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Public-Sector Employment, Wages and Human Capital Accumulation

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Public-sector employment, wages and human capital accumulation*

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Abstract

We set up a search and matching model with a private and a public sector to understand the effects of employment and wage policies in the public sector on unemployment and education decisions. The effects of wages and employment of skilled and unskilled public-sector workers on the educational composition of the labor force depend crucially on the structure of the labor market. An increase of skilled public-sector wages has a small positive impact on educational composition and larger negative impact on the private employment of skilled workers, if the two sectors are segmented. If search across the two sectors is random, it has a large positive impact on education and a large positive impact on skilled private employment. We highlight the usefulness of the model for policymakers by calculating the value of public-sector job security for skilled and unskilled workers.

JEL Classification: E24; J31; J45; J64.

Keywords: Public-sector employment; public-sector wages; unemployment; skilled workers; human capital accumulation, education decision, public-sector job security premium.

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1 Introduction

The government is a major employer of educated workers, hiring more than 25 percent of all college graduates in most OECD economies. However, in the public sector, the returns of education are typically lower. Using micro level data, several papers find that, on average, the public sector pays higher wages than the private sector but that the premium is higher for workers without a college degree. Examples include: Katz and Krueger (1991) for the United States; Postel-Vinay and Turon (2007) or Disney and Gosling (1998) for the United Kingdom; and Christofides and Michael (2013), Castro et al. (2013) and Giordano et al. (2011) for several European countries. Our objective is to study how public-sector employment and wage policies, heterogeneous across education groups, influence incentives to invest in formal education.

We set up a search model in which workers can search for jobs in either the private or the public sector. Employment and wages in the private sector are determined through the usual channels of free entry and Nash bargaining. In the public sector, by contrast, employment and wages are policy variables and, hence, taken as exogenous. We incorporate a human capital accumulation decision. Prior to entering the labor market, individuals decide whether to invest in education. Doing so is costly but yields returns, as highly-educated workers benefit from higher job-finding rates and wages in both sectors. Workers are heterogeneous with respect to their education costs. Thus, only a fraction of individuals – those whose benefits exceed the costs – invest in education.

We contribute to the labor market search literature that analyzes the role and effects of public-sector employment and wages. Burdett (2012) includes the public sector in a job-ladder framework where firms post wages. Bradley et al. (2017) further introduce on-the-job search and transitions between the two sectors to study the effects of public-sector policies on the distribution of private-sector wages. Albrecht et al. (2017) consider heterogeneous human capital and match specific productivity in a Diamond-Mortensen-Pissarides model. Michaillat (2014) shows that the crowding-out effect of public-sector employment is lower during recessions, giving rise to higher government spending multipliers. These papers' objective is to determine how public-sector employment and wage policies affect private employment, the unemployment rate and private wages. They assume that the unemployed search randomly across sectors, and, hence, public-sector policies affect the equilibrium only by affecting the outside option of the unemployed and their reservation wage.

Hörner et al. (2007) study the effect of turbulence on unemployment when wages in the public sector are insulated from this volatility. Quadrini and Trigari (2007) show how different exogenous business cycle rules affect unemployment volatility. Gomes (2015) emphasizes the role of public-sector wages in achieving the efficient allocation, while Afonso and Gomes (2014) highlight the interactions between private and public wages. These papers assume that the two sectors' labor markets are segmented, and that the unemployed choose which of the sectors to search in, depending on the government's hiring, separation and wage policies.

We add to this literature by contrasting the results under the two assumptions regarding search in the two sectors. In the first model, the two sectors are segmented as a consequence of the existence of barriers to enter the public sector. For workers choosing to pay the entry cost, searching for public-sector jobs strictly dominates all other options. These entry barriers may represent national entry exams or jobs that are restricted to workers with political capital. We think that the assumption of segmented markets portrays a realistic mechanism of selection into the public sector in several countries, documented empirically by Nickell and Quintini (2002) or experimentally by Bó et al. (2013) , lying at the heart of current policy discussions. High public wages attract many unemployed to queue for public sector jobs. Conversely, if wages are too low, few unemployed search in the public sector, which then faces recruitment problems. In several countries, like France or Spain, the general rule is that civil servants are recruited through competitive exams which clearly suggests that markets are segmented. However, we also examine the scenario where workers search randomly for jobs in both sectors. This could be a realistic assumption for some types of public-sector jobs or for some countries. In the US, for instance, the majority of federal government jobs are filled through an examination of the applicant's background, work experience, and education, not through a written civil service test. While this in itself does not rule out that markets are segmented, it is consistent with unemployed workers not specifically searching for a public-sector job, but getting one by chance.

With the exception of Albrecht et al. (2017), none of the above cited papers explicitly consider heterogeneity in terms of education. This is quite an oversight, given that the government hires predominantly workers with college degree and that the premia it pays varies substantially with education. Other papers that consider heterogeneity in a search and matching model include Gomes (2018) and Navarro and Tejada (2018). Gomes (2018) examines the effects of a public-sector wage reform that eliminates the wage premium for all types of public-sector workers. Navarro and Tejada (2018) study the interaction between public-sector employment and the minimum wage. Other papers model this heterogeneity in a frictionless labour market. Domeij and Ljungqvist (2016) study how the public-sector hiring of skilled and unskilled workers in Sweden and the US can explain the diverging evolutions of the skill premium in the two countries. Garibaldi et al. (2018) try to disentangle whether the higher skill intensity in the public sector is due to technology, wage compression or a higher level of underemployment. In a model of occupational choice, Gomes and Kuehn

(2017) analyze the effects of skill-biased hiring in the public sector on the occupational choice of entrepreneurs and on firm size. All of these papers take the education endowment as exogenous. By endogenizing the choice of education and showing under which conditions and how government employment and wage policies affect it, we contribute to the literature on the determinants of education that started with the seminal contributions of Mincer (1958), Ben-Porath (1967), Weisbrod (1962) and Becker (1975). More closely related contributions using search models include Charlot and Decreuse (2005), Charlot et al. (2005) and Charlot and Decreuse (2010). To the best of our knowledge, only Wilson (1982) considers the effects of public-sector employment on education, but within a neoclassical optimal taxation model.

We find that wage and employment policies in the public sector affect the educational composition and unemployment rate of skilled and unskilled workers. While the qualitative effects are similar in both labour market structures - segmented markets (with entry barriers) and random search (no entry barriers) - the mechanisms and the quantitative effects are different. An increase of skilled public-sector wages has a small positive impact on the proportion of highly educated and a larger negative impact on skilled private employment, if the two sectors are segmented. If search across the two sectors is random, it has a large positive impact on both education and skilled private employment. In segmented markets, when skilled public-sector wages increase, more people queue for public-sector jobs. The consequent decrease in the job-finding rate partially offsets the increase in the gains of education. In the extreme case, were the two sectors are segmented but entry into the public sector is free, the increase in unemployed queuing for public-sector jobs fully neutralizes the increase in the value of education and educational composition remains intact. However, if workers search randomly for jobs in both sectors such offsetting decrease in job-finding rate is not possible. The value of education goes up by more, leading to larger increases in the proportion of high-educated in the labor force. As unemployed search randomly for jobs, a higher pool of educated workers leads to a higher level of skilled private employment, but a lower level of unskilled private employment. Quantitatively, in the model calibrated to four economies - United States, United Kingdom, France and Spain - a 10 percent increase in skilled wages raises the number of educated workers by 0 to 0.18 percent under segmented markets, and by 2.5 to 6.9 percent under random search.

In policy discussions over public-sector pay, there is often the argument that public sector jobs have particular features that offer an extra-compensation to its workers. One of such features is job-security. We highlight the usefulness of our model by quantifying the value of job security in the public-sector, for skilled and unskilled workers. We do so by asking what fraction of their wage would private-sector workers be willing to sacrifice to have the job-separation rate of the public sector, or conversely, how much would public-sector workers have to be compensated in order to have the job-security of the private sector. As our model is populated by risk-neutral workers, this is a lower bound for the value of job security. We find that this premia varies substantially across countries and is different for workers with and without college. It is larger in countries where unemployment is more persistent and the unemployment benefits are low, and it is larger for unskilled workers. For the United States, for instance, the job security premium varies between 0.5 to 1.1 percent for skilled workers and between 1.7 to 5.9 percent for unskilled workers. In Spain, the values range from 2.5 to 7.0 percent.

The paper is structured as follows. Section 2 shows some evidence on the worker stocks and flows of the public- and private-sector and the wage differentials in the United States, United Kingdom, France and Spain. Section 3 presents the model economy with search and matching frictions. Section 4 describes the main results of the paper. Section 5 analyzes the setting in which search between the private and public sectors is random. In Section 6, we parameterize the model to each of the four economies and perform quantitative exercises. Section 7 concludes.

2 Evidence from microdata

2.1 Worker stocks and flows, by sector and education

Fontaine et al. (2018) establish a number of key facts about the US, UK, French and Spanish public and private sector labour market flows, using data from the US *Current Population Survey* (CPS) and the UK, French and Spanish *Labour Force Surveys* (LFS) over the past 15 years. We analyse more deeply the stocks and flows by education. These four countries have different but sizable levels of public-sector employment, variable industry composition of the public sector, different hiring methods and labour market institutions. All the common patterns that we find can then be considered general characteristics of the public sector.

By focussing on a representative survey, used to calculate official labour market statistics, we can pin down accurately the stocks of public- and private-sector employment, but more importantly the worker flows, particularly job-finding and job-separation rates. These flows are crucial to identify the key parameters of the model we propose. We use these statistics in the calibration of the model in Section 6.

We first extract the stocks and transition probabilities between private and public employment and unemployment, for workers with at least a college degree and for workers without a college degree. We follow the procedures describe in Fontaine et al. (2018), with a few exceptions. First, instead of dividing the population into three groups, we focus on two

Table 1: Public-sector employment stock and transition rates, by education

Variable	United States		United Kingdom		France		Spain	
	College	No college	College	No college	College	No college	College	No college
Public-sector employment (share of total employment)	0.254	0.116	0.358	0.192	0.281	0.181	0.275	0.095
Unemployment rate	0.032	0.073	0.033	0.057	0.057	0.103	0.110	0.208
Job-finding rate								
Private sector	0.213	0.328	0.325	0.268	0.285	0.217	0.239	0.214
Public sector	0.052	0.008	0.085	0.029	0.041	0.024	0.047	0.018
Job-separation rate								
Private sector	0.007	0.028	0.012	0.014	0.016	0.023	0.031	0.051
Public sector	0.005	0.016	0.005	0.005	0.005	0.010	0.014	0.036
Share in the labour force	0.27	0.73	0.46	0.54	0.32	0.68	0.34	0.66
Unemp. duration of new hires (private over public)	1.248	1.009	0.744	0.735	0.948	0.767	0.988	0.794

Note: Data extracted from the French, UK and Spanish LFS and the US CPS. Sample: 2003:1 to 2015:4 for France and UK, 2005:1 to 2015:4 for Spain, and 2003 Jan to 2017 Dec. Population aged 20 to 64. The job-finding and -separation rates are monthly for the US and quarterly for the UK, France and Spain.

education levels: college and no college.[1] Second, we restrict the sample to workers aged 20 to 64, whereas they focussed on workers aged 16 to 64. As our model does not incorporate a labour market participation decision, we abstract from the flows in and out of inactivity. Also, the direct flows between public and private sector are small, so we also abstract from on-the-job search and job-to-job transitions.

Table 1 shows the average stocks and transition rates for the four countries. The job-finding rate is the probability that an unemployed is employed in a particular sector in the following quarter. The job-separation rate is the probability that a worker employed in a particular sector is unemployed in the following quarter. The public sector is a sizable employer, especially for skilled workers. The US public sector employs 25 percent of the employed population with a college degree and only 12 percent of people without one. In the UK, the difference is even larger with 36 and 19 percent. In France, these are 29 and 18 percent. In Spain, these are 28 and 10 percent. Labour turnover is lower in the public sector. On the one hand, jobs are safer: the job-separation rate is roughly two to three times higher if working in the private sector. On the other hand, there are fewer hires in the public sector. The probability of finding a job in the private sector for college graduates is four to seven times the probability of finding a job in the public sector. It is eight to 12 times the probability of finding a job in the public sector for workers without college. The job-finding rates are increasing and job-separation rates are decreasing in education in both sectors.

How does the recruitment takes place in these countries? In France, the general rule

[1]For the United States, college graduates includes individuals with a Bachelors, Masters, Professional or Doctorate degree, but exclude individuals with Associate degrees or with some college but no degree.

is that civil servants are recruited through competitive exams, either external (reserved to competitors fulfilling certain conditions of diplomas or professional experience and age) or internal (reserved to civil servants in certain positions). See Meurs and Puhani (2018) for a detailed description of the process. The Spanish civil service has a similar recruitment method. Currently in Spain, the rule of thumb is that one candidate for a position requiring high-school needs 9 to 12 months to prepare for the exam, while for a position targeting a college graduate requires 18 to 24 months of preparation. In some specific occupations, such as notaries, the average time to prepare for the exam is more than 4 years. However, the public sector is wider than the civil service, and also includes workers in local government or in industries of education and health, each with specific channels of entry. In the particular case of France and Spain, many public-sector workers are hired with temporary contracts. In the US, civil service exams are required for certain groups including foreign service officers, customs, some secretarial and clerical, air traffic control, law enforcement, postal service, and for some entry level government jobs, but these are a minority. More than 80 percent of federal government jobs are filled through a competitive examination of the applicant's background, work experience, and education, not through a written civil service test. It is important to read the job announcement thoroughly to determine if a civil service exam, self certification, or a skill is necessary. In the UK, there are also no civil service exams, but standard recruitment methods alongside specific entry channels such as apprenticeships, graduate or internship programmes.

