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Volatility Forecast Combinations using Asymmetric Loss Functions*

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Abstract

The paper deals with the problem of model uncertainty in forecasting volatility using forecast combinations and a flexible family of asymmetric loss functions that allow for the possibility that an investor would attach different preferences to high vis-à-vis low volatility periods. Using daily as well as 5 minute data for US and major international stock market indices we provide volatility forecasts by minimizing the Homogeneous Robust Loss function of the Realized Volatility and the combined forecast. Our findings show that forecast combinations based on the homogeneous robust loss function significantly outperform simple forecast combination methods, especially during the period of the recent financial crisis.

Keywords: asymmetric loss functions, forecast combinations, realized volatility, volatility forecasting.
JEL Classifications: C53, C52, C58

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1 Introduction

Forecasting volatility is an important area of research in financial markets due to its role for pricing derivatives, asset allocations decisions, and risk management. This need for volatility forecasts has generated a vast literature on model and methods starting from the seminal work of Engle (1982) and Bollerslev (1986) on (G)ARCH volatility models. Andersen, Bollerslev, Christoffersen, and Diebold (2006) and more recently Hansen and Lunde (2011) provide excellent surveys on the most important developments on volatility forecasting. Despite the vast amount of empirical research on forecasting volatility, there is remarkably little consensus on which models are most salient in providing accurate predictions. As a result forecasters are faced with volatility model uncertainty about the choice of the family models (e.g. GARCH-type, (H)AR-RV, etc.) as well as model uncertainty about lags, leverage effects, functional forms, structural breaks, etc.

This problem of volatility model uncertainty has been highlighted by the recent financial crisis, especially in risk management where the volatility forecast is a key input to models such as the Value at Risk (VaR) and the Expected Shortfall (ES).[1] In particular, many financial institutions suffered large losses in the wake of the recent financial crisis. The Internationally Monetary Fund (IMF) in 2009 estimated that the total losses on US assets due to the financial crisis to be around 4.2 billion dollars. To us this uncertainty with respect to the appropriate volatility model suggests that forecast combinations can provide more a accurate forecast of volatility and thereby more robust and precise VaR and ES forecasts. This argument is consistent with Timmermann (2006) who emphasizes the benefits of forecast combinations since forecast combinations use evidence from all the models rather than relying on an individual model. More importantly, under certain conditions, forecast combinations can be robust to structural breaks as argued by Aiolfi and Timmermann (2006).

In this paper we deal with two objectives. First, we explicitly deal with model uncertainty with respect to the volatility specification. Specifically, we provide robust volatility forecasts based on volatility forecast combinations using a comprehensive model space rather than conditioning on a specific volatility model. We consider three different families of volatility models: (G)ARCH type models, high frequency realized volatility models and non-parametric rolling volatility models. Second, we study forecast combinations based on asymmetric loss functions as they are more appropriate than symmetric loss functions such as the Mean Square Error. For instance, in risk management an asymmetric loss function is more appropriate since under-prediction of volatility is more important and costly compared to over-prediction. This argument is especially relevant during high volatility periods and the recent financial crisis. Therefore, we investigate whether a framework that addresses model uncertainty and combines forecasts based on asymmetry in the loss function can improve our volatility predictions. Our empirical analysis uses daily

[1]Jorion (2009) argues that uncertainty associated with model specification is one possible reason for the failure of the existing risk management techniques.

and 5 minute data for S&P 500 and NASDAQ as well as for four other major international stock market indices including FTSE 100, DAX 30, CAC 40, and NIKKEI 225.

Combining volatility forecasts raises two challenges. First, note that the true conditional volatility is a latent variable, which needs to be estimated and as a result volatility is only observed with measurement error. This problem was recently revisited by Patton (2011) who shows that the effect of the measurement error can be large and it can vary substantially with the choice of loss function. This implies that the performance of the volatility forecast combination will generally depend on the effects of the error and the choice of loss function on the combination weights. Second, given that in risk management under-prediction of volatility is by far more important than over-prediction, asymmetric loss functions are more relevant than symmetric ones such as the Square Error.

Following Patton and Sheppard (2009), we address the above challenges using forecast combinations of volatility based on the Homogeneous Robust (HR) loss function, which has been found by Patton (2011) to enjoy several desirable properties. First, the measurement error due to the volatility estimation does not affect the rankings of the competing volatility forecasts. Second, it is a very flexible loss function that can take infinite shapes using a single parameter that determines its shape. For example, it nests the popular loss functions of the Square Error and QLIKE. For robustness we also consider the LINEX as an alternative asymmetric loss function. In general, our proposed forecast combination method is also in the spirit of Elliot and Timmermann (2004) who investigate forecast combinations under asymmetric loss functions. Elliot and Timmermann show that these asymmetries in the loss function have an important effect in optimal weights and improve the performance of the underlying forecasts.

As our ex-post estimator for the true conditional volatility we use the Realized Volatility (RV) proposed by Andersen, Bollerslev, Diebold, and Ebens (2001), Andersen, Bollerslev, Diebold, and Labys (2003), Bardorff-Nielsen and Shephard (2002b), and Bardorff-Nielsen and Shephard (2002a) based on intra-daily returns. According to Patton (2011) the use of unbiased and less noisy estimators such as RV may alleviate the measurement error problem partly.

Our paper is closely related to Patton and Sheppard (2009) who also study forecast combinations using the HR loss function. The main difference is that while they focus on forecasts based on high frequency realized estimators of volatility we have a richer model space that includes in addition to high frequency volatility specifications (G)ARCH and (H)AR-RV type models. Another difference is that while their analysis is limited to the volatility of IBM returns we study the volatility of returns on S&P 500, NASDAQ, and other major international stock market indices. In terms of the broader literature our paper is related to Fuertesa, Izzeldinb, and Kalotychoua (2009) who employ forecast combination of GARCH models and Liu and Maheu (2009) who employ Bayesian Model Averaging (BMA) to forecast realized volatility. However, these papers do not consider the effect of the estimation noise on the weights and do not consider forecast combinations using the

Homogeneous Robust loss function.

Our findings emphasize the importance of volatility forecast combinations and the use of asymmetric loss functions for obtaining the combined forecasts as well as for out-of-sample evaluation. The evidence is strongest for the loss functions that correspond to the parameters of the loss functions that penalize under-prediction more heavily than over-prediction and as expected they are especially useful in risk management given the importance of downside risk. In particular, we find that forecast combinations based on the Homogeneous Robust Loss function, and in particular with a shape similar to QLIKE, provide the smallest significant forecast losses in forecasting volatility. Among the individual models the best performing model is the LHAR-RV that captures the leverage effect of returns at daily, weekly and monthly frequencies during the most recent period up to 2010. Overall, we find that our methods are especially useful during the period of the recent financial crisis according to which our forecast combination methods using the asymmetric loss function provide substantial improvements in forecasting volatility.

The paper is organized as follows. Section 2 describes our methodology on forecast combinations using asymmetric loss functions. Section 3 describes our data. Section 4 describes the results on volatility forecast combinations for S&P 500. Section 5 presents a robustness analysis by extending our analysis to international stock market indices and LINEX and finally Section 6 concludes.

2 Robust Volatility Forecast Combinations

Our forecast combination strategy follows Patton and Sheppard (2009) who employ volatility forecast combinations based on the HR loss function of Patton (2011). They show that it is possible to consistently estimate the optimal combination weights from the data, by employing a robust loss function.

We proceed in stages. First, we construct a good estimator of the true conditional volatility. Using intra-daily 5 minute data we follow Andersen, Bollerslev, Diebold, and Ebens (2001), Andersen, Bollerslev, Diebold, and Labys (2003), Bardorff-Nielsen and Shephard (2002b), and Bardorff-Nielsen and Shephard (2002a) to estimate the *ex-post* Realized Volatility (RV).[2] RV is defined as the sum of squared intra-daily log returns $Y(t) = RV_t = \sum_{j=1}^{m} r_{t,j}^2$ and enjoys attractive properties. It is a consistent estimator of Quadratic Variation and exhibits a superior performance of the RV against the daily squared returns as documented in several papers (e.g.Andersen, Bollerslev, Diebold, and Labys (2001), Hansen and Lunde (2006), and Hansen and Lunde (2005)).

[2]The choice of 5 minute data is justified on the grounds of past empirical findings that show that at this frequency there is no evidence of microstructure noise; see for example Andersen, Bollerslev, Diebold, and Labys (2001).

Second, we consider combinations of individual forecasts over a large number of low and high-frequency volatility models. Table 1 defines the model space, M, which includes four broad families of volatility models: (i) We consider the simple Autoregressive models of Realized Volatility (AR-RV) using 1, 5, 10 and 22 lags, which correspond to the trading periods of one day, one week, two weeks, and one month, respectively. (ii) We use the Heterogeneous Autoregressive model of Realized Volatility (HAR-RV) (Corsi (2009)) and the Leverage HAR-RV (LHAR-RV)(Corsi and Reno (2009)). These models have been found to be successful in modeling the long memory behavior of volatility in a very simple and parsimonious way. HAR-RV is a simple AR-type model in the RV with the feature of considering volatilities realized over different interval sizes such as days, weeks, and months. LHAR-RV includes leverage effects that influence each market component separately, and appear aggregated at different horizons in the realized volatility dynamics. (iii) We use four GARCH-type volatility models: the GARCH of Bollerslev (1986), the TARCH or GJR-GARCH ofGlosten, Jagannathan, and Runkle (1993), which has an additional parameter to capture the leverage effect, the APARCH of Ding, Granger, and Engle (1993), which nests the two aforementioned models and does not restrict the power of returns to be equal to 2, and the EGARCH of Nelson (1991), which has logarithmic structure and therefore, provides positive volatility forecasts without imposing any restrictions to its parameters.[3] In this family we also include the RiskMetrics of J.P.Morgan, which can be viewed a special case of the IGARCH volatility model. (iv) Finally, we consider nonparametric models of volatility using Rolling Volatility estimators based on rolling windows of 30 and 60 daily observations.

