j] = [\alpha_i / q_i]$, $\forall i, j$. In this case, the good with the lowest *rate of excess taxation of production-pollution* is the one with the lowest production tax per its price and the good with the highest *rate of excess taxation of consumption-pollution* is the one with the highest consumption tax per its price. Thus, in this case, a small increase in the production tax on the n^{th} good, while reducing the production tax on the i^{th} good to keep government revenue constant, increases social welfare if the n^{th} (i^{th}) good carries the lowest (highest) production tax, and both goods are substitute in production with all other goods. Similar arguments hold for the case of reforming the consumption taxes and $(\beta_j / p_j) = (\beta_i / p_i), \forall i, j$.

It can be easily shown that in the presence of only production pollution, the effects of reforms in production and consumption taxes are qualitatively similar to the effects of such reforms in the presence of only consumption pollution.

Compared to the case without revenue constraint, the conditions of the reforms that bring towards uniformity the *rates of excess taxation of production- and consumption-pollution* are not sufficient for welfare improvement under the revenue constraint. We need the additional conditions that both taxes are revenue increasing and that the good whose production tax increases, carries also the lowest *production-tax impact factor*. These conditions suffice to ensure that increasing its production tax raises the tax revenue. Intuitively, the increase in the production tax of the good with the lowest *production-tax impact factor* decreases its production and (i) the direct effect on tax revenue is small while (ii) the indirect effect on tax revenue from the increase in production of all other goods is positive because of the assumption of the substitutability in production.

5. Concluding Remarks

Recent developments in the theory and practice of economic policy making acknowledge the adverse consequences of expanded economic activity on the quality of environment. Such environmental degradation must then be accounted for when evaluating the welfare and other economic effects of various economic policies. The literature on tax reforms within an integrated system of indirect taxes, offers a limited insight on the welfare and government revenue implications of such tax reforms in the presence of pollution. Thus, in this paper we revisit the question of reforming the structure of indirect taxes in the presence of production and consumption-generated pollution, and we identify sufficient conditions under which such tax reforms improve welfare with and without a binding government revenue constraint.

The sufficient conditions under which the various tax reforms improve welfare with or without constant government revenue are stated in the relevant Propositions of the paper. Here, instead of restating these conditions, we note some analytical features related to our results. First, the presence of production generated pollution does not alter the known results of consumption tax reforms alone. Second, regardless of a binding revenue constraint, the proposed welfare improving reforms of production taxes alone, or of consumption and production taxes combined, are those bringing towards uniformity the rates of excess taxation of pollution. The same feature holds for the case of consumption generated pollution and of reforming consumption taxes so as to bring the rates of excess taxation of pollution towards uniformity. For example, consider the case of reforming production taxes alone. When there is no binding revenue constraint, a welfare improving reform entails increasing the production tax on the good exhibiting the lowest rate of excess taxation of pollution in a way such that this rate does not increase beyond the second lowest rate of excess taxation of pollution. When there is a binding revenue constraint, such a reform is accompanied by appropriate changes in the production tax on another commodity so that government revenue is kept constant. Third, regardless of the source of pollution, two of the critical conditions supporting the results are: (i) the relationship in consumption and/or production between the good whose tax is changed to all other commodities, and (ii) under a binding revenue constraint, all reformed taxes are revenue increasing.

In the absence of pollution, the tax reform literature has shown that welfare improves, for example, by increasing the lowest consumption tax and reducing the highest one. In the present model with pollution, as a general rule for policy recommendation we can say that one has to be more careful about which consumption tax to raise and which one to lower. In particular, one has to take into consideration the size of the consumption tax as well as the pollution intensity of the consumption of the good.

We conclude the paper by pointing out a few limitations of our analysis. First, we only consider environmental tax as an instrument. In reality one finds a number of different instruments such as abatement subsidy, tradable permits, emission standards (which also come in different forms). Second, we do not model abatement in an exogenous manner. Third, we do not allow for discriminatory environmental policies within a sector. Finally, we consider a perfectly competitive framework although in reality many industries exhibit imperfect competition. The four limitations are somewhat related as analysis of different environmental policy instruments --- uniform and non-uniform --- and an endogenous and explicit treatment of abatement are usually analysed in imperfectly competitive framework (see, for example, Lahiri and Ono, 2007; Lahiri and Symeonidis, 2007). We plan to take up these limitations in a future research project.