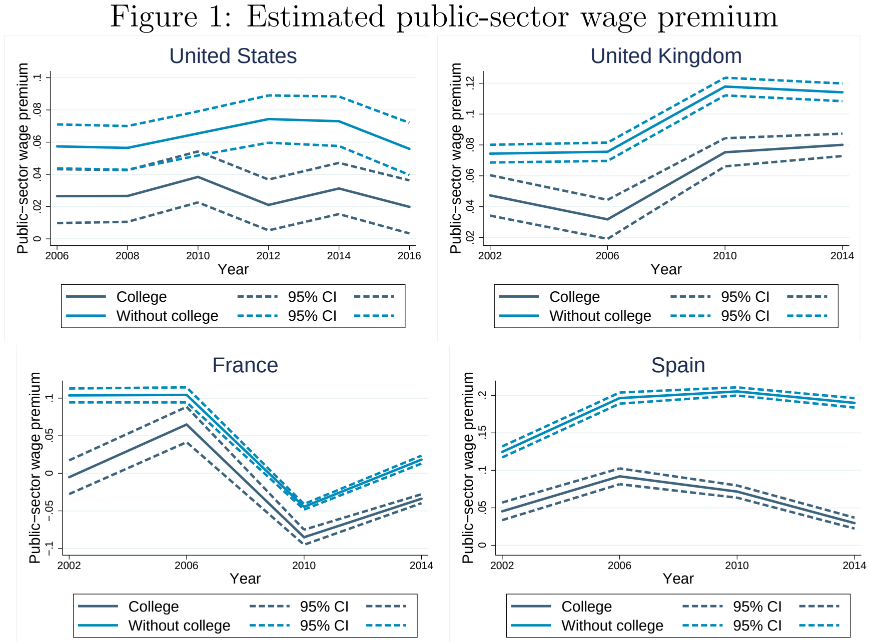
One of the difficulties with distinguishing whether search across sectors is random or directed towards one sector or the other is that the behaviour of the unemployed is unobservable. In the last row of Table 1, we calculate the ratio of unemployment durations of new hires in the private over that of the new hires in the public sector. For UK, France and Spain the number is lower than one, meaning that the unemployment duration is lower in the private sector, or in other words, queues are longer in the public sector. In the US the number is close to one. Notice that under random search, the unemployment duration should be equal for workers who just found a job in either the public or the private sector, so the ratio should be close to one. However, it could also be close to one under segmented markets. Consequently, differences in unemployment durations between new hires in the private and public sectors, as well as the existence of public-sector entry restrictions can give an indication as to whether the two sectors are segmented or not, but cannot give a definite answer. Without a better way to distinguish between the two labour market structures in the data, we set up a model under both assumptions in order to better understand the mechanisms and their quantitative implications.

2.2 Public-sector wage premium by education

We use microdata from the CPS and the *Structure of Earnings Survey* (SES) for the waves of 2002, 2006, 2010 and 2014, to reproduce some findings of the empirical literature estimating public-private wage differentials. We calculate the public-sector wage premium by education for these four countries. We first split the sample for college graduates and workers without college. We then run regressions of the log gross hourly earnings on a dummy for the public sector, controlling for region, gender, age, occupation, finer education categories and a part-time dummy for each the two groups and for each year of the survey. The estimated premia alongside with a 95 percent confidence intervals are shown in Figure 1.

The first result is that the premium for workers with lower qualifications is always higher than for college graduates in all countries. This reflects the wage compression across education groups that has been found in the literature. The second result is that the average premium and the compression varies substantially across countries. For instance, the estimated premia for the UK are consistently 3 to 4 percentage points higher than for the US. The third result is the premia varies across time. This can reflect either a different evolution in the private sector of skilled and unskilled wages, that was not incorporated in the public-sector pay scale, or a deliberate policy. For instance, in the beginning of the Euro-Area crisis

Figure 1: Estimated public-sector wage premium



Source: SES and CPS. We run regressions of the log gross hourly earnings on a dummy for the public sector, controlling for region, gender, age, occupation, education and a part-time dummy for workers with and without a college degree. For the European countries it is ran for 2002, 2006, 2010, 2014. For the US, each point is the estimation results for a 2-year sample.

in Spain, the highest public-sector wages had a cut of 10 percent while the lowest wages did not face direct cuts. One can see the effects of this policy in the graph. The estimated premium for college graduates in Spain fell from close to 10 percent in 2006 to 3 percent in 2014, while the estimated premium for workers without college stayed roughly constant. In France, both premia fell by close to 15 percentage points between 2006 and 2010.

The fact that the premia can change rather quickly is relevant for our interpretation of the estimates of the public-private wage differentials, and brings a note of caution in the quantitative section and the drawing of policy conclusions. One should be aware that they refer to an average of the policy between 2002 and 2014 and do not reflect the current policy.

3 Model with segmented markets

3.1 Preliminary considerations

The defining characteristic of the public sector is that it does not sell its goods or services - it supplies them directly to the population. There is no market price. Governments finance employment, not by selling goods, but by using the power of taxation. As such, the public sector does not maximize profits and the decisions regarding employment can reflect different government objectives. Even in determining public-sector wages (or wage growth) there is a discretionary component that can create wage differentials vis-à-vis the private sector, documented in the previous section. As such, the usual mechanisms that drive the private sector adjustments studied by economists do not map into the public sector.

Our modeling choices reflect this view of the public sector. As in other papers in the literature on public-sector employment, i.e. Bradley et al. (2017) or Albrecht et al. (2017), we assume that the government wage schedule is exogenous – it does not follow productivity and is not the outcome of bargaining. We view the public-sector wages not as an equilibrium outcome (i.e. private wages) but a policy variable (i.e. unemployment benefits or government spending). Notice that public-sector wages is a payment in units of private-sector goods (financed with taxation), not in units of public-sector goods, hence they are not necessarily dependent of the (marginal) productivity of the public sector. As such, public-sector wages might be influenced by several factors, such as unions, redistribution or elections. We do not take a stance as to why public-sector wages of skilled or unskilled workers are high or low. We take them as exogenous to match the data. Also, we assume that for the public sector to function, it requires an exogenous number of skilled and unskilled public-sector workers. Given a wage for each of these two skill groups, the government must maintain its employment level constant by hiring enough new workers to replace those that separate into unemployment or retire.

For the adamant reader, we show in Appendix C one simple model where government maximizes an objective function that incorporates government services and union preferences, subject to a budget constraint. This model endogeneizes both public-sector employment and wages as a function of the technology of production of government services, the budget, the power of unions and the relative union preference for unskilled public-sector workers. The exogenous changes in public-sector wages and employment of low- and high-educated workers that we consider, can be viewed as driven by changes in these factors.

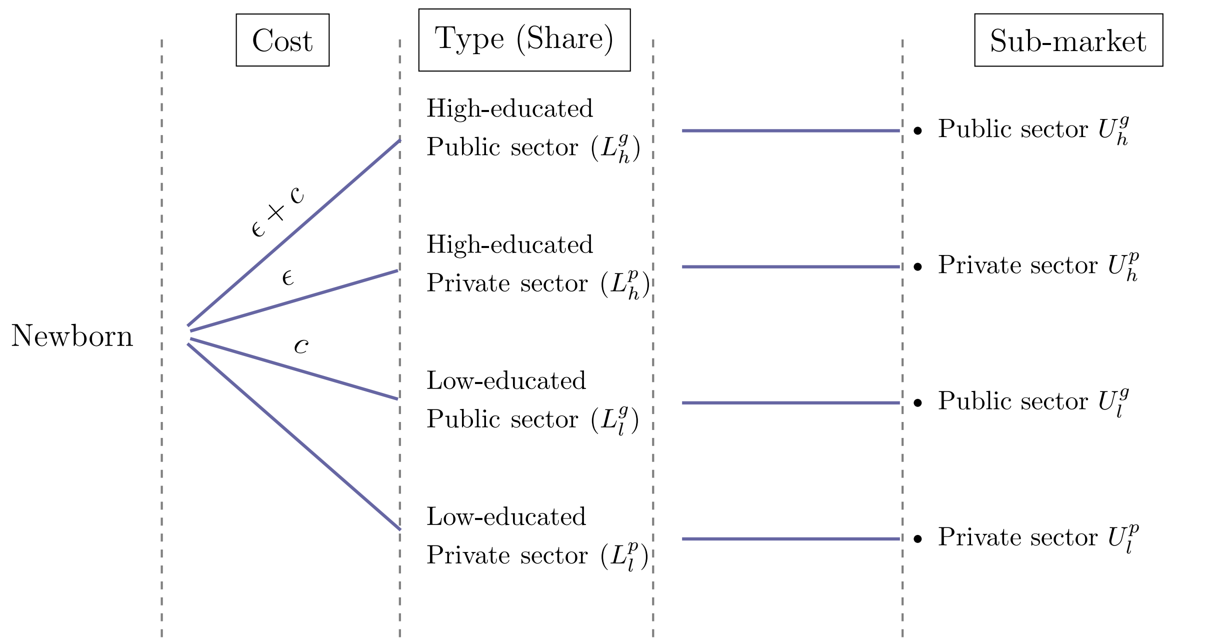
3.2 General setup

We consider a search and matching model with private-sector firms and a public sector. Workers can be either employed and producing or unemployed and searching for a job. Each private-sector firm is endowed with a single vacancy that can be vacant or filled (job). At each instant, τ individuals are born (enter the labor market) and die (retire) so that the working population is constant and normalized to unity. All agents are risk-neutral and discount the future at a common rate $r > 0$. Time is continuous.

An agent can be either low- or high-educated. All individuals are born low-educated, but prior to entering the labor market, they can become high-educated by paying a schooling cost ϵ. The schooling cost is distributed across individuals according to the cumulative distribution function $\Xi^{\epsilon}(\cdot)$ on $[0, \bar{\epsilon}]$. Heterogeneity with respect to schooling cost reflects either different learning abilities or the existence of financial constraints. High-educated individuals are more productive than low-educated individuals. Education is observable and there are no other dimensions of human capital, such as unobserved ability, match quality or on-the-job learning.

In parallel, prior to entering the labor force an individual must decide whether she wants to pursue a career in the public sector or not. To access jobs in the public sector an individual must pay cost c. The cost is distributed across individuals according to the cumulative distribution function $\Xi^{c}(\cdot)$ on $[0, \bar{c}]$. This cost may represent the cost of passing qualifying exams, networking or investing in political connections. Taking exams may be more costly to some individuals than to others, while having access to jobs in the public sector may be easier for some workers whose friends or family members, for instance, work in the public sector. If such barriers to entry exist, we show below that the two sectors, public and private, are segmented, because for the workers who choose to pay the entry cost, pursuing a career in the public sector strictly dominates all other options. In Section 5, we also explore the case where no barriers to entry into the public sector exist and workers search randomly for

Figure 2: Decision of newborn



both public- and private-sector jobs.

An endogenous proportion of the population (those whose schooling cost is sufficiently low) become high educated; another fraction (those whose entry cost is sufficiently low) attaches to the public sector. If both costs are low, workers become high-educated in the public sector, while if both costs are high, workers remain low-educated in the private sector. The rest may remain low educated and search in the public sector or become high educated and search in the private sector. The shares of each type in the population are L_l^p, L_l^g,L_h^p and L_h^g and add up to 1. Variables are indexed by the superscript $x = [g, p]$, where g refers to the public (government) sector and p to the private sector, and the subscript $i = [l, h]$, where h to high- and l to low-educated. Figure 2 depicts these four choices. In each of the two sectors there are two labor markets segmented by education. In the "high-education" market, both firms and government open vacancies suited for high-educated workers, whereas in the "low-education" market, vacancies are suited for low-educated workers; high-educated individuals in either sector (private or public) direct their search towards type-h jobs, whereas low-educated workers direct their search towards type-l jobs. In total, there are four active submarkets. The output y_i of any match depends only on the worker's education: high-educated individuals are more productive than low-educated individuals. That is, $(y_h > y_l)$. A searching (unemployed) worker of type-i receives a flow of income b_i, which can be considered the opportunity cost of employment.

3.3 The Private sector

Firms in each of the two submarkets open vacancies and search for workers until all rents are exhausted. The rate at which type-i workers find private-sector jobs of type i depends positively on the tightness, $\theta_i = \frac{v_i^p}{u_i^p}$, where v_i^p is the measure of private-sector vacancies of type i, and u_i^p is the number of type i workers that are unemployed and searching in the private sector. Workers of type i are hired into private-sector jobs (of type i) at Poisson rate $m(\theta_i)$, and private-sector firms fill type i vacancies at rate $q(\theta_i) = \frac{m(\theta_i)}{\theta_i}$.

Wages in the private sector, denoted as w_i^p, depend on match surplus, so they also differ by education level. They are determined by Nash bargaining, such that the worker gets a share β of the match surplus. With higher match surplus, firms expect to generate larger profits from creating jobs; firm entry is higher; and workers can more easily find jobs and also earn higher wages.

A vacant firm bears a recruitment cost κ_i specific to education, related to the expenses of keeping a vacancy open and looking for a worker. When a vacancy and a worker are matched, they bargain over the division of the produced surplus. The surplus that results from a match is known to both parties. After an agreement has been reached, production commences immediately. Matches in the private sector with type i workers dissolve at the rate s_i^p. Following a job destruction, the worker searches for a new match in the same submarket.

3.4 Government

To produce government services, the government employs a certain number of workers, high and low educated (e_h^g, e_l^g), which together with the public-sector wages, (w_h^g,w_l^g), are the exogenous policy variables. In each instant, the government has to hire enough workers to replace the workers that exogenously separate or retire. That means hiring $(s_h^g + \tau)e_h^g$ skilled and $(s_l^g + \tau)e_l^g$ unskilled workers, where s_i^g is the separation rate. The matching function in the public sector is $M_i^g = \min\{v_i^g, u_i^g\}$. To maintain its employment level, the government must be able to attract a number of searchers in each segment, u_i^g, at least equal to the number of job openings, v_i^g, meaning that $M_i^g = v_i^g$. Otherwise, public-sector services breakdown. As we will show in Lemma 2, this imposes a condition on public-sector wages to be high enough to attract at least the same number of searchers as of vacancies. We assume that the recruitment is part of the role of the government and is done by its workforce. Since the government's objective is to maintain employment levels (e_h^g, e_l^g) by hiring enough workers to replace those that separate or retire, it follows that $v_i^g = (s_i^g + \tau)e_i^g$. Workers of type-$i$ find public-sector jobs at rate $m_i^g = \frac{(s_i^g+\tau)e_i^g}{u_i^g}$.