Let $\widehat{w}_{m,t}$ be a weight of the m'th forecast based on an individual volatility model from M in period t . Let also define the vectors $w_t = (w_{t,1}, ..., w_{t,M})$ and $\widehat{h}_t = (\widehat{h}_{1,t}, \widehat{h}_{2,t}, ..., \widehat{h}_{M,t})'$. Then given M approximating volatility models, forecast combinations are weighted averages of the individual forecasts,

$$\widehat{h}_t(w_t) = \widehat{h}_t' w_t = \sum_{m=1}^{M} w_{m,t} \widehat{h}_{m,t}, \tag{2.1}$$

Then we obtain the HR forecast combination (HRFC) by choosing weights that minimize

[3]The Matlab codes and formulas of the GARCH-type models of this paper are based on the MFE toolbox (October 2009) written by Kevin Sheppard.

the HR loss function given in equation (2.2).

$$L^{HR}(Y_t, \widehat{h}_t(w_t); b) = \begin{cases} \frac{1}{(b+1)(b+2)}(Y_t^{b+2} - (\widehat{h}_t(w_t))^{b+2}) - \frac{1}{b+1} h_t^{b+1}(Y_t - (\widehat{h}_t(w_t)), & \text{if } b \notin \{-1,-2\} \\ \widehat{h}_t(w_t) - Y_t + Y_t \log \frac{Y_t}{\widehat{h}_t(w_t)}, & \text{if } b = -1 \\ \frac{Y_t}{\widehat{h}_t(w_t)} - \log \frac{Y_t}{\widehat{h}_t(w_t)} - 1, & \text{if } b = -2 \end{cases} \tag{2.2}$$

The HR loss function is in fact a family of loss functions controlled by a scalar parameter, b , that controls the shape of the function. It is robust to the "noise" caused by the estimation the true conditional volatility in the sense that it does not distort the relative rankings of competing volatility forecasts. The HR loss function is also homogeneous of degree b.[4] This class of loss functions nests two of the most popular loss functions, namely the Mean Square Error (MSE) ($b = 0$) and the QLIKE ($b = -2$). Another advantage of HR is that is in addition to the forecast error, it also depends on RV. This allows extra flexibility in periods of low or high volatility. We concentrate on Homogeneous Robust loss functions with non positive shape parameter ($b \leq 0$) that penalize under-prediction of volatility more heavily than over-prediction. Figure 1(a) shows the Homogeneous Robust loss function for various values of the shape parameter.

Following Timmermann (2006) we impose the convexity constraints $0 \leq \widehat{w}_{m,t} \leq 1$ and additivity constraint $\sum_{m=1}^{M} \widehat{w}_{m,t} = 1$. These constraints also enable the interpretability of the weights as a measure of the importance of each individual model. Specifically, we choose the HR weights as follows

$$\widehat{w}_t = \underset{0 \leq w_{t,m} \leq 1, \sum\limits_{m=1}^{M} w_{t,m} = 1}{\arg\min} L^{HR}(Y_t, \widehat{h}_t(w_t); b) \tag{2.3}$$

Note that in the case of the Mean Square Error, that is $b = 0$, HR forecast coincides with the constrained version of Granger and Ramanathan (1984) method. [5]

[4]According to Patton (2011) a loss function is homogeneous of degree k if it satisfies the property $L(\alpha RV_t, \alpha h_t) = \alpha^k L(RV_t, h_t)\ \forall \alpha > 0$.

[5]Unbiasedness of the constrained forecast combination requires unbiasedness of all the individual forecasts. This assumption is not necessary when we ignore the restriction and obtain an unconstrained volatility forecast combination. For robustness we also studied unconstrained forecast combination without finding substantial differences.

3 Data

Our database involves daily and 5 minute data of two major US and four international stock market indices from the US. Specifically we use the S&P 500 Index, NASDAQ Composite for the US stock markets, the German DAX 30, the UK FTSE 100, the French CAC 40 and the Japanese NIKKEI 225. The S&P 500 index covers the period February 1, 1983 to June 30, 2010 and it is the relatively longer historical time series available in our database. The other stock market indices are available from July 1, 2003 to June 30, 2010.

For the S&P 500 we consider the evaluation period from January 2, 2004 to June 30, 2010 as well as two subsamples that allow us to study the differential effect of the financial crisis. In particular, we split the S&P 500 Index sample in two subsamples using the event of the downgrade by Standard and Poor's as well as Moody's of 100 bonds backed by subprime mortgages that occurred on June 1, 2007. The first subsample covers the period from January 2, 2004 to May 31, 2007 and it is characterized by low volatility. During this period there is an increase in the number of subprime mortgages due to the drop of interest rates. The second subsample covers the period from June 1, 2007 to June 30, 2010. The most important event during this period is the drop in house prices, which caused the stock market crash and drove a number of financial institutions to bankruptcy (e.g. Lehman Brothers). This period is especially interesting in risk management since it consists of a lot of extreme events and it is characterized by rapid changes in volatility.

In terms of the other indices we only consider a high volatility subsample due to their shorter availability. The out of sample period for DAX 30 spans June 1, 2007 to June 30, 2010, for NASDAQ Composite, FTSE 100 and CAC 40, July 2, 2007 to June 30, 2010 and for NIKKEI 225, August 1, 2007 to June 30, 2010. Figures 2 and 3 show the annualized Realized Volatility based on 5 minute data of all the stock market indices for the corresponding out of sample evaluation period.

4 S&P 500 Volatility Forecasts

We provide rolling one-step ahead out-of-sample forecasts to evaluate the predictive ability of our methods. We choose to use a rolling estimation to ensure that our evaluation will be robust to the instability we observe in the financial series, especially due to the recent financial crisis. Specifically, we divide the sample of size T into an in-sample rolling window of size $m = 1000$ and an out-of-sample window of size $n = T - m$. At time $t = m$ we obtain the first one-step ahead forecast at time m using data $1, ..., m$ and compare it with Y_{m+1}. At time $t = m + 1$ we obtain the second one-step ahead forecast and compare it with the observed Y_{m+2}. This procedure is iterated to produce $n = T - m$ out-of-sample forecasts.

We assess the out-of-sample forecast accuracy of each volatility forecast combination method using the HR loss function given in equation (2.2). Given that the out-of-sample loss corresponding to the forecast at time t is given by $L^{HR}(Y_{t+1}, \widehat{h}_t(w_t); b)$, we can then define the out-of-sample mean losses as follows.

$$\overline{L}_{m,n} = \frac{1}{n} \sum_{t=m}^{T-1} L^{HR}(Y_{t+1}, \widehat{h}_t(w_t); b) \tag{4.4}$$

We should point out that in the case of HRFC both the forecast combination weights and the out-of-sample evaluation are based on exactly the same loss function and hence better performance is expected. [6]

4.1 Out-of-sample Evaluation

The out-of-sample evaluation of the volatility forecasts of the S&P500 using two approaches: the combination methods which are presented in Table 2 and the best performing individual models within each volatility family in Table 3.

In particular Table 2 shows the out-of-sample losses of volatility forecasts of the S&P500 using the Homogeneous Robust Loss Function for a range of values of the parameter b ranging from the MSE ($b = 0$) to the QLIKE loss ($b = -2$) and other values of b that yield less or more asymmetry than the QLIKE. The results are organized following the four combination methods: Mean, Median, Geometric Mean (GMean), and Homogeneous Robust Forecast Combinations (HRFC). We compare these combination methods for value of b in the full sample 1/2/2004 - 6/30/2010 and in the two subsamples before and after the wake of the recent financial crisis marked in 6/2007.

An important additional aspect of volatility forecast evaluation, which provides additional insights on the performance of combination methods is the performance of the combination weights for the four volatility families over different loss functions and sub-periods (marked by low and high volatility). These results are presented graphically in Figure 4.

In synthesizing the results of Table 2 and Figure 4 regarding the performance of the combination methods for the S&P500 we obtain the following main results.

First, in all three samples HRFC method always has smaller losses compared to the rest of the forecast combinations (Mean, Median, Geometric Mean) for a given value of b and across the range of b values considered. An explanation for the better performance of the HRFC method vis-à-vis the mean type methods is the fact that it allows varying forecasting weights. Therefore, turning to Figure 4, which presents the HR combinations weights

[6] A similar argument was made by Christoffersen and Jacobs (2004) who emphasized the consistency of the loss function in parameter estimation and evaluation of option valuation models. Instead, we emphasize the consistency of the forecast combination method and evaluation.

that correspond to the four families of volatility models (AR-RV, HAR-RV, GARCH and Nonparametric) we obtain the following results: The HAR-RV and LHAR-RV models obtain the highest weights compared to the other families of models in both the first subsample which is tranquil period (before June 2007) and in the beginning of the second subsample before 2008, for all the loss functions considered here (both symmetric and asymmetric). However, in the second sub-period and after 2008, in the presence of asymmetries in the loss function (QLIKE $b = -2$ and HR with $b = -3, -4$), the forecasting weights of the AR-RV models steadily increase compared to those before 2008 and to the HAR family of models. One explanation is that the AR-RV could be more robust to misspecification. Another interesting aspect of the combination weights obtained in Figure 4, using the MSE loss function in the second subsample relates to the fact that the GARCH family of models obtains the highest weights relative to the other models. This is due to the result that the EGARCH model is the best performing model given that it is a model which allows for leverage, captures asymmetry of news and does not impose parameter restrictions.