APPENDIX

I. Optimal taxes with pollution and with/-out binding revenue constraint

The Two-goods Case: We use the two-goods model as an illustrative example to derive the conditions for optimal taxes. As in the case without pollution, the property of uniformity of optimal policies does not imply uniqueness, for which one has to have some normalization of taxes. Now we assume two goods, 1 and 2, with good 1 being untaxed, i.e., $\tau_1 = 0$. Then, from the definition of $\sigma 's$ (see equation 5) we have:

$$\sigma_1 = \frac{\beta_1 \lambda}{p_1} \quad \text{and} \quad \sigma_2 = \frac{\tau_2}{p_2} + \frac{\beta_2 \lambda}{p_2}, \quad \lambda = \delta^{-1}\left(\tau_2 E_{p_2 r} - E_r\right). \tag{A.1}$$

Using the uniformity property of optimal $\sigma 's$, we set $\sigma_1 = \sigma_2$ in equations (A.7) to get:

$$\frac{\tau_2}{p_2} = \lambda\left(\frac{\beta_1}{p_1} - \frac{\beta_2}{p_2}\right). \tag{A.2}$$

Since $E_r > 0$, it is reasonable to assume that $\lambda < 0$. In fact if all consumption adjustments of a change in r fall on the first good, then $E_{p_2 r} = 0$ and λ is indeed negative. With this assumption from (A.8) we get:

$$\tau_2 \leq (\geq) 0 \qquad \text{according to whether} \quad \frac{\beta_1}{p_1} \geq (\leq) \frac{\beta_2}{p_2} \quad . \tag{A.3}$$

That is, the optimal consumption tax on good 2 is positive if and only if this good is more pollution-intensive than good 1.

II. Tax reforms under consumption pollution and a binding revenue constraint

(i). Changes only in consumption taxes τ_n and τ_i. Equations (5) and (13) become:

$$\Omega du = \sum_{j\neq i}^{K}(\sigma_j - \sigma_i) p_j E_{p_j p_i} d\tau_i + \sum_{j\neq n}^{K}(\sigma_j - \sigma_n) p_j E_{p_j p_n} d\tau_n ,$$

$$\lambda du + \left[E_{p_i} + \sum_{j\neq i}\left(\mu_j - \mu_i\right) p_j E_{p_j p_i} \right] d\tau_i + \left[E_{p_n} + \sum_{j\neq n}\left(\mu_j - \mu_n\right) p_j E_{p_j p_n} \right] d\tau_n = 0 , \qquad \text{(A.4)}$$

which produce the system of equations (16) in the main text.

(ii). Changes in consumption tax τ_i and production tax t_n. Equations (14) and (15) produce the following matrix system:

$$\begin{bmatrix} \Omega & -F_i \\ \lambda & G_i \end{bmatrix} \begin{bmatrix} du \\ d\tau_i \end{bmatrix} = \begin{bmatrix} -B_n \\ -D_n \end{bmatrix} dt_n , \qquad \text{(A.5)}$$

Note also that in this case, of only consumption pollution, $s_k = \varphi_k = \dfrac{t_k}{q_k}$.

III. Tax reforms under production pollution and a binding revenue constraint

With changes only in production taxes t_n and t_i, equations (5) and (13), become:

$$\Omega du = -\sum_{j\neq i}^{K}(s_j - s_i) q_j R_{q_j q_i} dt_i - \sum_{j\neq n}^{K}(s_j - s_n) q_j R_{q_j q_n} dt_n ,$$

$$\lambda du + \left[R_{q_i} + \sum_{j\neq i}^{K}\left(\varphi_i - \varphi_j\right) q_j R_{q_j q_i} \right] dt_i + \left[R_{q_n} + \sum_{j\neq n}^{K}\left(\varphi_n - \varphi_j\right) q_j R_{q_j q_n} \right] dt_n = 0 , \qquad \text{(A.6)}$$

which produce the system of equations (21) in the main text. Also, since consumption is a clean activity we have $\sigma_k = \mu_k = \dfrac{\tau_k}{p_k}$.

References

Abe, K., 1995, The target rates of tariffs and tax reforms, *International Economic Review* 36, 875-885.

Atkinson, A.B. and Stiglitz, J.E., 1976. The design of tax structure: Direct versus indirect taxation, *Journal of Public Economics*, 6, 55– 75.

Beghin, J., D. Holst and D. van der Mensbrugghe, 1997, Trade and pollution linkages: piecemeal reform and optimal intervention, *Canadian Journal of Economics* 30, 442-455.

Boadway, R. and M. Sato, 2009, Optimal tax design and enforcement with an informal sector, *American Economic Journal: Economic Policy* 1, 1-27.

Bovenberg, L., 1999, Green tax reform and the double dividend: An updated readers guide, *International Tax and Public Finance* 6, 421-442.

Bovenberg, L. and R.A. Mooij, 1994, Environmental levies and distortionary taxation, *American Economic Review* 84, 1085-1089.