We choose this matching function for simplicity and clarity. First, it makes the concept of queues in the public sector clearer. When there are more unemployed than vacancies, the vacancy filling rate for the government is 1, and all the unemployed in excess are queuing. As we will show, this makes the minimum wage required for the existence of the public sector an intuitive object, easy to calculate. Second, this assumption has been used in other papers, i.e. Quadrini and Trigari (2007) and there is evidence that the elasticity of matches with respect to unemployed is much lower in the public sector than in the private (Gomes (2015)). This does not mean that there are no matching frictions, only that they are one-sided. Nothing substantial would change in the model if the matching function in the public sector was Cobb Douglas: $M_i^g = (v_i^g)^\eta (u_i^g)^{1-\eta}$. In this case, the vacancy filling probability of the government would no longer be 1, and it would need to set endogenously the number of vacancies such that the total number of matches would equate exactly the number of workers that retire or separate – that is, $M_i^g = e_i^g(s_i^g + \tau)$. Solving for v_i^g we would obtain $v_i^g = (e_i^g(s_i^g + \tau))^{\frac{1}{\eta}} / (u_i^g)^{\frac{1-\eta}{\eta}}$. Still, the job-finding rate of the unemployed would be exactly the same, as well as the size of the queue, measured by $u_i^g - M_i^g$.

As mentioned above, we assume that a worker of type i produces y_i in both sectors. As will become clearer below, because public-sector employment is exogenous, the productivity of workers in the public sector is not important for the results to follow. Hence, we assume, without loss of generality, that workers are equally productive in the two sectors, but the results would still hold even if productivity was different across the two sectors.

Notice that in this setting, where the government has a fixed employment level, the separation rates s_i^g play a double role: they reflect the expected duration of the match but also determine the number of new hires. Higher separations reduce the value of employment in the public sector but, at the same time, increase the number of vacancies and make an unemployed more likely to find a job there. Also, we assume that the separation rates, as well as other labor market friction parameters, are exogenous. We ignore the issue of how the government finances its wage bill and assume that it can tax its citizens in a non-distortionary lump-sum tax.

3.5 Value functions, Free entry, Wages

Let U_i^p and E_i^p be the values (expected discounted lifetime incomes) associated with unemployment (searching for a job) and employment in the private sector of a worker of education level $i = [h, l]$. These are defined by:

$$(r+\tau)U_i^p = b_i + m(\theta_i)\left[E_i^p - U_i^p\right], \quad (1)$$

$$(r+\tau)E_i^p = w_i^p - s_i^p\left[E_i^p - U_i^p\right]. \quad (2)$$

The values associated with unemployment and employment in the public sector of a worker of education level $i = [h, l]$ are given by:

$$(r+\tau)U_i^g = b_i + m_i^g\left[E_i^g - U_i^g\right], \quad (3)$$

$$(r+\tau)E_i^g = w_i^g - s_i^g\left[E_i^g - U_i^g\right], \quad (4)$$

On the private-sector firm side, let J_i^p be the value associated with a job by a worker of type i and V_i^p be the value associated with posting a private-sector vacancy and searching for a type i worker to fill it, given by

$$rJ_i^p = y_i - w_i^p - (s_i^p + \tau)\left[J_i^p - V_i^p\right], \quad (5)$$

$$rV_i^p = -\kappa_i + q(\theta_i)\left[J_i^p - V_i^p\right]. \quad (6)$$

In equilibrium, free entry drives the value of a private vacancy to zero:

$$V_i^p = 0, \quad i = [h, l]. \quad (7)$$

Wages are determined by Nash bargaining between the matched firm and worker. The outside options of the firm and the worker are the value of a vacancy and the value of being unemployed, respectively. Let $S_i^p \equiv J_i^p - V_i^p + E_i^p - U_i^p$ denote the surplus of a match with a type i worker. With Nash bargaining, the wage w_i^p is set to a level such that the worker gets a share β of the surplus, and the share $(1-\beta)$ goes to the firm. This implies two equilibrium conditions of the following form:

$$\beta S_i^p = E_i^p - U_i^p \qquad (1-\beta)S_i^p = J_i^p - V_i^p. \quad (8)$$

Setting $V_i^p = 0$ in (6) and imposing the Nash bargaining condition in (8) gives:

$$\frac{\kappa_i}{q(\theta_i)} = (1-\beta)S_i^p. \quad (9)$$

Using (1)-(5) together with (8) and the free-entry condition $V_i^p = 0$, we can write:

$$S_i^p = \frac{y_i - b_i}{r + \tau + s_i^p + \beta m(\theta_i)}, \tag{10}$$

and the free-entry condition as

$$\frac{\kappa_i}{q(\theta_i)} = \frac{(y_i - b_i)(1-\beta)}{r + \tau + s_i^p + \beta m(\theta_i)}. \tag{11}$$

The equation in (11) gives the two job-creation conditions that set the expected costs of having a vacancy (left-hand-side) equal to the expected gains from a job (right-hand-side). It can be used to determine the equilibrium market tightness θ_i, the rates at which workers find jobs in the private sector, $m(\theta_i)$ and, in turn, the size of private-sector employment,

$$e_i^p = \frac{m(\theta_i) L_i^p}{s_i^p + \tau + m(\theta_i).} \tag{12}$$

Imposing the free-entry condition (9) for private-sector vacancy creation, the Nash bargaining solution implies that

$$w_i^p = b_i + \beta(y_i - b_i + \kappa_i \theta_i), \quad i = [h, l]. \tag{13}$$

Lemma 1 *Tightness and wages in the private sector, in both the low- and the high-education submarkets, are independent of the government employment and wage policies (e_i^g and w_i^g).*

This lemma is a useful intermediate result and follows directly from equations (11) and (13). It implies that government employment and wage policies affect the equilibrium only through the education decisions of the newborns or through the scale of the private sector (L_i^p). Given a constant tightness, policies that make the public sector more attractive will drain the unemployed from the private sector and reduce, one-to-one, the number of vacancies, leaving private wages unchanged.

3.6 Newborn‘s Decisions

We can summarize the four options of the newborn, depicted in Figure 2, as

$$(r+\tau)U_i^p = b_i + \frac{m(\theta_i)}{r + \tau + s_i^p + m(\theta_i)}[w_i^p - b_i], \quad i = [h, l], \tag{14}$$

$$(r+\tau)U_i^g = b_i + \frac{m_i^g}{r + \tau + s_i^g + m_i^g}[w_i^g - b_i], \quad i = [h, l]. \tag{15}$$

The newborn chooses the option that, given her ϵ and c, has the highest value:

$$Max\{U_l^p, U_h^p - \epsilon, U_l^g - c, U_h^g - c - \epsilon\}. \tag{16}$$

A worker of type $i = [h, l]$ and entry cost c chooses to search in the public sector only if the benefit, $U_i^g - U_i^p$, exceeds the cost – that is, only if $U_i^g - U_i^p \geq c$. The threshold level of c at which a worker of type i is indifferent between searching for a job in the public or in the private sector is, therefore, given by

$$\tilde{c}_i = U_i^g - U_i^p. \tag{17}$$

A worker of type i with cost c will choose to enter the public sector only if $c \leq \tilde{c}_i$. Otherwise, the worker will search for a job in the private sector.

Lemma 2 *There exist a public sector with two markets with employment levels e_h^g and e_l^g, provided that it pays sufficiently high wages $w_h^g \geq \underline{w}_h^g$ and $w_l^g \geq \underline{w}_l^g$.*

The exact expressions for the minimum level of wages in the public sector, $\underline{w}_h^g$ and $\underline{w}_l^g$, are in Appendix A. This lemma states that the public sector needs to pay a sufficiently high wages in order to attract enough job seekers to fill its vacancies and maintain constant employment levels e_h^g and e_l^g. These thresholds, $\underline{w}_h^g$ and $\underline{w}_l^g$, depend positively on private-sector wages, w_h^p and w_l^p, and unemployment benefits, b_h and b_l and is high enough to compensate workers for paying the entry cost c.

Lemma 3 *If $w_i^g \geq \underline{w}_i^g$ then $U_i^g > U_i^p$ and the two sectors of type i, the public and the private, are segmented.*

The proof is in Appendix A. This lemma implies that if a public sector with costly entry exists, which means that its wage is high enough to attract enough job searchers ($w_i^g \geq \underline{w}_i^g$), then it must be that the value of searching for a job in that public sector is higher than the value of searching for a job in the private sector. This also implies that for workers who paid the cost c, the option of searching for a job in the public sector strictly dominates all other options. These workers seek a career in the public sector; the rest search for jobs in the private sector.

3.7 Equilibrium Allocations

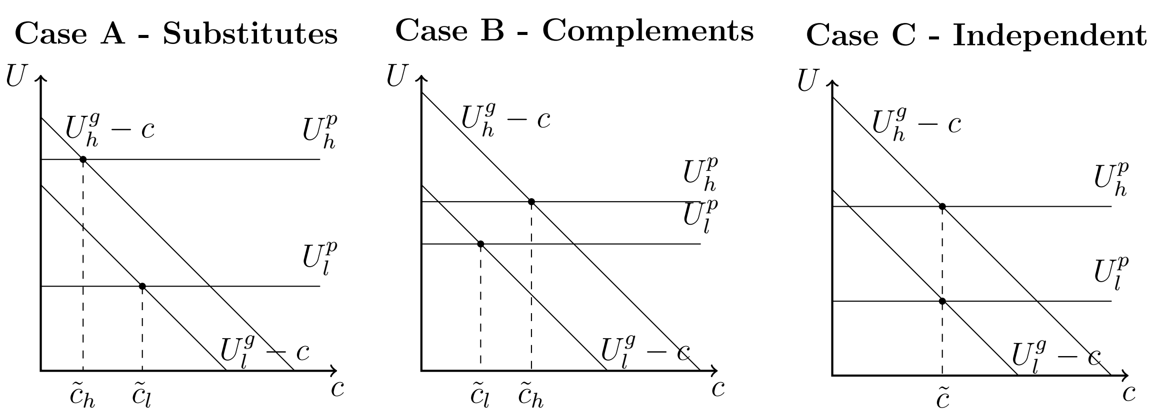
As shown in Figure 3, we can have three different cases, each with different implications for how the existence of a public-sector alters workers' incentives to invest in education.

Case A in Figure 3 describes a scenario in which a career in the public sector substitutes investing in education. The benefit from investing in education is smaller if the worker enters the public sector than if not. That is, $U_h^p - U_l^p > U_h^g - U_l^g$, and low-educated workers have more incentive than high-educated workers to join the public sector labour market ($\tilde{c}_h < \tilde{c}_l$). Case A could reflect a situation in which public-sector wages are flat across worker qualifications (wage compression) relatively to the private sector. In such cases, those seeking to attain jobs in the public sector have less incentive to invest in education, while those whose entry cost is high, and thus government jobs are less accessible, have more incentive to opt for education.

More specifically, the two thresholds $\tilde{c}_h$ and $\tilde{c}_l$ can be used to divide workers into three groups that differ in their incentives to obtain higher education. In the first group are workers whose entry cost c is low: $c < \tilde{c}_h (< \tilde{c}_l)$. For these workers, $U_c^g - c > U_i^p$ for $i = [h, l]$, and regardless of their education, targeting jobs in the public sector always yields a higher payoff than searching in the private sector. For these workers, the net benefit from investing in education is given by $\tilde{\epsilon}_g = U_h^g - U_l^g$. Next is the group of workers whose entry cost c lies between $\tilde{c}_h$ and $\tilde{c}_l$. For these workers, a job in the public sector is worthwhile only if they remain low-educated. That is, $U_l^g - c > U_l^p$, but $U_h^g - c < U_h^p$. If they invest in eduction they are better off searching in the private sector. Thus, their benefit from education is $\tilde{\epsilon}_m(c) \equiv U_h^p - (U_l^g - c)$, which is increasing in c. In the last group are the workers with $c > \tilde{c}_l (> \tilde{c}_h)$, who never obt for jobs in the public sector because the cost is too high. For these workers, $U_i^p > U_i^g - c$ for $i = [h, l]$. If they choose to become high-educated, they obtain a payoff of $\tilde{\epsilon}_p \equiv U_h^p - U_l^p$.

In the opposite case – case B in Figure 3 – education "complements" search in the public sector. That is, workers seeking a career in the public sector have more incentive to become

Figure 3: Decision Thresholds



high-educated ($U_h^g - U_l^g \geq U_h^p - U_l^p$), which also implies that high-educated workers have more incentive to join the public sector ($\tilde{c}_h > \tilde{c}_l$). Case B could arise when the public sector has many jobs available for skilled workers. As above, the low-entry-cost workers, those with $c < \tilde{c}_l (< \tilde{c}_h)$, always choose to target public-sector jobs and have a payoff $\tilde{\epsilon}_g$ from becoming high-educated. Conversely, there are workers with $c > \tilde{c}_h (> \tilde{c}_l)$, who never opt for government jobs and have a payoff $\tilde{\epsilon}_p$ from investing in education. In between are the workers with $\tilde{c}_l < c < \tilde{c}_h$. As Figure 3 shows, for these workers, $U_l^g - c < U_l^p$ and $U_h^g - c > U_h^p$; thus, they will enter the public sector if they become high-educated but will not if they remain low-educated. Investing in education brings them a benefit of $\tilde{\epsilon}_m(c) \equiv U_h^g - c - U_l^p$, which is decreasing in c.

Finally, case C is the knife-edge case in which the payoff from being high-educated is the same in both sectors. Targeting jobs in the public sector does not alter a worker's payoff from investing in education ($U_h^g - U_l^g = U_h^p - U_l^p$), and high- and low-educated workers both have equal incentives to search in the public sector ($\tilde{c}_h = \tilde{c}_l$). In this case, all workers obtain a payoff of $\tilde{\epsilon} = U_h^g - U_l^g = U_h^p - U_l^p$ from investing in education.