Second, in the full sample and second subsample characterized by the financial crisis, if an investor or policy maker has MSE preferences or a symmetric loss function ($b = 0$ in HR loss function), then her losses of the S&P500 volatility forecasting are higher vis-à-vis those of asymmetric preferences and HR loss functions with ($b < 0$). In fact the MSE loss functions performs 43 times worse than the QLIKE in the most recent financially turmoil period and vis-à-vis the other subsamples. The QLIKE ($b = -2$) provides the smallest losses in forecasting volatility in this most recent period. It is only during the tranquil period of 1/2/2004-5/31/2007, that MSE preferences yield low losses and even 4 times lower than those of the QLIKE method. Hence our findings suggest that during highly volatile periods the asymmetric loss (especially the QLIKE) functions provide considerably improved volatility forecasts compared to the tranquil periods. Similarly, in periods characterized by both high and low volatility the asymmetric loss functions and in particular the QLIKE performs around 3 times better than the MSE loss function.

Last but not least, we turn to Table 3 to get a deeper insight of the out-of-sample forecasting performance of individual models within the four families of volatility models. For the QLIKE and $b < -1$, the LHAR-RV and AR-RV (with 10 and 22 lags) perform better than the rest of the RV type models and among the GARCH models the Normal-EGARCH is the best performing model especially in the full- and high-volatility sample. However, comparing the out-of-sample losses of volatility forecasts of the combination methods in Table 2 and in particular HRFC we find that this combination method outperforms even the best performing individual forecasting models in Table 3 in all subsamples, for a given b value.

4.2 Conditional Predictive Ability

In this section we provide formal testing procedures to compare the out-of-sample performance of our forecast combination methods. In particular, we employ the CPA test of Giacomini and White (2006), which accounts for estimation uncertainty and is also valid for both nested and non-nested hypotheses. The null hypothesis of this test is given by $H_0 : E((L(Y_t, \widehat{h}_t^A(w_t); b) - L(Y_t, \widehat{h}_t^B(w_t); b))|\Im_t) \equiv E(\Delta L_{m,t+1}|\Im_t) = 0$, where $\Im_t$ is some information set. $\widehat{h}_t^A(w_t)$ and $\widehat{h}_t^B(w_t)$ refer to two different volatility forecasts based on either combinations or individual models. The CPA test statistic is a Wald-type statistic of the following form

$$CPA_{m,n} = n\overline{Z}'_{m,n}\widehat{\Omega}_n^{-1}\overline{Z}'_{m,n} \tag{4.5}$$

where $\overline{Z}_{m,n} \equiv n^{-1}\sum_{t=m}^{T-1} Z_{m,t+1}$, $Z_{m,t+1} \equiv \eta_t \Delta L_{m,t+1}$. $\widehat{\Omega}_n$ is the Newey and West (1987) HAC estimator of the asymptotic variance of $Z_{m,t+1}$. η_t is a q dimensional vector of test functions, which is chosen to embed elements of the information set that are expected to have potential explanatory power for the future difference in predictive ability. Here, we follow Giacomini and White (2006) and use $\eta_t = (1, \Delta L_t)'$, which corresponds to the difference of squared residuals in the last period. Under the null of equal conditional predictive ability $CPA_{m,n}$ asymptotically follows a χ_q^2 distribution.

To determine which method performs best we use the two-stage decision rule described in Giacomini and White (2006). Step 1: regress $\Delta L_{m,t+1} = L(Y_t, \widehat{h}_{t+1}^A) - L(Y_t, \widehat{h}_{t+1}^B)$ on the test function η_t over the out-of-sample period $t = m, ..., T-1$ and obtain the regression coefficients $\widehat{\delta}$. Apply the one-step ahead CPA test described above using 10% for size of the test. In case of rejection, proceed to Step 2. Step 2: Define the following decision rule based on the approximation $E(\Delta L_{m,t+1}) \approx \widehat{\delta}'\eta_T$: use $\widehat{h}_{t+1}^A$ if $\widehat{\delta}'\eta_T > 0$ and use $\widehat{h}_{t+1}^B$ if $\widehat{\delta}'\eta_T < 0$.

Table 4 presents the CPA tests that correspond to Table 2. As before we present the results for the full sample and the two subsamples of the S&P 500 and the various values of b that correspond to different preferences defined by the HR loss function. Each panel provides CPA tests for the four combination methods of Mean, Median, GMean, and Homogeneous Robust Forecast Combinations (HRFC). In general the HRFC significantly outperform the other three forecast combination methods with a few notable exception in the second subsample. In the cases of MSE preferences ($b = 0$) and extreme asymmetries ($b < -3.5$) GMean appears to have a superior performance. This finding is consistent with the fact that QLIKE preferences ($b = -2$) provide the smallest losses in forecasting volatility in this most recent period.

Next we provide evidence from an alternative asymmetric loss function as well as from other stock market indices.

5 Robustness

5.1 LINEX

For robustness we consider LINEX as an alternative class of asymmetric loss functions, which was introduced by Varian (1975) and later employed by Zellner (1986). LINEX can be solely expressed in term of forecast errors, $\widehat{e}_t = Y_t - \widehat{h}_t(w_t)$.

$$L^{LINEX}((Y_t, \widehat{h}_t(w_t); a) = \exp(a\widehat{e}_t) - a\widehat{e}_t - 1, \tag{5.6}$$

where a is a scalar parameter that controls the degree of asymmetry. As it is implied by its name, when $a > 0$ LINEX is approximately linear for negative errors and approximately exponential for positive errors, which implies that the loss for underprediction is larger than for overprediction. The converse is true for $a < 0$. As in the case of the HR loss function we focus on $a > 0$ because underprediction is more devastating than overprediction in the context of risk management. Figure 1(b) shows the LINEX loss function for different values of the shape parameter.

We present the LINEX results in Tables 5 and 6 that correspond to Tables 2 and 3, respectively. There are two differences. First, we obtain forecast combinations, which are based on LINEX (LFC) rather than on HRFC. And second, the out-of-sample evaluation of the volatility forecasts of the S&P500 for all combination methods is now based on the LINEX loss function.

First, our results show that in all three samples the LFC method provides the smallest losses compared to the other methods for almost all values of the shape parameter a. As the degree of asymmetry, a, increases we see that the losses become larger. Interestingly, the greater the value of a, the greater is the relative difference in losses between LFC and the rest of the methods, especially in the second subsample. For example, while the relative gains for LFC against GMean for $a = 0.005$ are about 15%, these gains grow up to 98% for $a = 0.5$.

In terms of the individual volatility models, Table 6 shows that best models are GARCH and HAR-RV. In the case of the first subsample HAR-RV models appear to be best performers for all values of a. In contrast, the high volatility subsample as well as the full sample HAR-RV are the best for rather asymmetric shapes of LINEX, $a \geq 0.1$. For $a \leq 0.1$ we find that GARCH type models exhibit superior performance.

Overall, the results show that forecast combinations based on an asymmetric loss functions, either LINEX or HR are preferable than simple combinations. Furthermore, different loss functions that correspond to different preferences may provide the investor with a lot of information depending on the degree of volatility.

5.2 Further Evidence from Other Indices

In this subsection we provide further evidence from other stock market indicators. We consider NASDAQ, which generally exhibits a higher volatility than S&P500 as well as four international stock market indicators, the German DAX 30, the UK FTSE 100, the French CAC 40 and the Japanese NIKKEI 225. Due to data availability, the out-of-sample evaluation focuses on the second subsample using a rolling window of 500 observations. A summary of our results is presented in Table 7, which includes results for both HR and LINEX loss functions. For conciseness we only present results for the HR loss function with $b = 0, -2, -4$ and for LINEX with $a = 0.005, 0.05, 0.5$. [7]

Starting from the HR loss function we find that HRFC provide the smallest losses against the competing forecast combination methods with a few notable exceptions. In the case, of $b = 0$ or MSE preferences we find that HRFC fails to give the smallest losses for FTSE 100, DAX 30, and NIKKEI 225. One reason for this difference is the fact these 3 international indices do not exhibit as much volatility as the stock market indicators of the US and Japan and therefore simple forecast combinations may as well work well for MSE preferences. Another reason is, as we argued earlier, MSE preferences may not be the appropriate loss function in periods of financial turmoil. Instead, asymmetric preferences may be more relevant since under-prediction of volatility is more important and costly compared to over-prediction. In support of this argument we find that QLIKE preferences provide the smallest losses for all of these indicators as it was the case for S&P500. The results for LINEX are similar, albeit weaker, which may be due to the fact that the estimated combination weights are distorted due to the measurement error since LINEX is not a robust loss of functions.

6 Conclusion

The paper addresses the problem of model uncertainty in forecasting volatility using a flexible family of asymmetric loss functions that allow for the possibility that an investor or a policy maker would attach different preferences to high vis-à-vis low volatility periods. In particular, we employ the Homogeneous Robust Loss function, which is a flexible function that takes a wide variety of shapes ranging from symmetric (Square Error) to asymmetric with a heavy penalty on under prediction (e.g. QLIKE). We show that forecast combinations with weights that are estimated by minimizing the Homogeneous Robust Loss function of the Realized Volatility and the combined forecast from a large model space, significantly outperform the majority of individual models and simple forecast combination methods.

Specifically, using US and major international stock market indices we show that forecast combinations based on Homogeneous Robust Loss function significantly outperform the majority of individual models and other simple forecast combination methods. More

[7]Full results are available upon request.

importantly, forecast combinations based on a loss function such as the QLIKE provides the smallest significant forecast losses in volatility forecasting during period of the financial crisis. Furthermore, our findings suggest that asymmetric loss functions such as QLIKE experience smaller losses than the MSE. Among the individual models the best performing model is the LHAR-RV that captures the leverage effect of returns at daily, weekly and monthly frequencies during the most recent period up to 2010. In sum, our findings emphasize the importance of volatility forecast combinations and the use of asymmetric loss functions for both obtaining the combination weights and out-of-sample evaluation.