Bovenberg, L. and R. van der Ploeg, 1994, Green policies and public finance in a small open economy, *The Scandinavian Journal of Economics* 96, 3, 343-363.

Copeland, B., 1994, International trade and the environment: policy reform in a polluted small open economy, *Journal of Environmental Economics and Management* 26, 44-65.

Diewert, W., A. Turunen-Red and A. Woodland, 1989, Productivity- and Pareto-improving changes in taxes and tariffs, *Review of Economic Studies* 56, 199-216.

Emran, S. and J. Stiglitz, 2005, On selective indirect tax reform in developing countries, *Journal of Public Economics* 89, 599-623.

European Union, 2010, Green Paper on the future of VAT: Towards a simpler, more robust and efficient VAT system, European Commission, Brussels, December 2010.

Goulder, L., 1995, Environmental taxation and the double dividend: A reader's guide, *International Tax and Public Finance* 2, 157-183.

Fullerton, D., A. Leicester, and S. Smith, 2008, Environmental taxes, NBER Working Papers 14197, National Bureau of Economic Research.

Hatta, T., 1977, A theory of piecemeal policy recommendation, *Review of Economic Studies* 44, 1-21.

Hatta, T., 1986, Welfare effects of changing commodity tax rates towards uniformity, *Journal of Public Economics* 29, 99-112.

Hatzipanayotou, P., M. Michael and S. Miller, 1994, Win-win indirect tax reform: a modest proposal, *Economics Letters* 44, 147-151.

Hatzipanayotou, P., S. Lahiri, and M. Michael, 2005, Reforms of environmental policies in the presence of cross-border pollution and public-private clean-up, *Scandinavian Journal of Economics* 107, 315-333.

Kayalica, O. and O. Kayalica, 2005, Transboundary pollution from consumption in a reciprocal dumping model, *Global Economy Review* 5, *Article* 7.

Keen, M., 1987, Welfare effects of commodity tax harmonization, *Journal of* Public Economics, 33, 107-114.

Keen, M. and S. Lahiri, 1998, The comparison between destination and origin principles under imperfect competition, *Journal of International Economics*, 45, 323-350.

Keen, M. and J. Ligthart, 2002, Coordinating tariff reduction and domestic tax reform, *Journal of International Economics* 56, 489-507.

Keen, M. and J. Ligthart, 2005, Coordinating tariff reduction and domestic tax reform under imperfect competition, *Review of International Economics* 13, 385-390.

Lahiri, S. and A. Nasimi, 2005, Commercial Policy Reform in Pakistan: Opening up the Economy under Revenue Constraint, *International Tax and Public Finance* 12, 7234-739.

Lahiri, S. and Y. Ono, 2007, Relative Emission Standard versus Tax under Oligopoly: The Role of Free Entry," *Journal of Economics*, 91, 107-128.

Lahiri, S. and G. Symeonidis, 2007, Piecemeal multilateral environmental policy reforms under asymmetric oligopoly," *Journal of Public Economic Theory*, 9, 885-899.

Lockwood, B., D. Meza, and G. Myles, 1994, When are origin and destination regimes equivalent? *International Tax and Public Finance*, 1, 5-24.

Michael, M., P. Hatzipanayotou and S. Miller, 1993, Integrated reforms of tariffs and consumption taxes, *Journal of Public Economics* 52, 417-428.

Majumdar, S., 2004, Revenue implications of trade liberalization under imperfect competition, *Economics Letters* 82, 83-89.

Naito, T., 2005, Revenue-neutral environmental tariff reform, growth and welfare, *Review of International Economics* 13, 985-996.

Naito, T., 2006, Growth, revenue, and welfare effects of tariff and tax reform: Win-win-win strategies, *Journal of Public Economics* 90, 1263-1280.

Naito, T. and K. Abe, 2008, Welfare- and revenue-enhancing tariff and tax reform under imperfect competition, *Journal of Public Economic Theory* 10, 1085-1094.

Neary, J., P., 1998, Pitfalls in the theory of international trade policy: Concertina reform of tariffs and subsidies to high technology industries, *Scandinavian Journal of Economics* 100, 187-206.

Neary, P. and F. Ruane, (1988), International capital mobility, shadow prices and the cost of protection, *International Economic Review* 29, 571-585.

Saez, E., 2002, The desirability of commodity taxation under non-linear income taxation and heterogeneous tastes, *Journal of Public Economics*, 83, 217-230.

Turunen-Red, A. and A. Woodland, 2004, Multilateral reforms on trade and environmental policy, *Review of International Economics* 12, 321-336.

Wilson D. J., 1991, Optimal public good provision with limited lump-sum taxation, *American Economic Review* 81, 153-166.

World Bank, 2002, *World development Indicators 2002*, World Bank, Washington, DC.