A worker invests in education only if the benefit exceeds the cost (ϵ). The education benefit can be either $\tilde{\epsilon}_p, \tilde{\epsilon}_m$ or $\tilde{\epsilon}_g$, depending on the worker's public-sector entry cost (c). It can be easily verified that these thresholds are given by:

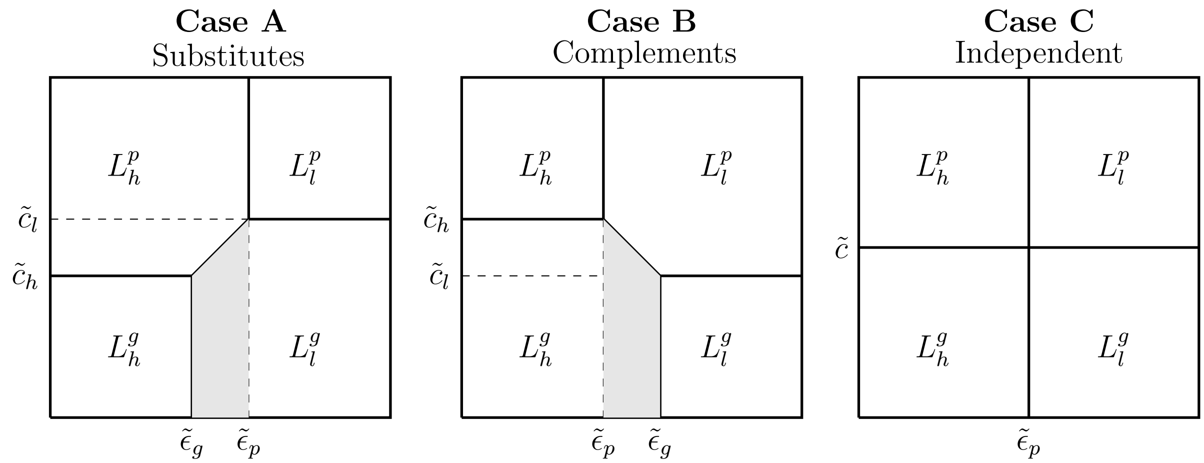
$$\tilde{\epsilon}_g = \tilde{\epsilon}_p + \tilde{c}_h - \tilde{c}_l, \tag{18}$$

$$\tilde{\epsilon}_m(c) = \tilde{\epsilon}_p + c - \tilde{c}_l \quad c \in [\tilde{c}_h, \tilde{c}_l], \text{ if } \tilde{c}_h < \tilde{c}_l \text{ (case A)}, \tag{19}$$

$$\tilde{\epsilon}_m(c) = \tilde{\epsilon}_p + \tilde{c}_h - c \quad c \in [\tilde{c}_l, \tilde{c}_h], \text{ if } \tilde{c}_h > \tilde{c}_l \text{ (case B)}. \tag{20}$$

Figure 4 illustrates how the education and public-sector entry cutoffs relate under the three cases. In case A, education substitutes for search in the public sector, and those most likely to invest in education have a high public-sector entry cost: $\tilde{\epsilon}_g \leq \tilde{\epsilon}_m \leq \tilde{\epsilon}_p$ and $\tilde{c}_h < \tilde{c}_l$. In this case, $\tilde{\epsilon}_m$ is increasing one-to-one with c. In case B, education complements search in the public sectors, and the benefit from education is higher for those whose public-sector entry cost is low: $\tilde{\epsilon}_g \geq \tilde{\epsilon}_m \geq \tilde{\epsilon}_p$ and $\tilde{c}_h > \tilde{c}_l$. In this case, $\tilde{\epsilon}_m$ is decreasing one-to-one with c. In case C, incentives to invest in education are independent of workers' public-sector entry cost, and equal fractions of workers searching in the private- and of workers searching in the public-sector invest in education: $\tilde{\epsilon}_g = \tilde{\epsilon}_m = \tilde{\epsilon}_p$ and $\tilde{c}_h = \tilde{c}_l = \tilde{c}$. Workers' cutoffs determine their selection into four groups: the high- and low-educated who target public-sector jobs (L_h^g and L_l^g), and the high- and low-educated who search in the private sector (L_h^p and L_l^p), as depicted in Figure 4. For each of the cases A, B and C, we can measure each of these four groups' share in the labor force as:

Figure 4: Cutoffs and allocations



$$\text{Case A, } \tilde{c}_h < \tilde{c}_l, \quad \begin{cases} L_h^g = \Xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_h) \\ L_l^g = (1-\Xi^\epsilon(\tilde{\epsilon}_g))\Xi^c(\tilde{c}_h) + \int_{\tilde{c}_h}^{\tilde{c}_l}(1-\Xi^\epsilon(\tilde{\epsilon}_m(c)))d\Xi^c(c) \\ L_h^p = \int_{\tilde{c}_h}^{\tilde{c}_l}\Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1-\Xi^c(\tilde{c}_l))\,\Xi^\epsilon(\tilde{\epsilon}_p) \\ L_l^p = (1-\Xi^\epsilon(\tilde{\epsilon}_p))(1-\Xi^c(\tilde{c}_l)) \end{cases} \tag{21}$$

$$\text{Case B, } \tilde{c}_h > \tilde{c}_l \quad \begin{cases} L_h^g = \Xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_l) + \int_{\tilde{c}_l}^{\tilde{c}_h}\Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) \\ L_l^g = (1-\Xi^\epsilon(\tilde{\epsilon}_g))\Xi^c(\tilde{c}_l) \\ L_h^p = (1-\Xi^c(\tilde{c}_h))\,\Xi^\epsilon(\tilde{\epsilon}_p) \\ L_l^p = (1-\Xi^\epsilon(\tilde{\epsilon}_p))(1-\Xi^c(\tilde{c}_h)) + \int_{\tilde{c}_l}^{\tilde{c}_h}(1-\Xi^\epsilon(\tilde{\epsilon}_m(c)))d\Xi^c(c) \end{cases} \tag{22}$$

$$\text{Case C, } \tilde{c}_h = \tilde{c}_l = \tilde{c} \quad \begin{cases} L_h^g = \Xi^\epsilon(\tilde{\epsilon}_p)\Xi^c(\tilde{c}) \\ L_l^g = (1-\Xi^\epsilon(\tilde{\epsilon}_p))\Xi^c(\tilde{c}) \\ L_h^p = (1-\Xi^c(\tilde{c}))\,\Xi^\epsilon(\tilde{\epsilon}_p) \\ L_l^p = (1-\Xi^c(\tilde{c}))(1-\Xi^\epsilon(\tilde{\epsilon}_p)) \end{cases} \tag{23}$$

$L_h = L_h^g + L_h^p$ gives the share of high-educated in the labor force and $L_l = 1 - L_h = L_l^g + L_l^p$ the share of low-educated. $L^g = L_h^g + L_l^g$ gives the share of labor force that is attached to the public sector and $L^p = L_h^p + L_l^p = 1 - L^g$ the share of labor force that is attached to the private sector.

Using (8)-(11) and (14)-(15), we can write the cutoffs as:

$$\tilde{c}_i = \frac{1}{r+\tau}\left[\frac{\frac{(s_i^g+\tau)e_i^g}{u_i^g}}{r+\tau+s_i^g+\frac{\mu(s_i^g+\tau)e_i^g}{u_i^g}}[w_i^g-b_i]-\frac{\beta\kappa_i\theta_i}{(1-\beta)}\right], \quad i=[h,l], \tag{24}$$

$$\tilde{\epsilon}_p = \frac{1}{r+\tau}\left[b_h-b_l+\frac{\beta\kappa_h\theta_h}{(1-\beta)}-\frac{\beta\kappa_l\theta_l}{(1-\beta)}\right]. \tag{25}$$

Definition 1 *A steady-state equilibrium consists of a set of cut-off costs $\{\tilde{c}_h, \tilde{c}_l, \tilde{\epsilon}_p, \tilde{\epsilon}_g\}$, private sector tightness $\{\theta_h, \theta_l\}$, and unemployed searching in each submarket $\{u_h^p, u_l^p, u_h^g, u_l^g\}$, such that, given the exogenous government policies $\{w_h^g, w_l^g, e_h^g, e_l^g\}$, the following apply.*

1. *Private-sector firms satisfy the free-entry condition (11) $i=[h,l]$.*
2. *Private-sector wages are the outcome of Nash Bargaining (13) $i=[h,l]$.*
3. *Newborns decide optimally their investments in education and choice of sector (equation 16), and the population shares are determined by the equations (21), (22) or (23), depending on the case.*
4. *Flows between private employment and unemployment are constant:*

$$(s_h^p+\tau)e_h^p = m(\theta_h)u_h^p, \tag{26}$$

$$(s_l^p+\tau)e_l^p = m(\theta_l)u_l^p. \tag{27}$$

5. *Population add-up constraints are satisfied:*

$$L_i^p = e_i^p + u_i^p, \tag{28}$$

$$L_i^g = e_i^g + u_i^g, \tag{29}$$

$$L_h^p + L_l^p + L_h^g + L_l^g = 1. \tag{30}$$

4 Main results

This section details the main results, under 7 propositions. The propositions summarize how the existence and policies of a public sector affect: i) the composition of the labor force in terms of education and ii) employment. The first two propositions summarize the effects on educational composition and employment in a special case where the two sectors are segmented but there are no costs to entering the public sector. The next four propositions summarize results in the benchmark model where the two sectors are segmented because entry into the public sector is costly: two are on the educational composition and two are on employment. The last proposition explains how the public sector affects the educational composition of the private-sector labor force and employment. All the derivations and proofs are shown in Appendix A, including the proof that the equilibrium exists and is unique.

4.1 A special case: no entry costs and segmented markets

We first consider the case where, as above, the two sectors are segmented, but there are no costs to entering the public sector. In this special case, the presence or policies of a public sector do not distort newborns' decision to invest in education and generate only employment effects. We examine this special case separately for two reasons. First, it helps demonstrate how mobility between the two sectors can neutralize the effects of policies, when the two sectors are segmented. Second, it allows us to isolate the pure employment effects of the public sector on the low- and high-educated workers, from the changes in employment that arise due to changes in the educational composition of the labor force.

Free entry into the public sector ensures

$$U_i^g = U_i^p, \quad i = [h, l]. \tag{31}$$

This condition determines the number of searchers in the public sector, u_i^g, which is the variable that compensates any asymmetry in the value of the job in the two sectors. An increase of the value of a public-sector job, E_i^g, (driven by either higher wages or lower separations) raises the number of unemployed searching in the public sector and lowers their job-finding probability (m_i^g), such that its effect on U_i^g is neutralized. This condition also ensures that $U_h^g - U_l^g = U_h^p - U_l^p$ meaning that $\tilde{\epsilon}_g = \tilde{\epsilon}_p$ and the payoff from being high-educated is the same in both sectors. The presence of the public sector does not distort newborns' decision to invest in education and the population shares are determined by the equations

$$L_h = \Xi^\epsilon(\tilde{\epsilon}_p) \tag{32}$$

$$L_l = 1 - \Xi^\epsilon(\tilde{\epsilon}_p) \tag{33}$$

where the cutoff education costs $\tilde{\epsilon}_p$ is as given in (25).

Proposition 1 *With segmented markets and no costs to entering the public sector, the presence of a public sector does not alter the educational composition of the labor force. Moreover, the educational composition of the labor force is independent of public-sector policies.*

If no barriers to entering the public-sector exist, free entry of searchers into the public sector equalizes any differences in the education premium across the two sectors. A larger education premium in the public sector means that a larger share of high-educated workers is searching for jobs in the public sector. The resulting decrease in the job-finding rate offsets the larger education premium and makes the benefit from investing in equation equal across the two sectors. Free entry into the public sector also ensures that the benefit from investing in education is independent of public-sector policies. Consider, for instance, an increase in w_h^g that raises the value of a high-education job in the public sector. Consequently, some high-educated unemployed direct their search towards the public sector. The increased congestion due to the arrival of additional job seekers in the public sector lowers the job-finding rate, and pushes the value of searching for a high-education job in the public sector back to its initial level. Incentives to invest in education remain intact.

Now, let $e_i = e_i^g + e_i^p$ be the total number of workers of type i who are employed. If no public sector exists (e_i^g=0) then $e_i = e_i^p$ and all workers are employed in the private sector. The presence of a public sector drains some of these workers away from the private sector and into the public sector. In this special case, where the public sector does not affect the educational composition of the labor force, this may increase or decrease the total employment of workers of type i (e_i), depending on public-sector wage and lay-off policies. Specifically,

Proposition 2 *With segmented markets and no costs to entering the public sector, the existence of a public sector for workers of type i with wage $w_i^g = \underline{w}_i^g$*

- *increases e_i if $s_i^g < s_i^p$*
- *decreases e_i if $s_i^g > s_i^p$ and*
- *does not affect e_i if $s_i^g = s_i^p$.*

If $w_i^g > \underline{w}_i^g$ the existence of the public sector

- *decreases e_i if $s_i^g \geq s_i^p$*
- *may decrease or increase e_i if $s_i^g < s_i^p$.*

We show in Appendix A that in the presence of a public sector we can write the total employment of type i workers as

$$e_i = \frac{m(\theta_i)L_i}{s_i^p + \tau + m(\theta_i)} + \frac{e_i^g(s_i^p - s_i^g)}{s_i^p + \tau + m(\theta_i)} - \frac{Q_i m(\theta_i)}{s_i^p + \tau + m(\theta_i)} \tag{34}$$

The first term in the above expression captures the effect of changes in labor force shares due to changes in the education decision. The employment of type i workers is larger when their share in the labor force (L_i) is larger. The second and third terms capture, respectively, the role of public-sector job security and public-sector queues. In particular, $Q_i = L_i^g - e_i^g - e_i^g(s_i^g + \tau)$ denotes the size of the "queue" in public-sector submarket i. $Q_i > 0$ means that the public sector attracts a larger number of unemployed than that needed to fill its vacancies. We further show that public-sector queues exist only if the wage is too high. If $w_i^g = \underline{w}_i^g$ then $Q_i = 0$ meaning that there are as many unemployed workers seeking to find jobs in the public sector as vacancies, whereas, if $w_i^g > \underline{w}_i^g$ then $Q_i > 0$ and the existence of a public sector involves an additional negative effect on employment. As shown in Proposition 1, if there are no barriers to entry into the public sector, the education shares (L_i) are independent of the public policies. Hence, in this case, the public sector can affect employment only through job security and queues. If, in addition, $w_i^g = \underline{w}_i^g$, then there are no queues and only job security matters. The public sector increases total employment if it offers greater job security ($s_i^g < s_i^p$) and vice versa, whereas if job stability is the same in both sectors ($s_i^g = s_i^p$) total employment is independent of the presence of a public sector. If, on the other hand, $w_i^g > \underline{w}_i^g$, then the public attracts too many job searchers with an additional negative impact on employment. In this case the presence of a public sector unambiguously decreases total employment if job separation rates in the public sector are not lower than those in the private sector ($s_i^g \geq s_i^p$). But even if public-sector jobs are more stable than private jobs, the presence of the public sector may have a negative impact on employment, if public-sector wages are too high.