Figure 1: We show various shapes of the Homogeneous Robust and LINEX loss functions. The horizontal axis corresponds to the values volatility forecast and the vertical axis to the losses using alternative loss functions. The true conditional variance is assumed to be equal to $\sigma_t^2 = 10$.

(a) Homogeneous Robust Loss Function

(b)LINEX Loss Function

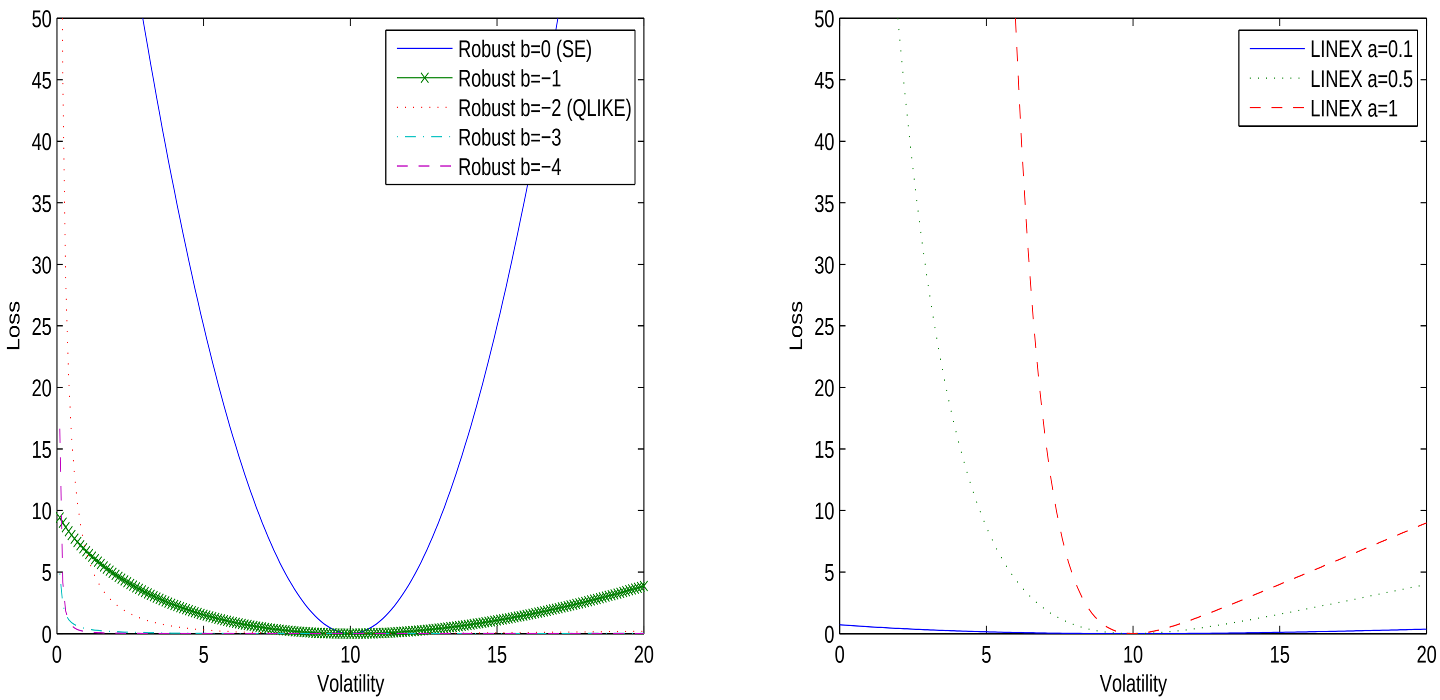


Figure 2: This figure shows the annualized Realized Volatility based on 5-min calendar-time trade prices for S&P 500 stock market index using the formula $\sigma_t = \sqrt{252 RV_t}$.

(a) S&P 500: 1/2/2004 - 5/31/2007

(b) S&P 500: 6/1/2007 - 6/30/2010

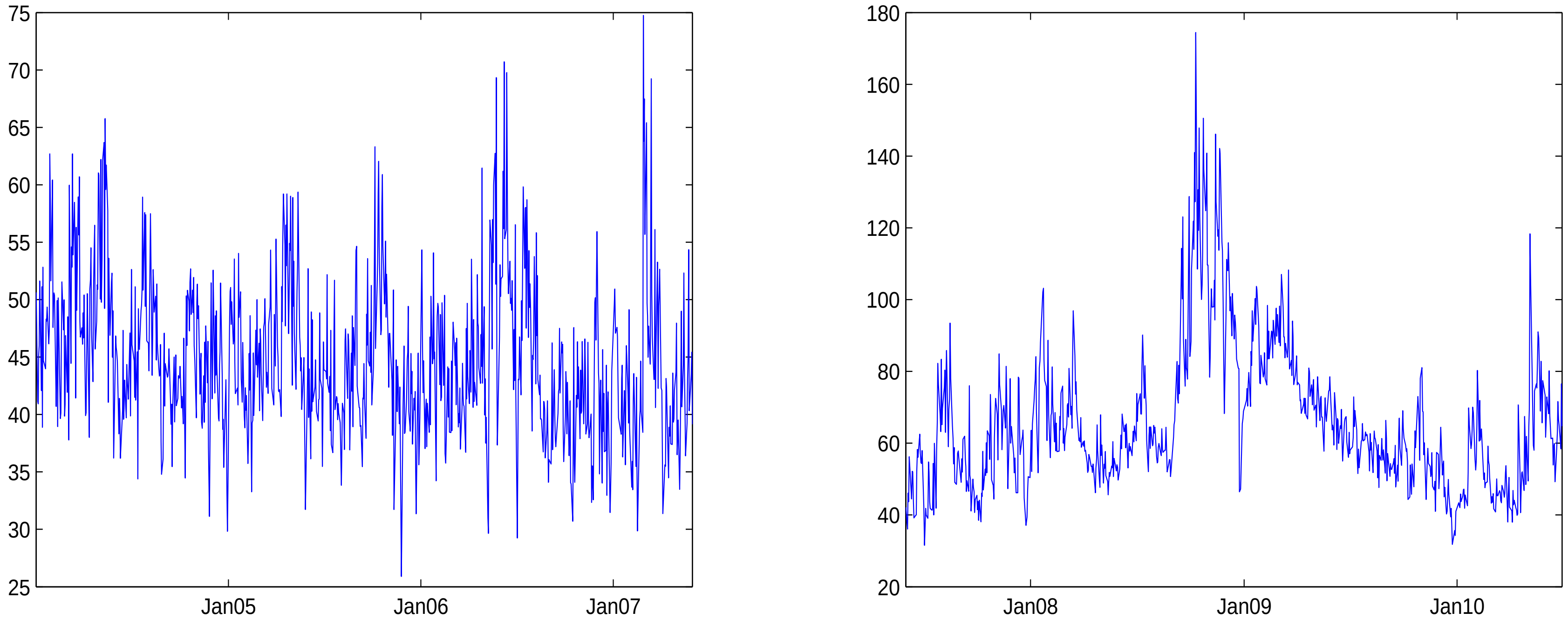


Figure 3: This figure shows the annualized Realized Volatility based on 5-min calendar-time trade prices for NASDAQ, FTSE 100, DAX 30, CAC 40, and NIKKEI 225 using the formula $\sigma_t = \sqrt{252 RV_t}$.

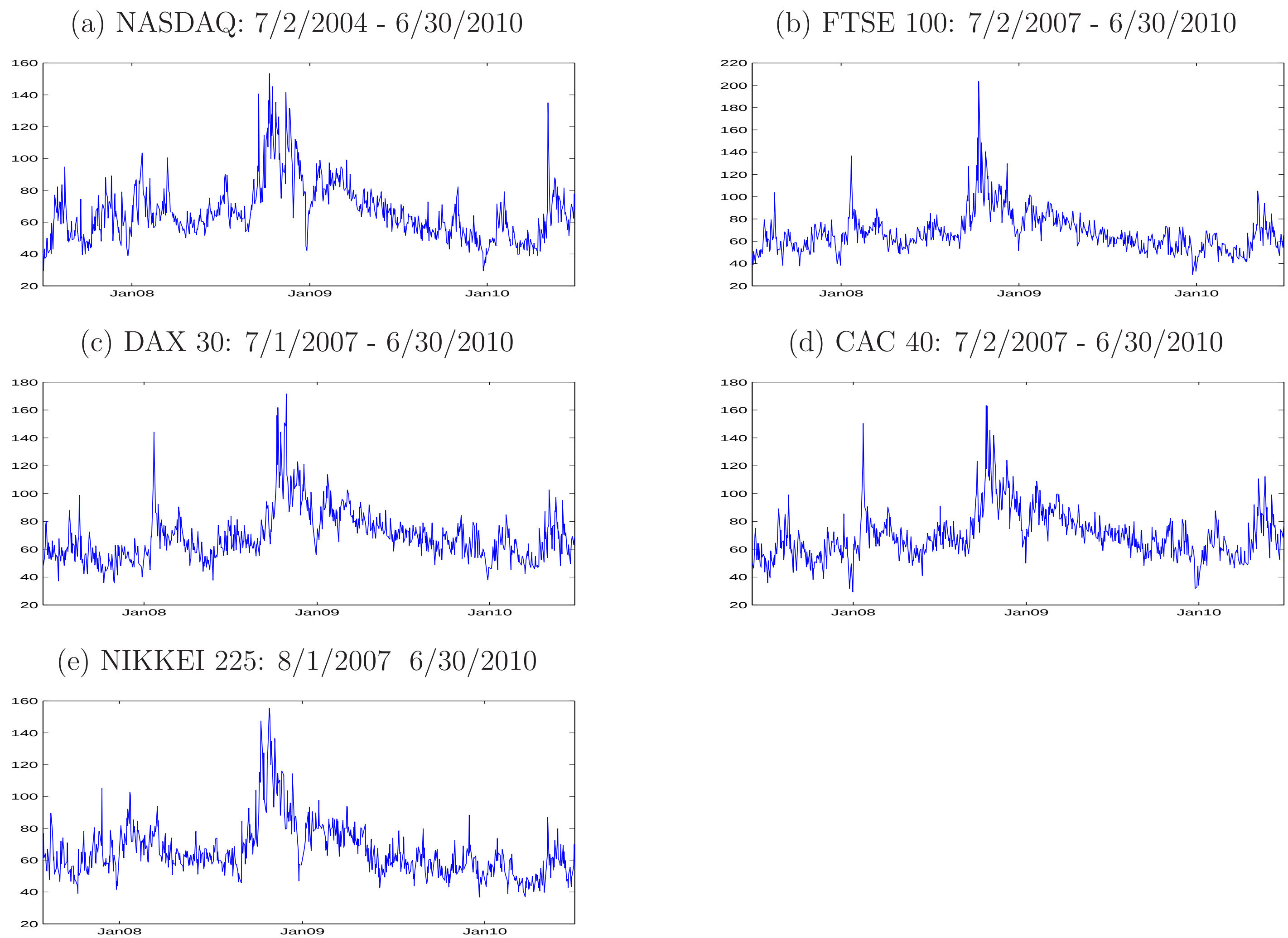
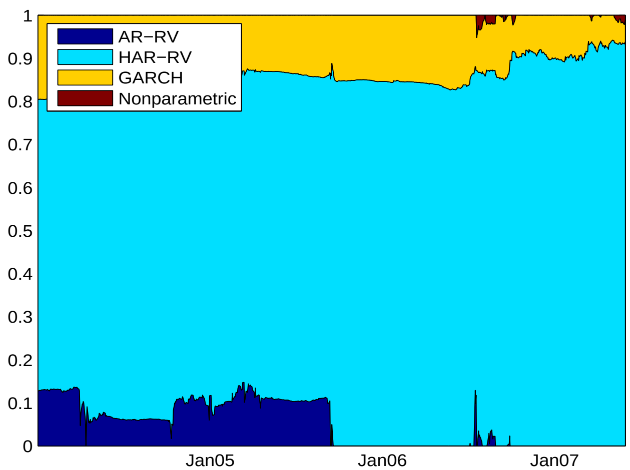
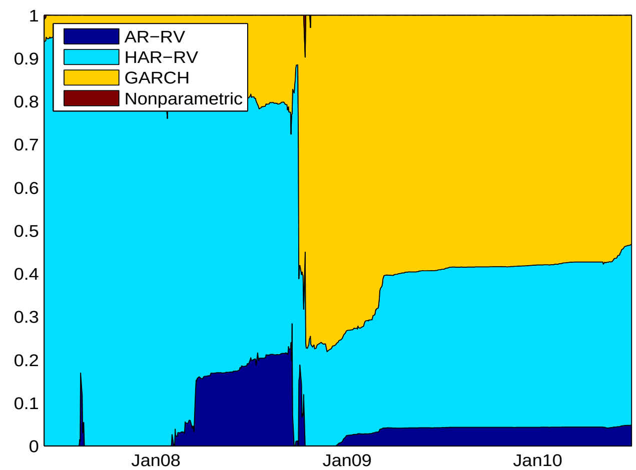


Figure 4: These figures present the HR combination weights that correspond to the four families of volatility models: AR-RV, HAR-RV, GARCH, and Nonparametric for the whole out-of-sample period. The weights are presented for the two subsamples of 1/2/2004 - 5/31/2007 (1st subsample) and 6/1/2007 - 6/30/2010 (2nd subsample) for $b = 0$ (MSE) and $b = 2$ (QLIKE), and $b = 4$.

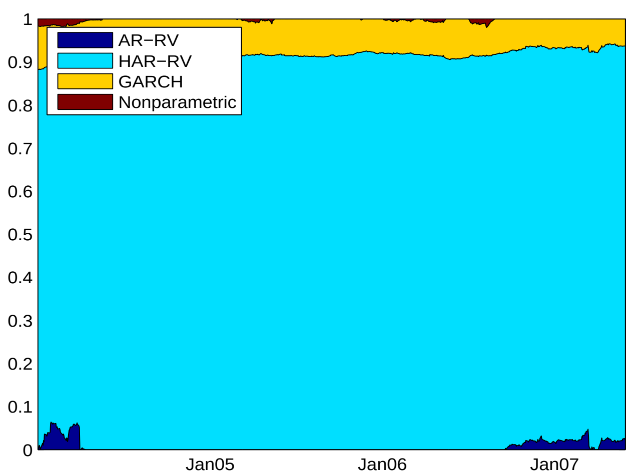
(i) $b = 0$:(MSE) 1/2/2004 - 5/31/2007



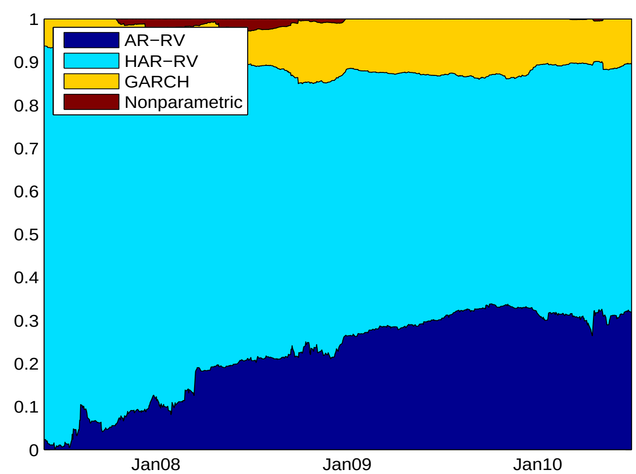
(ii) $b = 0$: (MSE) 6/1/2007 - 6/30/2010



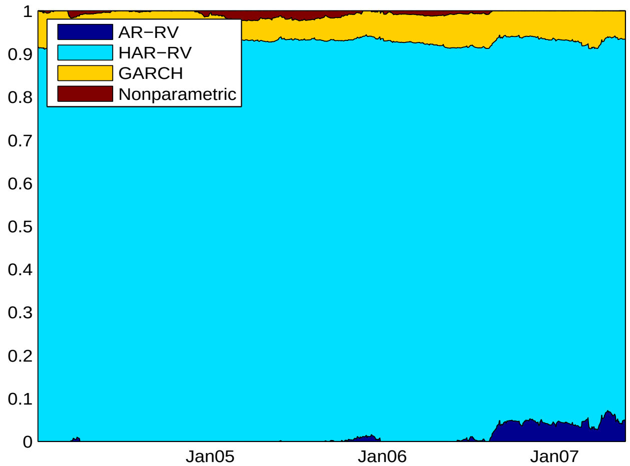
(iii) $b = -2$: (QLIKE) 1/2/2004 - 5/31/2007



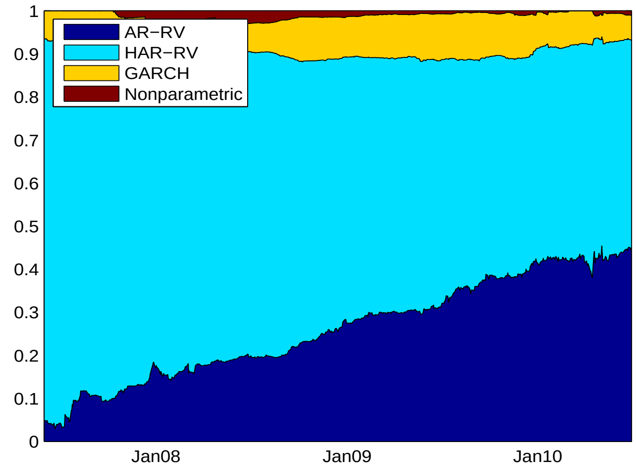
(iv) $b = -2$: (QLIKE) 6/1/2007 - 6/30/2010



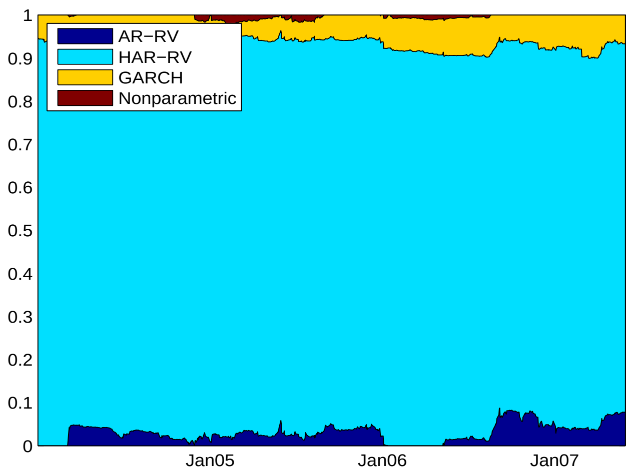
(v) $b = 3$: 1/2/2004 - 5/31/2007



(vi) $b = -3$: 6/1/2007 - 6/30/2010



(vii) $b = 4$: 1/2/2004 - 5/31/2007



(viii) $b = -4$: 6/1/2007 - 6/30/2010

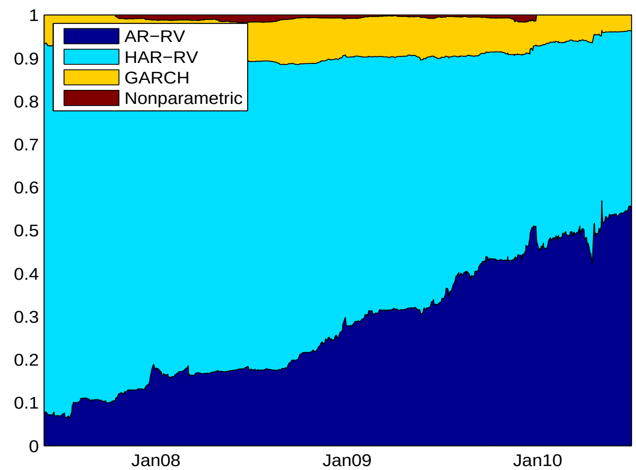


Table 1: Model Space - Individual Volatility Models

This table shows the individual volatility models included in the model space. For the HAR-RV and LHAR-RV models $RV_t^{(d)} = RV_t$, $RV_t^{(w)} = \frac{1}{5}(RV_t + RV_{t-1} + ... + RV_{t-4})$ and $RV_t^{(m)} = \frac{1}{22}(RV_t + RV_{t-1} + ... + RV_{t-21})$. For the LHAR-RV model $r_t^{(d)-} = r_t I\{r_t < 0\}$, $r_t^{(w)-} = \frac{1}{5}(r_t + r_{t-1} + ... + r_{t-4}) I\{r_t + r_{t-1} + ... + r_{t-4} < 0\}$ and $r_t^{(m)-} = \frac{1}{22}(r_t + r_{t-1} + ... + r_{t-21}) I\{r_t + r_{t-1} + ... + r_{t-21} < 0\}$. For the RiskMetrics model $\lambda = 0.94$.