If barriers to entering the public sector exist, then limited entry into the public sector prevents the education premium from being equalized across the two sectors. As discussed above, in this case the public sector influence newborns' decision to invest in education and the labor force shares. Consequently, the existence of a public sector and changes in policies affect total employment, not only through job stability and job queues, but also through their impact on education decisions. We next discuss how the public sector affects the educational composition of the labor force and the employment of high- and low-educated workers when barriers to entering the public sector exist.

4.2 Effects on educational composition

When entry into the public sector is costly, differences in the education premium between the private and public sector (or equivalently, differences in the public-sector premium for high- and low-educated workers) are important in determining how the public sector affects the choice of education and in turn, the composition of the labor force in terms of education.

Proposition 3 *If entry into the public sector is costly, the existence of the public-sector*

- *decreases L_h (increases L_l) if $\tilde{c}_h < \tilde{c}_l$ – Case A/substitutes*
- *increases L_h (decreases L_l) if $\tilde{c}_h > \tilde{c}_h$ – Case B/compliments*
- *does not affect L_h (and L_l) if $\tilde{c}_h = \tilde{c}_l$ – Case C/independent.*

In Case A, where for some workers a career in the public sector substitutes obtaining education, then the low-educated can benefit the most from entering the public sector. For workers whose entry cost is low, it is worthwhile to substitute education for a low-education job in the public sector. As a result, there are fewer high-educated people. In the opposite case, when education compliments search for government jobs (case B), if the option to search in the public sector is allowed, some workers, those whose cost of entering the public sector is low, have more incentive to become educated and there are more high educated people.

The shaded areas in Figure 4 illustrate the decrease and increase, respectively, in the fraction of high-educated workers in the labor force once a public-sector is introduced. Under Case A, the shaded area represents the fraction of people that would have become educated in the absence of the public sector, but prefer to remain uneducated and search for low-education jobs in the public-sector. Under Case B, the shaded area represents the fraction of people that would have remained uneducated, when only search in the private sector was allowed, but now prefer to get education and search for government jobs.

Let us next consider the effect of government policies.

Proposition 4 *If entry into the public sector is costly, an increase in w_h^g, e_h^g or a decrease in w_l^g, e_l^g raises the proportion of the high-educated in the labor force (raises L_h and decreases L_l).*

Any improvement in the value of working in the public sector cannot be fully neutralised since the presence of entry costs reduces the inflow of searchers into the public sector. As a result, government policies that increase the benefit from investing in education now induce a higher fraction of the labor force to become high-educated.

4.3 Effects on employment

Besides job stability and job queues, the public sector also affects total employment though its impact on the education decision. As explained above, this occurs when barriers to entry into the public sector prevent the education premium from being equalized across the two sectors. To see this consider the case where $s_i^p = s_i^g$ and $w_i^g = \underline{w}_i^g$ for $i = [h, l]$, which implies that private and public jobs are equally stable and that there are no queues for jobs in the public sector. In terms of equation (34) this means setting the last two terms equal to zero so that only changes in the labor force shares L_i matter for employment changes. The proposition that follows shows that even in this case the public sector will affect the employment of high- and low-educated by changing their labour force shares.

Proposition 5 *The existence of a public sector with entry costs, $s_i^g = s_i^p$, and $w_i^g = \underline{w}_i^g$ for $i = [h, l]$,*

- *decreases e_h and increases e_l if $\tilde{c}_h < \tilde{c}_l$ – Case A/substitutes*
- *increases e_h and decreases e_l if $\tilde{c}_h > \tilde{c}_h$ – Case B/compliments*
- *does not affect e_h and e_l if $\tilde{c}_h = \tilde{c}_l$ – Case C/independent.*

As summarised in Proposition 3, the public sector may increase, decrease or leave the proportion of high-educated in the labor force unchanged, depending on how the education premium differs between the two sectors (whether in Case A, B or C). The employment of high- and low-educated workers changes accordingly: with a higher share of high-educated workers the number of employed high educated workers increases while that of low-educated decreases and vice versa. If, on the other hand, the public sector does not distort the education decision (e.g., if we have Case C: $\tilde{c}_h = \tilde{c}_l$), then its consequences on employment will reflect differences in job stability and the creation of job queues, as summarized in Proposition 2. In the general case, effects on employment work through all of these channels: educational composition, job stability and job queues. If public-sector jobs are more stable than private-sector jobs, the existence of the public sector will have a more positive (or less negative impact) on the employment of each of the two education types. If, on the other hand, public-sector wages are high ($w_i^g > \underline{w}_i^g$), the effects on employment will be more negative (or less positive). It follows that more generous public-sector wage policies have a negative impact on employment.

Proposition 6 *An increase in w_i^g increases L_i^g and decreases L_j^g, L_i^p, L_j^p. It also decreases e_i and e_j and the total employment rate ($e = e_i + e_j$). $i = [h, l]$, $j = [h, l]$ and $i \neq j$.*

A higher wage in the public sector market i makes the value of searching for a job there higher and shifts workers away from all other sectors and into this market. By shifting workers away from the private sectors and into the public sector waiting for jobs, a higher public-sector wage lowers the employment of both types of workers, as well as the total employment rate.

An increase in the size of the public sector i, e_i^g, has similar effects. It drains workers away from all other sub-markets, thus it decreases e_j and increases L_i^g. However, the impact of such an increase on the employment of type i workers is not necessarily negative because of the direct employment effect.

4.4 Effects on the private sector

Proposition 7 *The existence of the public-sector improves (worsens) the educational composition of the private-sector labor force and employment: increases (decreases)* $\frac{L_h^p}{L_l^p}$ *and* $\frac{e_h^p}{e_l^p}$, *if* $\tilde{c}_h < \tilde{c}_l$ ($\tilde{c}_l < \tilde{c}_h$) *– Case A (Case B). The existence of the public-sector has no impact on the educational composition of the private-sector if* $\tilde{c}_h = \tilde{c}_l$ *– Case C.*

As summarised in Proposition 3 the public sector may decrease or increase the share of high-educated in the labor force overall, but its effects on private-sector composition go in the opposite direction. If the payoff from having a job in the public sector is relatively higher for low-educated workers (Case A) then those more likely to stay attached to the private sector are those whose education cost is low. Allowing for the option to enter the public sector therefore increases the share of low-educated in the labor force overall, but lowers the proportion of low-educated in the private sector and as a consequence, it also lowers the proportion of low-educated among those employed in the private sector. In this case, the ratio of high- to low-educated is higher in the private than in the public sector. If on the other hand, a job in the public sector is relatively more attractive to workers that have high-education (case B) then with the introduction of a public sector, a relatively higher fraction of workers with low education cost opt for government jobs. The share of high-educated in the labor force overall increases, but among those that stay attached to the private sector a higher fraction is low-educated. If the benefit of being educated is the same in both sectors (case C), then the opening of the pubic sector drains high- and low-educated workers from the private sector in equal proportions leaving the educational composition unchanged.

5 Random search between the two sectors

In this section, we discuss and compare the effects of government policies on employment and human capital under the alternative model assumption that search between the two sectors is random. We further compare the benchmark model with segmented markets to the alternative model introduced here in the quantitative exercise in Section 6.

We assume that workers search randomly for jobs that suit their skill type in the two sectors, which ultimately implies that there are no costs to entering the public sector. As shown in Lemma 3, if entry into the public sector is costly, then those choosing to enter it search exclusively there, while those choosing not to pay the entry cost can only get a job in the private sector. In other words, the existence of entry costs rules out the possibility of random search between the two sectors. To allow for random search we therefore need to assume free entry into the public sector.

A matching function $m(v_i, u_i)$ determines the total number of matches between workers and jobs and $m(\theta_i)$, where $\theta_i = \frac{v_i}{u_i}$, gives the rate at which workers match with (either private or government) vacancies. Since they search randomly for jobs, the total number of vacancies available to them, consists of both private-sector v_i^p and government v_i^g vacancies. They find jobs in the private sector at rate $m(\theta_i)\nu_i^p$ and in the public sector at rate $m(\theta_i)(1-\nu_i^p)$, where $\nu_i^p = \frac{v_i^p}{v_i}$ is the fraction of private-sector vacancies in the total number of type i vacancies ($v_i = v_i^p + v_i^g$).

The key difference between the model with random search and segmented markets is the value of unemployment. It changes to take into account that workers now can match randomly with either private or government jobs.

$$(r + \tau)U_i = b_i + m(\theta_i)\nu_i^p \left[E_i^p - U_i\right] + m(\theta_i)(1 - \nu_i^p) \left[E_i^g - U_i\right]. \tag{35}$$

Under segmented markets, tightness (job creation) in the private sector is independent of any government policy (see Lemma 1) because the outside option (unemployment value) of workers searching for private-sector jobs and their wages are independent of government policy. Under random search, by contrast, the outside option of searching workers also includes the possibility of finding a public-sector job. As can be seen by equation (35), the outside option of workers is a convex combination of the value a public-sector job (E_i^g) and the value of a private-sector job (E_i^p) with weights reflecting the relative number of vacancies in the two sectors. Thus, public-sector wages, employment opportunities, and separation probabilities affect private-sector wages. More specifically, the private-sector wage of a worker of skill type i is given by

$$w_i^p = b_i + \beta\left[y_i - b_i + \nu_i^p \theta_i \kappa_i\right] + (1-\beta) D_i (w_i^g - b_i), \tag{36}$$

where $D_i = \frac{(1-\nu_i^p)m(\theta_i)}{r+\tau+s_i^g+(1-\nu_i^p)m(\theta_i)}$ measures how much public-sector wages influence private-sector wage bargaining. A free-entry condition as in (9) determines the number of vacancies in the private sector. But now the match surplus, $S_i^p = \frac{p_i - w_i^p}{r+s_i^p+\tau}$, which decreases as the wage increases, depends also on public-sector policy. The full set of equations describing the model with random search, a formal definition of a steady-state equilibrium and conditions for existence of a steady-state equilibrium are in Appendix B.

Under random search, the effects of government policies work through the outside option of workers and its impact on private-sector wages. We show in Appendix B that:

Proposition 8 *If search between the private and public sector is random, then for $i = [h, l]$, an increase in w_i^g or e_i^g increases private-sector wages (w_h^p and w_l^p) and lowers market tightness (θ_h and θ_l).*

Under segmented markets, any negative employment effects of policies are due to decreases in the size of private-sector labor force and increases in the queues for public-sector jobs. Under random search, by contrast, the public sector hurts job creation in the private sector by putting upward pressure on wages thereby lowering firm profits and inducing firms to open fewer vacancies per unemployed worker. In both cases, negative effects on total employment arise when the value of employment in the public sector is relatively high.

5.0.1 The effect of government policies on human capital

As discussed above, when the two sectors are segmented, an increase in the education premium generates offsetting decreases in the job-finding rate by inducing more workers to search in the public sector. The lower the public-sector entry cost, the larger these offsetting decreases are. However, when workers search randomly between the two sectors such offsetting decreases in the job-finding rate are not possible. Hence, under random search, policies that increase the payoff from being educated, such as increasing the public-sector wage premium or increasing public-sector employment of the high- vs. low-educated, encourage workers to become educated and raise the proportion of high-educated workers in the labor force (L_h). More formally, we show in Appendix B that:

Proposition 9 *If search between the private and public sector is random, an increase in w_i^g or e_i^g increases L_i and decreases L_j. $i = [h, l]$, $j = [h, l]$ and $i \neq j$.*

6 Quantitative analysis

The objective of our numerical exercise is threefold. First, we want to inspect the quantitative effects of changes in government policies on the educational composition and employment. As discussed earlier, the effects of policies on the education choice (and thus on the employment of the high- and low-educated workers) depend on whether education and public-sector jobs are substitutes (Case A with $\tilde{c}_l > \tilde{c}_h$) or complements (Case B with $\tilde{c}_h > \tilde{c}_l$). Their effects on employment depend also on differences in job security between the two sectors and on the size of public-sector wages. The model is parsimonious enough to be calibrated for four countries – United States, United Kingdom, France and Spain – that have different but sizable public sectors. Secondly, we want to compare the quantitative results in the benchmark model with those in the random search model proposed in Section 5. Finally, we also quantify the value of job security in the public-sector, for skilled and unskilled workers.

6.1 Parameterization

We parameterize both versions of the model, with segmented markets and with random search, for the four countries. We match the US economy at a monthly frequency, drawing largely on the *CPS* for the period 2006-2016. We also calibrate the version for UK, French and Spain at a quarterly frequency, drawing from their LFS and the SES. Most of the data was discussed in Section 2 and shown in Table 1. A set of parameters is directly fixed to values taken from the data, while a second set of parameters targets steady-state values. Table 2 lists all the parameters and their values and Table 3 targeted and non-targeted steady-state variables for the segmented markets model. Table 4 lists them for the random search model.