Model Number	Model	Formula
1	AR(1)-RV	$h_{t+1} = \hat{\alpha} + \hat{\beta}_1 RV_t$
2	AR(5)-RV	$h_{t+1} = \hat{\alpha} + \hat{\beta}_1 RV_t + ... + \hat{\beta}_5 RV_{t-4}$
3	AR(10)-RV	$h_{t+1} = \hat{\alpha} + \hat{\beta}_1 RV_t + ... + \hat{\beta}_{10} RV_{t-9}$
4	AR(22)-RV	$h_{t+1} = \hat{\alpha} + \hat{\beta}_1 RV_t + ... + \hat{\beta}_{22} RV_{t-21}$
5	HAR-RV	$\log h_{t+1} = \hat{\alpha} + \hat{\beta}_d \log RV_t^{(d)} + \hat{\beta}_w RV_t^{(w)} + \hat{\beta}_m \log RV_t^{(m)}$
6	LHAR-RV	$\log h_{t+1} = \hat{\alpha} + \hat{\beta}_d \log RV_t^{(d)} + \hat{\beta}_w RV_t^{(w)} + \hat{\beta}_m \log RV_t^{(m)} + \gamma_d r_{t-1}^{(d)-} + \gamma_w r_{t-1}^{(w)-} + \gamma_m r_{t-1}^{(m)-}$
7	Normal GARCH(1,1)	$h_{t+1} = \omega + \alpha r_t^2 + \beta h_t,\ r_t = \epsilon_t \sqrt{h_t},\ \epsilon_t \sim N(0,1)$
8	Normal TARCH(1,1,1)	$h_{t+1} = \omega + \alpha r_t^2 + \beta h_t + \theta r_t^2 1_{\{r_t<0\}},\ r_t = \epsilon_t \sqrt{h_t},\ \epsilon_t \sim N(0,1)$
9	Normal EGARCH(1,1,1)	$\log h_{t+1} = \omega + \alpha \left[\frac{\lvert r_t \rvert}{\sqrt{h_t}} - E\left(\frac{\lvert r_t \rvert}{\sqrt{h_t}}\right)\right] + \beta \log h_t + \theta \frac{r_t}{\sqrt{h_t}},\ r_t = \epsilon_t \sqrt{h_t},\ \epsilon_t \sim N(0,1)$
10	Normal APARCH(1,1,1)	$\left(\sqrt{h_{t+1}}\right)^\delta = \omega + \alpha \left(\lvert r_t \rvert - \theta r_t\right)^\delta + \beta \left(\sqrt{h_t}\right)^\delta,\ r_t = \epsilon_t \sqrt{h_t},\ \epsilon_t \sim N(0,1)$
11	t GARCH(1,1)	$h_{t+1} = \omega + \alpha r_t^2 + \beta h_t,\ r_t = \epsilon_t \sqrt{h_t},\ \epsilon_t \sim t_\nu$
12	t TARCH(1,1,1)	$h_{t+1} = \omega + \alpha r_t^2 + \beta h_t + \theta r_t^2 1_{\{r_t<0\}},\ r_t = \epsilon_t \sqrt{h_t},\ \epsilon_t \sim t_\nu$
13	t EGARCH(1,1,1)	$\log h_{t+1} = \omega + \alpha \left[\frac{\lvert r_t \rvert}{\sqrt{h_t}} - E\left(\frac{\lvert r_t \rvert}{\sqrt{h_t}}\right)\right] + \beta \log h_t + \theta \frac{r_t}{\sqrt{h_t}},\ r_t = \epsilon_t \sqrt{h_t},\ \epsilon_t \sim t_\nu$
14	t APARCH(1,1,1)	$\left(\sqrt{h_{t+1}}\right)^\delta = \omega + \alpha \left(\lvert r_t \rvert - \theta r_t\right)^\delta + \beta \left(\sqrt{h_t}\right)^\delta,\ r_t = \epsilon_t \sqrt{h_t},\ \epsilon_t \sim t_\nu$
15	RiskMetrics	$h_{t+1} = \lambda h_t + (1-\lambda) r_t^2$
16	Rolling 30 days	$h_{t+1} = \frac{1}{30} \sum_{j=t-30}^{t} r_j^2$
17	Rolling 60 days	$h_{t+1} = \frac{1}{60} \sum_{j=t-60}^{t} r_j^2$

Table 2: Volatility Robust Forecast Combinations for the S&P 500 based on the Homogeneous Robust Loss Function

This table shows out-of-sample losses of volatility forecasts of the S&P 500 using the Homogeneous Robust Loss Function for a range of values of the scalar parameter b, which includes the MSE loss function (b = 0) and the QLIKE loss function (b = -2). We present results for four volatility forecast combination methods: Mean, Median, Geometric Mean (GMean), and Homogeneous Robust Forecast Combination (HRFC). We report results for three out-of-sample evaluation periods: 1/2/2004 - 6/30/2010 (full sample), 1/2/2004 - 5/31/2007 (1st subsample), and 6/1/2007 - 6/30/2010 (2nd subsample).

	Full Sample				1st Subsample				2nd Subsample			
	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC
b												
0	3.969	4.819	3.565	3.129	0.050	0.054	0.045	0.029	8.277	10.057	7.433	6.535
-0.5	0.638	0.753	0.563	0.426	0.038	0.041	0.035	0.022	1.297	1.536	1.143	0.869
-1	0.280	0.315	0.246	0.172	0.061	0.066	0.056	0.035	0.520	0.589	0.455	0.321
-1.5	0.191	0.206	0.171	0.119	0.104	0.110	0.097	0.062	0.286	0.311	0.252	0.182
-2	0.202	0.212	0.185	0.133	0.186	0.195	0.174	0.117	0.219	0.231	0.197	0.150
-2.5	0.289	0.300	0.271	0.203	0.352	0.365	0.333	0.237	0.220	0.229	0.202	0.165
-3	0.499	0.513	0.475	0.374	0.705	0.724	0.674	0.511	0.273	0.282	0.256	0.222
-3.5	0.978	0.998	0.944	0.782	1.507	1.535	1.457	1.178	0.398	0.409	0.380	0.347
-4	2.131	2.160	2.078	1.814	3.468	3.509	3.385	2.904	0.662	0.677	0.641	0.617

Table 3: Volatility Forecast of Individual Volatility Models for the S&P 500 based on the Homogeneous Robust Loss Function

This table shows out-of-sample losses of volatility forecasts of the S&P 500 using the Homogeneous Robust Loss Function for a range of values of the scalar parameter b, which includes the MSE loss function (b = 0) and the QLIKE loss function (b = -2). We present results for four families of volatility models and report the best: AR-RV (1-4), HAR-RV (5-6), GARCH (7-15), Nonparametric (N-P) (16-17) - the reference number in the parenthesis corresponds to the model in Table 1. We report results for three out-of-sample evaluation periods: 1/2/2004 - 6/30/2010 (full sample), 1/2/2004 - 5/31/2007 (1st subsample), and 6/1/2007 - 6/30/2010 (2nd subsample).

	Full Sample				1st Sample				2nd Sample			
	AR-RV	HAR-RV	GARCH	N-P	AR-RV	HAR-RV	GARCH	N-P	AR-RV	HAR-RV	GARCH	N-P
b												
0	4.901(3)	3.758(5)	3.124(9)	8.869(16)	0.034(4)	0.028(6)	0.063(15)	0.071(17)	10.249(3)	7.854(5)	6.468(9)	18.535(16)
-0.5	0.653(3)	0.564(5)	0.528(9)	1.383(16)	0.025(4)	0.022(6)	0.047(15)	0.053(16)	1.342(3)	1.159(5)	1.046(9)	2.844(16)
-1	0.239(3)	0.223(6)	0.259(9)	0.553(16)	0.041(4)	0.035(6)	0.074(15)	0.081(16)	0.457(3)	0.429(6)	0.450(9)	1.072(16)
-1.5	0.145(3)	0.133(6)	0.198(9)	0.325(16)	0.071(4)	0.063(6)	0.122(15)	0.130(16)	0.226(3)	0.210(6)	0.267(9)	0.539(16)
-2	0.149(3)	0.143(6)	0.222(9)	0.292(16)	0.131(4)	0.119(6)	0.212(15)	0.221(16)	0.168(3)	0.170(6)	0.215(9)	0.370(16)
-2.5	0.218(3)	0.216(6)	0.320(9)	0.367(16)	0.260(4)	0.242(6)	0.392(15)	0.397(16)	0.172(3)	0.188(6)	0.221(9)	0.333(16)
-3	0.393(3)	0.395(6)	0.542(9)	0.577(16)	0.549(3)	0.524(6)	0.770(15)	0.766(16)	0.221(3)	0.254(6)	0.278(9)	0.370(16)
-3.5	0.809(3)	0.822(6)	1.040(9)	1.065(16)	1.239(3)	1.210(6)	1.615(9)	1.589(16)	0.336(4)	0.395(5)	0.405(10)	0.489(16)
-4	1.853(3)	1.897(6)	2.221(9)	2.233(16)	3.010(3)	2.989(6)	3.616(9)	3.578(16)	0.582(4)	0.697(5)	0.672(10)	0.754(16)

Table 4: Comparison of Forecast Combinations based on the Homogeneous Robust Loss Function for the S&P 500

This table compares the performance of the simple Forecast Combinations Mean, Median, Geometric Mean as well as the Homogeneous Robust Forecast Combination (HRFC) for a range of values of the scalar parameter b using the Conditional Predictive Ability (CPA) test (Giacomini and White, 2006). We report results for three out-of-sample evaluation periods: 1/2/2004 - 6/30/2010 (full sample), 1/2/2004 - 5/31/2007 (1st subsample), and 6/1/2007 - 6/30/2010 (2nd subsample). The entries are the p-values of the CPA test using the Newey and West (1987) variance estimator and the number in parenthesis the proportion of times that the method in the column outperforms the method in the row based on the rule described inGiacomini and White (2006). A plus (minus) sign indicates that method in the column (row) significantly outperforms the method in the row (column). Bold entries correspond to the cases that unconstrained forecast combinations based on the Homogeneous Robust loss function significantly outperform the other methods of forecasting volatility for a confidence level of 10%.

Homogeneous Robust Loss for $b = 0$

	Full Sample				1st Subsample				2nd Subsample			
	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC
Mean		0.120	0.074+	0.162		0.000-	0.000+	0.000+		0.095-	0.068+	0.150
		(0.017)	(0.965)	(0.940)		(0.165)	(0.970)	(0.986)		(0.026)	(0.942)	(0.920)
Median	0.120		0.014+	0.116	0.000+		0.000+	0.000+	0.095+		0.009+	0.100
	(0.983)		(0.985)	(0.964)	(0.835)		(0.918)	(0.974)	(0.974)		(0.979)	(0.936)
GMean	0.074-	0.014-		0.373	0.000-	0.000-		0.000+	0.068-	0.009-		0.370
	(0.035)	(0.015)		(0.930)	(0.030)	(0.082)		(0.998)	(0.058)	(0.021)		(0.903)
HRFC	0.162	0.116	0.373		0.000-	0.000-	0.000-		0.150	0.100	0.370	
	(0.060)	(0.036)	(0.070)		(0.014)	(0.026)	(0.002)		(0.080)	(0.064)	(0.097)	

Homogeneous Robust Loss for $b = -0.5$

	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC
Mean		0.091-	0.021+	0.038+		0.000-	0.000+	0.000+		0.074-	0.019+	0.038+
		(0.012)	(0.975)	(0.945)		(0.196)	(0.964)	(0.984)		(0.017)	(0.974)	(0.929)
Median	0.091+		0.001+	0.036+	0.000+		0.000+	0.000+	0.074+		0.001+	0.031+
	(0.988)		(0.974)	(0.961)	(0.804)		(0.900)	(0.962)	(0.983)		(0.970)	(0.941)
GMean	0.021-	0.001-		0.143	0.000-	0.000-		0.000+	0.019-	0.001-		0.161
	(0.025)	(0.026)		(0.932)	(0.036)	(0.100)		(0.993)	(0.026)	(0.030)		(0.909)
HRFC	0.038-	0.036-	0.143		0.000-	0.000-	0.000-		0.038-	0.031-	0.161	
	(0.055)	(0.039)	(0.068)		(0.016)	(0.038)	(0.007)		(0.071)	(0.059)	(0.091)	

Homogeneous Robust Loss for $b = -1$

	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC
Mean		0.091-	0.021+	0.038+		0.000-	0.000+	0.000+		0.039-	0.001+	0.003+
		(0.012)	(0.975)	(0.945)		(0.227)	(0.952)	(0.979)		(0.000)	(0.990)	(0.997)
Median	0.091+		0.001+	0.036+	0.000+		0.000+	0.000+	0.039+		0.000+	0.002+
	(0.988)		(0.974)	(0.961)	(0.773)		(0.878)	(0.954)	(1.000)		(0.948)	(0.972)
GMean	0.021-	0.001-		0.143	0.000-	0.000-		0.000+	0.001-	0.000-		0.020+
	(0.025)	(0.026)		(0.932)	(0.048)	(0.122)		(0.992)	(0.010)	(0.052)		(0.920)
HRFC	0.038-	0.036-	0.143		0.000-	0.000-	0.000-		0.003-	0.002-	0.020-	
	(0.055)	(0.039)	(0.068)		(0.021)	(0.046)	(0.008)		(0.003)	(0.028)	(0.080)	

Homogeneous Robust Loss for $b = -1.5$

	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC
Mean		0.001-	0.000+	0.000+		0.000-	0.000+	0.000+		0.008-	0.000+	0.000+
		(0.013)	(0.973)	(0.965)		(0.254)	(0.944)	(0.975)		(0.009)	(0.968)	(0.960)
Median	0.001+		0.000+	0.000+	0.000+		0.000+	0.000+	0.008+		0.000+	0.000+
	(0.987)		(0.916)	(0.987)	(0.746)		(0.872)	(0.959)	(0.991)		(0.920)	(0.983)
GMean	0.000-	0.000-		0.000+	0.000-	0.000-		0.000+	0.000-	0.000-		0.000+
	(0.027)	(0.084)		(0.999)	(0.056)	(0.128)		(0.987)	(0.032)	(0.080)		(0.999)
HRFC	0.000-	0.000-	0.000-		0.000-	0.000-	0.000-		0.000-	0.000-	0.000-	
	(0.035)	(0.013)	(0.001)		(0.025)	(0.041)	(0.013)		(0.040)	(0.017)	(0.001)	

Table continued on next page ...

Table 4 continued

Homogeneous Robust Loss for $b = -2$

	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC
Mean		0.000- (0.112)	0.000+ (0.945)	0.000+ (0.971)		0.000- (0.268)	0.000+ (0.933)	0.000+ (0.978)		0.000- (0.091)	0.000+ (0.939)	0.000+ (0.959)
Median	0.000+ (0.888)		0.000+ (0.918)	0.000+ (0.964)	0.000+ (0.732)		0.000+ (0.868)	0.000+ (0.967)	0.000+ (0.909)		0.000+ (0.939)	0.000+ (0.955)
GMean	0.000- (0.055)	0.000- (0.082)		0.000+ (0.982)	0.000- (0.067)	0.000- (0.132)		0.000+ (0.985)	0.000- (0.061)	0.000- (0.061)		0.001+ (0.974)
HRFC	0.000- (0.029)	0.000- (0.036)	0.000- (0.018)		0.000- (0.022)	0.000- (0.033)	0.000- (0.015)		0.000- (0.041)	0.000- (0.045)	0.001- (0.026)	

Homogeneous Robust Loss for $b = -2.5$

	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC
Mean		0.000- (0.078)	0.000+ (0.964)	0.000+ (0.976)		0.000- (0.273)	0.000+ (0.941)	0.000+ (0.984)		0.007- (0.010)	0.000+ (0.966)	0.002+ (0.979)
Median	0.000+ (0.922)		0.000+ (0.940)	0.000+ (0.961)	0.000+ (0.727)		0.000+ (0.878)	0.000+ (0.974)	0.007+ (0.990)		0.000+ (0.978)	0.000+ (0.963)
GMean	0.000- (0.036)	0.000- (0.060)		0.000+ (0.978)	0.000- (0.059)	0.000- (0.122)		0.000+ (0.988)	0.000- (0.034)	0.000- (0.022)		0.041+ (0.991)
HRFC	0.000- (0.024)	0.000- (0.039)	0.000- (0.022)		0.000- (0.016)	0.000- (0.026)	0.000- (0.012)		0.002- (0.021)	0.000- (0.037)	0.041- (0.009)	

Homogeneous Robust Loss for $b = -3$

	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC
Mean		0.000- (0.044)	0.000+ (0.973)	0.000+ (0.976)		0.000- (0.265)	0.000+ (0.947)	0.000+ (0.987)		0.128 (0.003)	0.000+ (0.981)	0.041+ (0.995)
Median	0.000+ (0.956)		0.000+ (0.958)	0.000+ (0.964)	0.000+ (0.735)		0.000+ (0.892)	0.000+ (0.984)	0.128 (0.997)		0.000+ (0.991)	0.043+ (0.968)
GMean	0.000- (0.027)	0.000- (0.042)		0.000+ (0.977)	0.000- (0.053)	0.000- (0.108)		0.000+ (0.988)	0.000- (0.019)	0.000- (0.009)		0.244 (0.999)
HRFC	0.000- (0.024)	0.000- (0.036)	0.000- (0.023)		0.000- (0.013)	0.000- (0.016)	0.000- (0.012)		0.041- (0.005)	0.043- (0.032)	0.244 (0.001)	

Homogeneous Robust Loss for $b = -3.5$

	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC
Mean		0.000- (0.020)	0.000+ (0.977)	0.000+ (0.981)		0.000- (0.247)	0.000+ (0.962)	0.000+ (0.988)		0.438 (0.015)	0.000+ (0.985)	0.156 (0.970)
Median	0.000+ (0.980)		0.000+ (0.970)	0.000+ (0.972)	0.000+ (0.753)		0.000+ (0.907)	0.000+ (0.986)	0.438 (0.985)		0.001+ (1.000)	0.339 (0.985)
GMean	0.000- (0.023)	0.000- (0.030)		0.000+ (0.982)	0.000- (0.038)	0.000- (0.093)		0.000+ (0.989)	0.000- (0.015)	0.001- (0.000)		0.421 (0.228)
HRFC	0.000- (0.019)	0.000- (0.028)	0.000- (0.018)		0.000- (0.012)	0.000- (0.014)	0.000- (0.011)		0.156 (0.030)	0.339 (0.015)	0.421 (0.772)	