In the US, around 26.6 percent of the population have a college degree, of which 24.6 percent work in the public sector ($e_h^g = 0.0656$). Out of the remaining population with no college degree, 10.8 percent work in the public sector ($e_l^g = 0.0792$). We set the separation rates by education and sector: $s_h^g = 0.005$, $s_h^p = 0.007$, $s_l^g = 0.0158$ and $s_l^p = 0.0283$. We consider, in the private sector, a Cobb-Douglas matching function with matching efficiency ζ_i and matching elasticity with respect to the unemployment of η_i. As the matching efficiency and the cost of posting vacancies are not separable, we normalize the matching efficiencies $\zeta_h = \zeta_l = 1$. The costs of posting vacancies, κ_h and κ_l, are set to target the unemployment rate of 3.2 percent for college graduates and 7.3 percent for non-college graduates. The matching elasticities are set to the common value of 0.5, and the Hosios condition is assumed to hold ($\eta_h = \eta_l = \beta = 0.5$). The unemployment benefits b_h and b_l are set to match a replacement rate of 0.29 and 0.43, an average of estimates for the replacement rate of

Table 2: Parametrization of segmented markets model

Fixed parameters	**United States**	**United Kingdom**	**France**	**Spain**
Public-sector employment				
Skilled	$e_h^g = 0.066$	$e_h^g = 0.170$	$e_h^g = 0.095$	$e_h^g = 0.105$
Unskilled	$e_l^g = 0.079$	$e_l^g = 0.110$	$e_l^g = 0.137$	$e_l^g = 0.079$
Job-separation rates (private)				
Skilled	$s_h^p = 0.007$	$s_h^p = 0.012$	$s_h^p = 0.016$	$s_h^p = 0.031$
Unskilled	$s_l^p = 0.028$	$s_l^p = 0.014$	$s_l^p = 0.023$	$s_l^p = 0.051$
Job-separation rates (public)				
Skilled	$s_h^g = 0.005$	$s_h^g = 0.005$	$s_h^g = 0.005$	$s_h^g = 0.014$
Unskilled	$s_l^g = 0.016$	$s_l^g = 0.005$	$s_l^g = 0.010$	$s_l^g = 0.036$
Matching elasticities	$\eta_h = \eta_l = 0.5$			
Matching efficiencies	$\zeta_h = \zeta_l = 1$			
Bargaining power of workers	$\beta = 0.5$			
Discount rate	$r = 0.004$	$r = 0.0120$		
Retirement rate	$\tau = 0.002$	$\tau = 0.0056$		
Other parameters				
Cost of posting vacancies				
Skilled	$\kappa_h = 17.250$	$\kappa_h = 3.901$	$\kappa_h = 7.146$	$\kappa_h = 11.821$
Unskilled	$\kappa_l = 3.781$	$\kappa_l = 5.117$	$\kappa_l = 5.599$	$\kappa_l = 7.612$
Unemployment benefits				
Skilled	$b_h = 0.475$	$b_h = 0.330$	$b_h = 0.659$	$b_h = 0.430$
Unskilled	$b_l = 0.405$	$b_l = 0.397$	$b_l = 0.562$	$b_l = 0.429$
Productivity skilled	$y_h = 1.697$	$y_h = 1.387$	$y_h = 1.475$	$y_h = 1.442$
Public-sector entry cost distribution				
Mean	$\mu^c = 2.911$	$\mu^c = 1.537$	$\mu^c = 0.223$	$\mu^c = 1.774$
Variance	$\sigma^c = 0.466$	$\sigma^c = 0.054$	$\sigma^c = 0.001$	$\sigma^c = 0.035$
Education cost distribution				
Mean	$\mu^\epsilon = 5.133$	$\mu^\epsilon = 3.204$	$\mu^\epsilon = 4.275$	$\mu^\epsilon = 4.242$
Variance	$\sigma^\epsilon = 0.622$	$\sigma^\epsilon = 1.656$	$\sigma^\epsilon = 2.312$	$\sigma^\epsilon = 2.830$
Public-private wages ratio				
Skilled	$\frac{w_h^g}{w_h^p} = 1.038$	$\frac{w_h^g}{w_h^p} = 1.067$	$\frac{w_h^g}{w_h^p} = 1.001$	$\frac{w_h^g}{w_h^p} = 1.053$
Unskilled	$\frac{w_l^g}{w_l^p} = 1.050$	$\frac{w_l^g}{w_l^p} = 1.092$	$\frac{w_l^g}{w_l^p} = 1.038$	$\frac{w_l^g}{w_l^p} = 1.176$

workers with 150 and 67 percent of the average wage, according to OECD. Additionally, $r = 0.004$ and $\tau = 0.002$ target a yearly interest rate of about four percent and an average working life of 40 years.

We use CPS data from the 2002 to calculate the college premium and the public-sector wage premium by education. We normalize $y_l = 1$ and set y_h to target a private-sector college premium of close to 72 percent found by regression of the log gross hourly earnings on a dummy for college education, using the sub-set of private-sector workers. We set the public-sector wages to target the public-sector wage premium for college and non-college workers shown in Section 2. The public-sector wages of the two types are set such that $\frac{w_h^g}{w_h^p} = 1.027$ and $\frac{w_l^g}{w_l^p} = 1.064$. We do not fix the premium to these numbers, but instead use them as a target, but allow the minimization routine to deviate from these for values within the 95 percent confidence, to improve the fit in other dimensions.

Table 3: Steady-state values of variables in segmented markets model

Targets (data value)	United States	United Kingdom	France	Spain
Unemployment rate				
Skilled	0.032 (0.032)	0.033 (0.033)	0.057 (0.057)	0.112 (0.110)
Unskilled	0.073 (0.073)	0.057 (0.057)	0.106 (0.103)	0.213 (0.208)
Replacement rates				
Skilled	0.290 (0.290)	0.248 (0.248)	0.470 (0.474)	0.331 (0.331)
Unskilled	0.425 (0.425)	0.418 (0.418)	0.595 (0.598)	0.483 (0.498)
Unemp. duration - private over public				
Skilled	1.248 (1.248)	0.744 (0.744)	0.985 (0.948)	0.994 (0.988)
Unskilled	1.009 (1.009)	0.735 (0.735)	0.585 (0.767)	0.683 (0.794)
Public-private wage ratio				
Skilled	1.038 (1.027)	1.067 (1.059)	1.001 (0.985)	1.053 (1.060)
Unskilled	1.050 (1.064)	1.092 (1.096)	1.038 (1.045)	1.176 (1.179)
Skilled-unskilled wage premium	1.720 (1.720)	1.401 (1.401)	1.483 (1.474)	1.463 (1.434)
Tertiary education costs (fraction of consumption)	0.047 (0.474)	0.031 (0.031)	0.022 (0.026)	0.018 (0.019)
Share of college graduates	0.266 (0.266)	0.460 (0.460)	0.320 (0.320)	0.337 (0.340)
Other steady-state variables				
Value of employment				
Private - Skilled	272.3	72.57	75.87	67.10
Private - Unskilled	154.9	51.46	50.75	44.72
Public - Skilled	285.0	78.03	77.23	73.35
Public - Unskilled	165.5	56.85	53.12	51.86
Total employment				
Skilled	0.258	0.445	0.301	0.299
Unskilled	0.680	0.509	0.608	0.522
Thresholds (ϵ_x)				
Private	114.4	20.79	24.29	21.08
Public	117.3	20.91	24.29	21.20
Thresholds (c_i)				
Skilled	13.315	4.573	1.249	5.806
Unskilled	10.491	4.453	1.249	5.684
Public sector required minimum wage over actual wage ($\underline{w}_i^g/w_i^g$)				
Skilled	0.938	0.916	0.961	0.865
Unskilled	0.918	0.884	0.918	0.763

The distribution of education costs is assumed to be lognormally distributed. To calibrate the mean and the standard deviation, we target the fraction of the labour force with a college degree and the total cost of tertiary education, which according to the OECD represents 3.8 percent of US private consumption. Finally, the distribution of costs of joining the public sector is assumed to be lognormally distributed. The parameters of this distribution determine the size of queues in the public sector. However, the searching behaviour of the unemployed is in general unobservable. We calculate the average unemployment durations of workers who have found a job in the private sector and of those who have found a job in the public sector. Under segmented markets, the ratio of the two is equivalent to the inverse ratio of the average conditional job-finding rates, which is found 1.25 for workers with college

Table 4: Parametrization of random search model

	United States	**United Kingdom**	**France**	**Spain**
Other parameters				
Cost of posting vacancies				
Skilled	$\kappa_h = 12.486$	$\kappa_h = 2.228$	$\kappa_h = 6.990$	$\kappa_h = 8.819$
Unskilled	$\kappa_l = 3.177$	$\kappa_l = 3.989$	$\kappa_l = 5.145$	$\kappa_l = 6.451$
Unemployment benefits				
Skilled	$b_h = 0.479$	$b_h = 0.334$	$b_h = 0.665$	$b_h = 0.431$
Unskilled	$b_l = 0.408$	$b_l = 0.402$	$b_l = 0.569$	$b_l = 0.452$
Productivity skilled	$y_h = 1.696$	$y_h = 1.378$	$y_h = 1.476$	$y_h = 1.410$
Education cost distribution				
Mean	$\mu^\epsilon = 5.152$	$\mu^\epsilon = 3.260$	$\mu^\epsilon = 3.939$	$\mu^\epsilon = 4.176$
Variance	$\sigma^\epsilon = 0.631$	$\sigma^\epsilon = 1.738$	$\sigma^\epsilon = 1.655$	$\sigma^\epsilon = 2.741$
Public-private wages ratio				
Skilled	$\frac{w_h^g}{w_h^p} = 1.027$	$\frac{w_h^g}{w_h^p} = 1.069$	$\frac{w_h^g}{w_h^p} = 0.986$	$\frac{w_h^g}{w_h^p} = 1.060$
Unskilled	$\frac{w_l^g}{w_l^p} = 1.064$	$\frac{w_l^g}{w_l^p} = 1.090$	$\frac{w_l^g}{w_l^p} = 1.049$	$\frac{w_l^g}{w_l^p} = 1.181$
Targets (data value)				
Unemployment rate				
Skilled	0.032 (0.032)	0.033 (0.033)	0.057 (0.057)	0.110 (0.110)
Unskilled	0.073 (0.073)	0.057 (0.057)	0.103 (0.103)	0.208 (0.208)
Replacement rates				
Skilled	0.290 (0.290)	0.248 (0.248)	0.474 (0.474)	0.331 (0.331)
Unskilled	0.425 (0.425)	0.418 (0.418)	0.598 (0.598)	0.498 (0.498)
Public-private wage ratio				
Skilled	1.027 (1.027)	1.069 (1.059)	0.986 (0.985)	1.060 (1.060)
Unskilled	1.064 (1.064)	1.090 (1.096)	1.049 (1.045)	1.181 (1.179)
Skilled-unskilled wage premium	1.720 (1.720)	1.401 (1.401)	1.474 (1.474)	1.435 (1.434)
Tertiary education costs (fraction of consumption)	0.047 (0.474)	0.031 (0.031)	0.026 (0.026)	0.018 (0.019)
Share of college graduates	0.266 (0.266)	0.460 (0.460)	0.320 (0.320)	0.340 (0.340)
Other SS variables				
Value of employment				
Private - Skilled	275.9	74.21	76.07	68.47
Private - Unskilled	157.2	52.51	51.40	46.60
Public - Skilled	280.6	78.53	76.22	72.09
Public - Unskilled	160.7	56.73	53.62	50.01
Total employment				
Skilled	0.258	0.445	0.302	0.302
Unskilled	0.680	0.509	0.610	0.523
Threshold (ϵ_x)	116.5	21.89	23.70	21.00

and 1.01 for workers without college. This means than in the US it is faster to find a job in the public sector than the private, particularly for college graduates.

We use the same procedure to calibrate the model for the remaining three countries. The targets are matched perfectly with the exception of the unemployment duration ratio for unskilled workers that is lower than the one found in the data for France and Spain. In the baseline steady-state, education compliments a career in the public sector in the US, UK and Spain, meaning that these economies are in Case B. Despite the public-sector premium being higher for workers without a college degree, there are many more jobs available in the public sector for college graduates. In France, the difference between the two cutoffs c_h and c_l is very small, meaning that it is close to Case C. The last rows of Table 3 show the minimum wage required for the existence of the public sector relative to the actual wage. The gap between the minimum wage and the actual wage is higher for unskilled workers in all countries. It is particularly high in Spain where the minimum wage is 15 to 24 percent lower that the actual wage. In France this gap is only 4 and 8 percent.

For the random search model, the distribution of public-sector entry cost is nonexistent, so we drop the ratio of the duration of unemployment which, by definition, must equal 1. As in the segmented markets model, the value of employment is always higher in the public sector than in the private.

6.2 The effects of skilled-biased policies

We now compare the results for the four countries in the segmented market and random search models. Table 5 shows the effects of five different policies: i) a ten-percent increase in skilled public-sector wages; ii) a ten-percent increase in unskilled public-sector wages; iii) a ten-percent increase in skilled public-sector employment; iv) a ten-percent increase in unskilled public-sector employment; v) no public sector. It shows the percentage change in the number of skilled workers and the percentage points difference in skilled and unskilled unemployment rate.

In line with Propositions 4 and 9, under both labor market structures, policies that improve the public-sector value of the high- relative to the low-educated (such as increasing wages or employment of high-educated workers) increase the proportion of high-educated in the labor force and vice versa. But our quantitative results show that random search amplifies the effects of policies on educational composition. When increasing skilled wages by ten percent, the share of high-educated in the labor force goes up by at most 0.2 percent in the segmented markets model, as opposed to 2.5 to 7 percent in the random search model. When increasing unskilled wages by ten percent, the share of high-educated education decreases by less than 0.1 percent in segmented markets, as opposed to 1 to 2.3 percent in the random search model. Education responds differently in the two models because, as discussed above, under segmented markets offsetting increases in job queues prevent the value of obtaining education from increasing. When search is random, by contrast, such adjustments in the job-finding rate are not possible, leading to relatively larger changes in the educational composition.

Under both labor market structures, wage policies that increase the public-sector value for high- or low-educated workers, involve a negative effect on total employment. In the segmented market model by attracting too many job seekers into the public sector and creating longer queues of unemployed. A ten percent increase in the wages of high-educated public-sector employees in the US, decreases the employment of high-educated workers by 2 percent and of low-educated workers by 0.06 percent. In the random search model increases in public-sector pay lower firm profits and decrease private-sector job creation. However, in the random search model the changes in educational composition tend to overshadow these negative job creation effects. Hence, changes in the employment of the two education groups reflect mainly changes in their proportions in the labor force. A ten percent increase in the wages of high-educated public-sector employees in the US increases their employment by about 5.5 percent, because it increases their proportion in the labor force by about 7 percent. The employment of the low-educated falls by 2.5 percent.