Homogeneous Robust Loss for $b = -4$

	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC	Mean	Median	GMean	HRFC
Mean		0.002- (0.010)	0.000+ (0.980)	0.002+ (0.984)		0.000- (0.231)	0.000+ (0.974)	0.001+ (0.995)		0.659 (0.021)	0.000+ (0.992)	0.249 (0.120)
Median	0.002+ (0.990)		0.000+ (0.975)	0.001+ (0.980)	0.000+ (0.769)		0.000+ (0.933)	0.000+ (0.992)	0.659 (0.979)		0.010+ (0.999)	0.641 (0.947)
GMean	0.000- (0.020)	0.000- (0.025)		0.010+ (0.985)	0.000- (0.026)	0.000- (0.067)		0.003+ (0.995)	0.000- (0.008)	0.010- (0.001)		0.423 (0.044)
HRFC	0.002- (0.016)	0.001- (0.020)	0.010- (0.015)		0.001- (0.005)	0.000- (0.008)	0.003- (0.005)		0.249 (0.880)	0.641 (0.053)	0.423 (0.956)	

Table 5: Volatility Forecast Combinations for the S&P 500 based on the LINEX Loss Function

This table shows out-of-sample losses of volatility forecasts of the S&P 500 using the LINEX loss function for a range of values of the scalar parameter a. We present results for four volatility forecast combination methods: Mean, Median, Geometric Mean (GMean), and LINEX loss function (LFC). We report results for three out-of-sample evaluation periods: 1/2/2004 - 6/30/2010 (full sample), 1/2/2004 - 5/31/2007 (1st subsample), and 6/1/2007 - 6/30/2010 (2nd subsample). E stands for times 10 to the power of.

	Full Sample				1st Subsample				2nd Subsample			
	Mean	Median	GMean	LFC	Mean	Median	GMean	LFC	Mean	Median	GMean	LFC
a												
0.001	1.99E-06	2.41E-06	1.79E-06	1.57E-06	2.50E-08	2.70E-08	2.27E-08	1.72E-08	4.15E-06	5.04E-06	3.73E-06	3.28E-06
0.005	5.02E-05	6.10E-05	4.56E-05	3.88E-05	6.25E-07	6.76E-07	5.68E-07	3.69E-07	1.05E-04	1.27E-04	9.50E-05	8.11E-05
0.01	2.03E-04	2.47E-04	1.87E-04	1.54E-04	2.50E-06	2.70E-06	2.27E-06	1.48E-06	4.24E-04	5.16E-04	3.89E-04	3.23E-04
0.05	6.08E-03	7.71E-03	6.25E-03	3.86E-03	6.25E-05	6.75E-05	5.68E-05	3.72E-05	1.27E-02	1.61E-02	1.31E-02	8.05E-03
0.1	4.04E-02	6.08E-02	5.07E-02	1.76E-02	2.50E-04	2.70E-04	2.28E-04	1.50E-04	8.45E-02	1.27E-01	1.06E-01	3.68E-02
0.5	1.35E+05	1.73E+06	7.64E+05	1.37E+04	6.30E-03	6.78E-03	5.78E-03	4.74E-03	2.84E+05	3.64E+06	1.60E+06	2.88E+04
1	2.99E+13	4.90E+15	9.51E+14	4.46E+12	2.60E-02	2.78E-02	2.41E-02	2.83E-02	6.28E+13	1.03E+16	2.00E+15	9.36E+12

Table 6: Volatility Forecast of Individual Volatility Models for the S&P 500 based on the LINEX Loss Function

This table shows out-of-sample losses of volatility forecasts of the S&P 500 using the LINEX Loss Function for a range of values of the scalar parameter a. We present results for four families of volatility models and report the best: AR-RV (1-4), HAR-RV (5-6), GARCH (7-15), Nonparametric (N-P) (16-17) - the reference number in the parenthesis corresponds to the model in Table 1. We report results for three out-of-sample evaluation periods: 1/2/2004 - 6/30/2010 (full sample), 1/2/2004 - 5/31/2007 (1st subsample), and 6/1/2007 - 6/30/2010 (2nd subsample).

	Full Sample				1st Sample				2nd Sample			
	AR-RV	HAR-RV	GARCH	N-P	AR-RV	HAR-RV	GARCH	N-P	AR-RV	HAR-RV	GARCH	N-P
a												
0.001	2.45E-06(3)	1.89E-06(5)	1.57E-06(9)	3.76E-06(16)	1.69E-08(4)	1.42E-08(6)	3.15E-08(15)	3.54E-08(17)	5.13E-06(3)	3.96E-06(5)	3.25E-06(9)	9.27E-06(16)
0.005	6.16E-05(3)	4.89E-05(5)	4.00E-05(9)	1.11E-04(16)	4.22E-07(4)	3.56E-07(6)	7.87E-07(15)	8.84E-07(17)	1.29E-04(3)	1.02E-04(5)	8.29E-05(9)	2.32E-04(16)
0.01	2.49E-04(3)	2.04E-04(5)	1.64E-04(9)	4.45E-04(16)	1.69E-06(4)	1.43E-06(6)	3.15E-06(15)	3.54E-06(17)	5.20E-04(3)	4.26E-04(5)	3.41E-04(9)	9.29E-04(16)
0.05	7.55E-03(2)	6.33E-03(6)	5.52E-03(9)	1.27E-02(16)	4.23E-05(4)	3.59E-05(6)	7.86E-05(15)	8.84E-05(17)	1.58E-02(2)	1.32E-02(6)	1.15E-02(9)	2.67E-02(16)
0.1	5.15E-02(2)	2.10E-02(6)	4.01E-02(13)	9.93E-02(16)	1.70E-04(4)	1.45E-04(6)	3.14E-04(15)	3.54E-04(17)	1.08E-01(2)	4.40E-02(6)	8.37E-02(13)	2.08E-01(16)
0.5	2.69E+05(2)	6.07E+00(6)	4.72E+04(14)	1.47E+07(16)	4.45E-03(4)	3.87E-03(6)	7.81E-03(15)	8.89E-03(17)	5.66E+05(2)	1.27E+01(6)	9.90E+04(14)	3.09E+07(16)
1	1.18E+14(2)	1.75E+04(6)	3.63E+12(14)	3.53E+17(16)	1.93E-02(4)	1.73E-02(6)	3.16E-02(15)	3.63E-02(16)	2.48E+14(2)	3.66E+04(6)	7.62E+12(14)	7.42E+17(16)

Table 7: Volatility Forecast Combinations for Other Stock Markets

This table shows out-of-sample losses of volatility forecasts of NASDAQ, FTSE 100, DAX 30, CAC 40, and NIKKEI 225 using the homogeneous robust loss function for $b = 0, -2, -4$ and the LINEX loss function for $a = 0.005, 0.05, 0.5$. We present results for four volatility forecast combination methods: Mean, Median, Geometric Mean (GMean), and Homogeneous Robust Forecast Combination (HRFC). We report results for the evaluation periods as follows: for DAX 30 6/1/2007 - 6/30/2010, for NASDAQ Composite, FTSE 100 and CAC 40, 7/2/2007 - 6/30/2010, and for NIKKEI 225, 8/1/2007 - 6/30/2010.

	Homogeneous Robust				LINEX			
	Mean	Median	GMean	HRFC	Mean	Median	GMean	LFC
	$b = 0$ (MSE)				$a = 0.005$			
NASDAQ	6.994	8.346	5.896	4.324	8.77E-05	1.04E-04	7.43E-05	5.44E-05
FTSE 100	18.816	17.079	16.813	24.010	2.61E-04	2.41E-04	2.38E-04	3.39E-04
DAX 30	8.588	7.996	7.780	9.485	1.10E-04	1.03E-04	1.00E-04	1.21E-04
CAC 40	7.993	7.608	6.985	7.677	1.01E-04	9.71E-05	8.95E-05	9.84E-05
NIKKEI 225	10.648	11.282	7.288	4.206	1.31E-04	1.38E-04	8.99E-05	5.34E-05
	$b = -2$ (QLIKE)				$a = 0.05$			
NASDAQ	0.259	0.276	0.231	0.155	9.28E-03	1.07E-02	8.26E-03	5.89E-03
FTSE 100	0.181	0.176	0.166	0.127	1.52E-01	1.53E-01	1.57E-01	1.37E-01
DAX 30	0.175	0.170	0.161	0.130	1.48E-02	1.47E-02	1.45E-02	1.54E-02
CAC 40	0.170	0.169	0.157	0.132	1.28E-02	1.27E-02	1.21E-02	1.36E-02
NIKKEI 225	0.323	0.335	0.268	0.138	1.14E-02	1.20E-02	8.24E-03	5.99E-03
	$b = -4$				$a = 0.5$			
NASDAQ	0.681	0.691	0.663	0.577	1.22E+02	1.94E+02	1.54E+02	7.74E+01
FTSE 100	0.385	0.379	0.376	0.345	4.95E+17	5.73E+17	7.65E+17	7.45E+16
DAX 30	0.300	0.293	0.291	0.271	6.90E+04	1.12E+05	1.00E+05	5.41E+04
CAC 40	0.461	0.457	0.454	0.445	4.46E+04	6.47E+04	5.73E+04	1.91E+05
NIKKEI 225	0.434	0.437	0.412	0.296	1.83E+01	3.66E+01	2.94E+01	1.68E+01

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