The effects of increasing public-sector employment of either high- or low-educated workers are more similar across the two labor market structures. An increase in the size of employment of public-sector i (e_i^g) increases the share of type-i workers in the labor force, and if markets are segmented, attracts more job seekers into public-sector i. Some of them occupy the new jobs in the public sector. Only those in excess of the number of new jobs are unemployed waiting for jobs. Thus, compared to wage increases, public-sector employment increases generate smaller increases in queues. Consequently, under both labor market structures, the impact of changes in the size of public-sector employment on employment overall reflects mainly changes in the educational composition of the labor force. For instance, in both models a 10 percent increase in the public-sector employment of the high-educated leads to an increase in their total employment, whereas, a 10 increase in their wage decreases their employment in the segmented markets model, by creating longer queues, and increases it in the random search model, by increasing their proportion in the labor force.

As shown in Table 2, in our calibration, France is close to Case C ($\tilde{c}_h = \tilde{c}_l$ and $\tilde{\epsilon}_p = \tilde{\epsilon}_g$) meaning that the public sector premium in France is the same for both types of workers (or equivalently, the education premium is the same in both sectors). All other countries are in case B ($\tilde{c}_h > \tilde{c}_l$ and $\tilde{\epsilon}_p < \tilde{\epsilon}_g$), where education compliments search in the public sector, but presumably for different reasons. The UK represents the case where the public sector is a major source of job opportunities for high-educated workers since it hires a large portion of the high-educated labor force. In the US, on the other hand, high-educated workers can find job in the public sector much faster than in the private sector, which also acts to increase the benefit from investing in education. In Spain also, the public sector strengthens incentives to invest in education especially under random search, by offering to high-educated workers

Table 5: Effects of policies under different models/calibrations

Policy	**United States**		**United Kingdom**		**France**		**Spain**	
	SM	**RS**	**SM**	**RS**	**SM**	**RS**	**SM**	**RS**
Increase on skilled wages by 10 percent								
%Δ fraction of skilled	0.18%	6.85%	0.02%	2.49%	0.00%	4.00%	0.01%	2.67%
Δ employment skilled	-2.03%	5.54%	-2.43%	0.76%	-2.87%	2.69%	-3.13%	1.09%
Δ employment unskilled	-0.06%	-2.49%	-0.0%	-2.13%	-0.00%	-1.88%	-0.01%	-1.39%
Δ u. rate skilled	2.14 p.p.	1.19 p.p.	2.36 p.p.	1.63 p.p.	2.70 p.p.	1.19 p.p.	2.79 p.p.	1.37 p.p.
Δ u. rate unskilled	0.00 p.p.	0.01 p.p.	0.00 p.p.	0.01 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.01 p.p.
Increase on unskilled wages by 10 percent								
%Δ fraction of skilled	-0.07%	-2.34%	-0.01%	-1.47%	-0.00%	-2.08%	-0.01%	-1.02%
Δ employment skilled	-0.07%	-2.34%	-0.01%	-1.48%	-0.00%	-2.05%	-0.01%	-1.01%
Δ employment unskilled	-1.49 %	-0.03%	-1.72%	0.20%	-3.23%	-0.62%	-2.15%	-0.64%
Δ u. rate skilled	-0.00 p.p.	0.00 p.p.	-0.00 p.p.	0.01 p.p.	-0.00 p.p.	-0.02 p.p.	-0.00 p.p.	-0.01p.p
Δ u. rate unskilled	1.41 p.p.	0.81 p.p.	1.63 p.p.	0.98 p.p.	2.89 p.p.	1.42 p.p.	1.69 p.p.	0.92 p.p
Increase on skilled employment by 10 percent								
%Δ fraction of skilled	0.20%	0.28%	0.02%	0.26%	0.00%	0.02%	0.01%	0.29%
Δ employment skilled	0.27%	0.26%	0.06%	0.20%	0.10%	0.12%	0.23%	0.35%
Δ employment unskilled	-0.07%	-0.10%	-0.02%	-0.23%	-0.00%	-0.01%	-0.01%	-0.15%
Δ u. rate skilled	-0.07 p.p.	0.02 p.p.	-0.03 p.p.	0.06 p.p.	-0.10 p.p.	-0.10 p.p.	-0.19 p.p.	-0.05p.p
Δ u. rate unskilled	-0.00 p.p.	0.00 p.p.	-0.00 p.p.	0.00 p.p.	-0.00 p.p.	-0.00 p.p.	0.00 p.p.	0.00p.p
Increase on unskilled employment by 10 percent								
%Δ fraction of skilled	-0.05%	-0.19%	-0.01%	-0.16%	-0.00%	-0.15%	-0.00%	-0.18%
Δ employment skilled	-0.05%	-0.19%	-0.01%	-0.16%	0.00%	-0.15%	-0.00%	-0.18%
Δ employment unskilled	0.09%	0.04%	0.05%	0.10%	0.01%	0.08%	-0.02%	-0.02%
Δ u. rate skilled	-0.00 p.p.	0.00 p.p.	-0.00 p.p.	0.00 p.p.	-0.00 p.p.	-0.00 p.p.	-0.00 p.p.	-0.00 p.p
Δ u. rate unskilled	-0.06 p.p.	0.03 p.p.	-0.04 p.p.	0.04 p.p.	-0.01 p.p.	-0.01 p.p.	0.01 p.p.	0.09 p.p
No public sector								
%Δ fraction of skilled	-0.86%	-1.15%	-0.09%	-1.63%	0.00%	1.44%	-0.05%	-1.21%
Δ employment skilled	-1.22%	-0.96%	-0.33%	-1.00%	0.00%	0.42%	-2.12%	-1.64%
Δ employment unskilled	-0.07%	0.71%	-0.25%	1.80%	-0.13%	-0.73%	0.27%	1.85%
Δ u. rate skilled	0.35 p.p.	-0.18 p.p.	0.23 p.p.	-0.62 p.p.	0.00 p.p.	0.95 p.p.	1.83 p.p.	0.39p.p
Δ u. rate unskilled	0.35 p.p.	-0.27 p.p.	0.31 p.p.	-0.38 p.p.	0.12 p.p.	0.05 p.p.	-0.19 p.p.	-0.97p.p

access to more secure and better paying jobs in the public sector. The fact that all countries but France are in case B explains why eliminating the public sector lowers the proportion of high-educated in all countries but France. Because France is in Case C, as expected, in the segmented markets model eliminating the French public sector leaves the educational composition intact, in line with Proposition 3. However, in the random search model it increases the proportion of educated suggesting that the public sector in France improves by relatively more the outside option of low-educated workers. This is consistent with a much larger portion of government employment in France being low-educated. We see from Table 2 that about 60% of government employees in France are low-educated as opposed to about 40% in the UK and Spain and 54% in the US. Moreover, about 18% of low-educated workers in France are government employees as opposed to 9.5% in Spain (see Table 1)

Job security seems to be a feature of the public sector that is important in increasing employment in all countries. While in the random search model changes in employment follow the changes in the labor force composition, in the segmented markets model, we see strong negative employment effects from eliminating the public sector. In Spain eliminating the public sector decreases the proportion of high-educated in the labor force by only 0.05%. Nevertheless, it decreases their employment by more than 2%. The excess decrease is due to restricting access of workers to government jobs that are safer and last longer. Although in the absence of a public-sector the proportion of low-educated increases in all countries (except France where it remains the same), the employment of low-educated falls, reflecting the impact of lower job security in the private sector. The only exception is Spain, where results indicate that job queues are also important there. As mentioned above, in Spain actual public-sector wages, especially of low-educated workers, are much larger than the minimum wage (see Table 4), suggesting the existence of long queues for public-sector jobs. This explains, why in the case of Spain, eliminating the public sector, increases the employment of low-educated by 0.27 percent, despite lowering their proportion in the labor force.

6.3 Job-security premium

In this sub-section we illustrate the potential of our model to inform policymakers. The argument that public-sector jobs are safer is often used in policy discussions over public-sector wages. However, there have been no attempts to quantify this value and it is not clear whether it is taken in account by pay-review bodies determining the public-sector pay scale. Fontaine et al. (2018) illustrate, using a two-equation model, the principle of how we can use a model with search and matching frictions to evaluate the value of job security. We can use the same principle in our fully-developed model.

We can ask private-sector workers what fraction of their wage they would be willing to pay to have the same job-separation rate as in the public sector (Ψ_i^p). Alternatively, we can ask public-sector workers how much they would need to be compensated to be given the same job-separation rate as in the private sector (Ψ_i^g). To calculate both values, we maintain the value of employment and unemployment at the baseline value. Using (2) and (4), respectively, we can derive the following expressions for the value of job security:

$$\Psi_i^p = \frac{(s_i^g - s_i^p)[E_i^p - U_i^p]}{w_i^p} \times 100, \quad i = [h, l] \tag{37}$$

$$\Psi_i^g = \frac{(s_i^p - s_i^g)[E_i^g - U_i^g]}{w_i^g} \times 100, \quad i = [h, l]. \tag{38}$$

As we have linear utility, safer jobs raise the expected duration of a job and reduce the

Table 6: Job-security premium different models/calibrations

	United States		United Kingdom		France		Spain	
	SM	**RS**	**SM**	**RS**	**SM**	**RS**	**SM**	**RS**
Lower bound - baseline case with risk neutrality								
Percentage of wage private-sector workers would sacrifice for same job-security of the public sector								
Skilled	-0.60%	-0.51%	-1.01%	-0.76%	-1.72%	-1.69%	-3.83%	-3.35%
Unskilled	-1.81%	-1.66%	-1.52%	-1.34%	-1.86%	-1.77%	-2.75%	-2.48%
Percentage of wage public-sector workers would accept for the same job-security of the private sector								
Skilled	0.49%	1.13%	1.38%	2.89%	1.80%	1.85%	4.19%	7.85%
Unskilled	1.91%	5.91%	2.20%	4.93%	3.28%	4.64%	4.42%	6.94%
Upper bound - with risk aversion $\sigma = 2$ and no savings								
Percentage of wage private-sector workers would sacrifice for the same job-security of the public sector								
Skilled	-1.72%	-1.56%	-2.77%	-2.70%	-2.04%	-1.74%	-6.99%	-8.91%
Unskilled	-3.06%	-3.24%	-2.28%	-2.22%	-2.22%	-2.02%	-6.76%	-7.52%
Percentage of wage public-sector workers would accept for the same job-security of the private sector								
Skilled	1.47%	2.94%	3.34%	5.59%	1.99%	2.27%	7.22%	15.24%
Unskilled	3.44%	5.96%	3.26%	5.23%	3.70%	3.87%	8.52%	14.73%

Note: In the model with risk aversion we change equations (1-4) such that the instantaneous payoffs are $\frac{b_i^{1-\sigma}}{1-\sigma}$ and $\frac{(w_i^x)^{1-\sigma}}{1-\sigma}$. However we maintained the same wage determination rule 13. The model was re-calibrated for each country following the procedure described in Section 6.1.

expected time spent in unemployment. The monetary value of job-security reflects how painful unemployment is, both in terms of payoff, b_i, but also persistence in the form of job-finding rates in each sector. Hence, the values of Ψ_i^p and Ψ_i^g should be different. The linear utility also implies that we can interpret this value as a lower bound of the job-security premium, because it does not incorporate the fact that risk-averse agents would find unemployment more damaging. In order to have an idea of the upper bound for this premium, we extend the model by changing the instantaneous payoff function to $\frac{b_i^{1-\sigma}}{1-\sigma}$ for the unemployment and to $\frac{(w_i^x)^{1-\sigma}}{1-\sigma}$ is employed. This is an upper bound because the model does not allow savings for precautionary motives. We set $\sigma = 2$.

Table 6 shows our estimates for job-security premium. Across countries, the value of job-security is related to how dynamic the labour market is, with the US and UK being the countries with lower premia and France and Spain the countries with higher premia. With the exception of Spain, the job-security premium is higher for workers without college. In the US, for instance, private-sector workers with college would sacrifice 0.5 percent of their wage, while workers without college would sacrifice more than 1.5 percent. In Spain, the difference between job-separation rates across sectors is also quite high for college graduates so they would be willing to sacrifice 3 to 4 percent of their wage, while no-college workers would only pay 2.5 percent. When measured by the compensation needed to give to public-sector workers to reduce their job-security, the numbers are larger, in particular to unskilled workers. The values would vary from 2 percent in the US to close to 7 percent in Spain. The values for UK and France are in between.

When measured by the private-sector workers, the premium is slightly lower in the random search model, but it is higher when measured by the public-sector workers. Regarding the model with risk aversion, the values are roughly two to three larger than in the model with risk-neutral agents.

7 Conclusion

This paper provides two benchmark models to understand how public-sector hiring and wage policies affect education decisions and employment.

Previous literature has highlighted the problems of setting high public-sector wages. For example, Gomes (2015) show that they generate higher unemployment. Cavalcanti and Santos (2017) argue that higher wages might lead to misallocation of resources with a lower entrepreneurship rate. Chassamboulli and Gomes (2018) argue that they might foster rent-seeking activities of unemployed trying to get a public-sector job through political or personal connections. Garibaldi et al. (2018) demonstrate that high wages for workers with low qualifications might generate under-employment. We highlight another potential negative effect. Higher public-sector wages for unskilled workers lead to lower incentives for education.

In this paper, we have compared two benchmark models that have been used in the literature: segmented market versus random search. While the main results are consistent across the two models, the mechanisms and the quantitative implications of policies are different. In segmented markets public-sectors policies that increase the value of education generate offsetting decreases in job finding rates by inducing workers to move away from the private and into the public sector, where they will queue up waiting for jobs. Thus such policies have a relatively small effect on education and larger effects on unemployment. However, when workers search randomly for jobs in both sectors, such offsetting decreases in job finding rates are not possible. In this case, incentives to obtain education increase by more, while effects on unemployment are relatively small.

If we want to develop a model that can be used for policy analysis and forecasting, it is imperative that we realistically model the labor market. In reality, the search structure might be different across countries and in different occupations. In this regard, it is important to develop the empirical research on public-sector hiring procedures and the unemployed search behaviour to determine which model is more suitable to use.

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COMPANION APPENDIX

Public-sector employment and wages and human capital accumulation

Andri Chassamboulli and Pedro Gomes

Appendix A: Proofs of propositions

- A.1 Lemma 2
- A.2 Lemma 3
- A.3 Proof of existence and uniqueness
- A.4 Proposition 1
- A.5 Proposition 2
- A.6 Proposition 3
- A.7 Proposition 4
- A.8 Proposition 5
- A.9 Proposition 6
- A.10 Proposition 7

Appendix B: Random search

- B.1 Setup
- B.2 Definition of equilibrium
- B.3 Proof of existence and uniqueness
- B.4 Proof of proposition 8
- B.5 Proof of proposition 9

Appendix C: Endogenizing public-sector employment and wages

A Proofs of propositions

A.1 Lemma 2

We consider that the public-sector labour market for workers of type i breaks down if the government is not able to hire enough workers to replace the workers that have lost their job. At the limit, it means the government needs to post a wage, defined as $\underline{w}_i^g$, such that it attracts at least $(s_i^g + \tau)e_i^g$ job searches. This means that at $w_i^g = \underline{w}_i^g$ the cutoffs $\tilde{c}_h, \tilde{c}_l$ are such that, using equations (21)-(23), we get $L_i^g - e_i^g \equiv u_i^g = (s_i^g + \tau)e_i^g$ and the job-finding rate is 1 ($m_i^g = 1$). Let $\underline{\tilde{c}}_h, \underline{\tilde{c}}_l$ be that cutoffs that satisfy this condition. Applying $m_i^g = 1$ to (15) and then setting $U_i^g - U_i^p = \underline{\tilde{c}}_i$ gives

$$b_i + \frac{1}{r + \tau + s_i^g + 1}[\underline{w}_i^g - b_i] = (r + \tau)(U_i^p + \underline{\tilde{c}}_i)$$

Substituting the $(r + \tau)U_i^p$ by equation (14) we get

$$\underline{w}_i^g = (r + \tau + s_i^g + 1)\left(\frac{m(\theta_i^*)}{r + \tau + s_i^p + m(\theta_i^*)}[w_i^{p,*} - b_i] + b_i + (r + \tau)\underline{\tilde{c}}_i\right)$$

where θ_i^* and $w_i^{p,*}$ are the equilibrium tightness and wages in the private sector.

A.2 Lemma 3

Proof. Based on Lemma 2 (see also subsection A.1) if $w_i^g \geq \underline{w}_i^g$ then $L_i^g - e_i^g \geq (s_i^g + \tau)e_i^g$ which means that $L_i^g > 0$. From (22)-(23) we can verify that this implies $\tilde{c}_i > 0$ and from (17) that this, in turn, implies $U_i^g > U_i^p$. ■

A.3 Proof of Existence and Uniqueness of a Steady-State Equilibrium

Proof. It can be easily verified that the two free-entry conditions in (11) pin down a unique set of equilibrium values for θ_h and θ_l. Substituting these values into (25) we get the unique equilibrium value for $\tilde{\epsilon}_p$. To complete the proof of existence and uniqueness we need to show that with the equilibrium values of θ_h, θ_l and $\tilde{\epsilon}_p$ substituted in, the two threshold conditions in (24), $\tilde{c}_h = U_h^g - U_h^p$ and $\tilde{c}_l = U_l^g - U_l^p$ only cross once in the $[\tilde{c}_h, \tilde{c}_l]$ plane giving a unique set of equilibrium values for $\tilde{c}_h$ and $\tilde{c}_l$.

Let as write (24) as:

$$\tilde{c}_i = \frac{1}{r + \tau}\left[A_i - \frac{\beta\kappa_i\theta_i}{(1 - \beta)}\right], \quad i = [h, l] \tag{39}$$

where

$$A_i \equiv \frac{\frac{(s_i^g + \tau)e_i^g}{L_i^g - e_i^g}}{r + \tau + s_i^g + \frac{(s_i^g + \tau)e_i^g}{L_i^g - e_i^g}}(w_i^g - b_i) \tag{40}$$

By total differentiation of (39) we can derive their slopes:

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_h = U_h^g - U_h^p} = \frac{\frac{\partial A_h}{\partial L_h^g}\frac{\partial L_h^g}{\partial \tilde{c}_l}\frac{1}{r+\tau}}{1 - \frac{\partial A_h}{\partial L_h^g}\frac{\partial L_h^g}{\partial \tilde{c}_h}\frac{1}{r+\tau}} > 0 \tag{41}$$

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_l = U_l^g - U_l^p} = \frac{1 - \frac{\partial A_l}{\partial L_l^g}\frac{\partial L_l^g}{\partial \tilde{c}_l}\frac{1}{r+\tau}}{\frac{\partial A_l}{\partial L_l^g}\frac{\partial L_l^g}{\partial \tilde{c}_h}\frac{1}{r+\tau}} > 0 \tag{42}$$

Both slopes are positive, since, as can be easily verified from (40) and (21)-(23), $\frac{\partial A_i}{\partial L_i^g} < 0$, $\frac{\partial L_i^g}{\partial \tilde{c}_i} > 0$ and $\frac{\partial L_i^g}{\partial \tilde{c}_j} < 0$. But it can also be shown that $\frac{\partial L_i^g}{\partial \tilde{c}_i} > -\frac{\partial L_i^g}{\partial \tilde{c}_j} > 0$ so that:

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_h = U_h^g - U_h^p} < 1$$
$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_l = U_l^g - U_l^p} > 1$$

This completes the proof of existence and uniqueness. The two loci only cross once in the $[\tilde{c}_h, \tilde{c}_l]$ plane giving a unique set of equilibrium values for $\tilde{c}_h$ and $\tilde{c}_l$. ■

A.4 Proof of Proposition 1

Proof. We see from (25) that the cutoff e^p depends only on private sector tightness θ_h and θ_l, which as shown in Lemma 1 (see subsection A.1) are independent of public-sector policies.

■

A.5 Proof of Proposition 2

Proof. Using (26), (27), and (28) we can solve for e_i^p. Then setting $e_i = e_i^p + e_i^g$ gives:

$$e_i = e_i^g + \frac{m(\theta_i)L_i^p}{s_i^p + \tau + m(\theta_i)} \tag{43}$$

Note that $L_i^p = L_i - L_i^g$ and let $Q_i = L_i^g - e_i^g - e_i^g(s_i^g + \tau)$ denote the size of the "queue" in public sector i. There are $L_i^g - e_i^g$ workers searching for jobs in sector in public-sector i and only $e_i^g(s_i^g + \tau)$ vacancies. So only $e_i^g(s_i^g + \tau)$ will be hired, the rest will queue up waiting for new vacancies. We can write the total employment rate as

$$e_i = \frac{m(\theta_i)L_i}{s_i^p + \tau + m(\theta_i)} + \frac{e_i^g(s_i^p - s_i^g)}{s_i^p + \tau + m(\theta_i)} - \frac{Q_i m(\theta_i)}{s_i^p + \tau + m(\theta_i)} \tag{44}$$

In the absence of a public sector total employment equals total employment in the private sector. Let the superscript "ng" denote the absence of a public-sector. Total employment in that case is given by

$$e_i^{ng} = \frac{m(\theta_i)L_i^{ng}}{s_i^p + \tau + m(\theta_i)} \tag{45}$$

Subtracting one from the other we get

$$e_i - e_i^{ng} = \frac{m(\theta_i)(L_i - L_i^{ng})}{s_i^p + \tau + m(\theta_i)} + \frac{e_i^g(s_i^p - s_i^g)}{s_i^p + \tau + m(\theta_i)} - \frac{Q_i m(\theta_i)}{s_i^p + \tau + m(\theta_i)} \tag{46}$$

The first term in the above expression captures the impact of changes in the educational composition of the labor force. The second term shows the impact of higher public-sector job security, while the last term shows the impact of public-sector queues. As shown in Proposition 1, when entry into the public sector is free, the educational composition of the labor force is independent of the public sector: $L_h = L_h^{ng} = \Xi^{\epsilon}(\tilde{\epsilon}_p)$, $L_l = L_l^{ng} = 1 - \Xi^{\epsilon}(\tilde{\epsilon}_p)$. In the case, the existence of a public sector will affect total employment only through job security and queues. As shown in Section A.1, if $w_i^g = \underline{w}_i^g$ then $Q_i = 0$, i.e., there are no queues for public-sector jobs. In this particular case, of free entry into the public sector and no public-sector queues the existence of a public sector will alter employment only if public-sector jobs differ from private jobs in terms of job stability. If $s_i^g < s_i^p$ then $e_i < e_i^{ng}$, if $s_i^g > s_i^p$ then $e_i < e_i^{ng}$ and if $s_i^g = s_i^p$ then $s_i^g = s_i^p$. If, on the other hand, entry into the public sector is free and $w_i^g > \underline{w}_i^g$, then $Q_i > 0$ and the existence of the public sector generates an additional negative effect on total employment. In this case, it is apparent from the expression above that $e_i < e_i^{ng}$ if $s_i^g \geq s_i^p$, while if $s_i^g < s_i^p$ then $e_i < e_i^{ng}$ or $e_i > e_i^{ng}$ depending on which of the two effects dominates: the negative effect of longer queues or the positive effect of job security.

■

A.6 Proof of Proposition 3

Proof. From the expressions for L_h^p and L_h^g in (21)-(23) we obtain

$$
\begin{aligned}
L_h &= \Xi^c(\tilde{c}_h)\Xi^\epsilon(\tilde{\epsilon}_g) + \int_{\tilde{c}_h}^{\tilde{c}_l} \Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1-\Xi^c(\tilde{c}_l))\,\Xi^\epsilon(\tilde{\epsilon}_p),\ \text{if } \tilde{c}_h < \tilde{c}_l \qquad (47)\\
L_h &= \Xi^c(\tilde{c}_l)\Xi^\epsilon(\tilde{\epsilon}_g) + \int_{\tilde{c}_l}^{\tilde{c}_h} \Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1-\Xi^c(\tilde{c}_h))\,\Xi^\epsilon(\tilde{\epsilon}_p),\ \text{if } \tilde{c}_l < \tilde{c}_h \qquad (48)\\
L_h &= \Xi^\epsilon(\tilde{\epsilon}_p),\ \text{if } \tilde{c}_l = \tilde{c}_h \qquad (49)
\end{aligned}
$$

While if a public sector does not exist (denoted by the superscript "ng"), then $L_h^{ng} = \Xi^\epsilon(\tilde{\epsilon}_p)$. Subtracting, L_h^{ng} from (47), (48) and (49), respectively, we obtain

$$
\begin{aligned}
L_h - L_h^{ng} &= \int_{\tilde{c}_h}^{\tilde{c}_l} \left[\Xi^\epsilon(\tilde{\epsilon}_p) - \Xi^\epsilon(\tilde{\epsilon}_m(c))\right] d\Xi^c(c) + \Xi^c(\tilde{c}_h)\left[\Xi^\epsilon(\tilde{\epsilon}_p) - \Xi^\epsilon(\tilde{\epsilon}_g)\right] > 0,\ \text{if } \tilde{c}_h < \tilde{c}_l \qquad (50)\\
L_h - L_h^{ng} &= -\int_{\tilde{c}_l}^{\tilde{c}_h} \left[\Xi^\epsilon(\tilde{\epsilon}_m(c)) - \Xi^\epsilon(\tilde{\epsilon}_p)\right] d\Xi^c(c) - \Xi^c(\tilde{c}_l)\left[\Xi^\epsilon(\tilde{\epsilon}_g) - \Xi^\epsilon(\tilde{\epsilon}_p)\right] < 0,\ \text{if } \tilde{c}_h > \tilde{c}_l \qquad (51)\\
L_h - L_h^{ng} &= 0,\ \text{if } \tilde{c}_h = \tilde{c}_l. \qquad (52)
\end{aligned}
$$

As shown above, $\tilde{\epsilon}_g \geq \tilde{\epsilon}_m \geq \tilde{\epsilon}_p$, if $\tilde{c}_h > \tilde{c}_l$, while $\tilde{\epsilon}_g \leq \tilde{\epsilon}_m \leq \tilde{\epsilon}_p$, if $\tilde{c}_h < \tilde{c}_l$, implying that the terms in the brackets of (50) and (51) are positive. ■

A.7 Proof of Proposition 4

Proof. Using (24) we can derive for $x_i = [w_i^g, s_i^g, e_i^g]$, $i = [h, l]$, $j = [h, l]$ and $j \neq i$ that

$$
\begin{aligned}
\frac{d\tilde{c}_i}{dx_i} &= \frac{\frac{\partial A_i}{\partial x_i}}{r + \tau - \frac{\partial A_i}{\partial L_i^g}\left[\frac{\partial L_i^g}{\partial \tilde{c}_i} + B_j \frac{\partial L_i^g}{\partial \tilde{c}_j}\right]} \qquad (53)\\
\frac{d\tilde{c}_j}{dx_i} &= B_j \frac{d\tilde{c}_i}{dx_i} \qquad (54)
\end{aligned}
$$

where A_j is as defined in (40) above and

$$
B_j = \frac{\frac{\partial A_j}{\partial L_j^g}\frac{\partial L_j^g}{\partial \tilde{c}_i}}{r + \tau - \frac{\partial A_j}{\partial L_j^g}\frac{\partial L_j^g}{\partial \tilde{c}_j}} \qquad (55)
$$

It can be easily verified from (40) that $\frac{\partial A_j}{\partial L_j^g} < 0$ $\left(\frac{\partial A_i}{\partial L_i^g} < 0\right)$ and from (21)-(23) that $\frac{\partial L_j^g}{\partial \tilde{c}_j} > -\frac{\partial L_j^g}{\partial \tilde{c}_i} > 0$ $\left(\frac{\partial L_i^g}{\partial \tilde{c}_i} > -\frac{\partial L_i^g}{\partial \tilde{c}_j} > 0\right)$ so that $1 > B_j > 0$ $(1 > B_i > 0)$. These imply that the denominator of (53) is positive. From (40) we also know that the numerator of (53) is also positive since $\frac{\partial A_i}{\partial x_i} > 0$. We can therefore conclude that:

$$
\frac{d\tilde{c}_i}{dx_i} > 0 \text{ and } \frac{d\tilde{c}_j}{dx_i} > 0 \text{ for } x_i = [w_i^g, s_i^g, e_i^g],\ i = [h, l],\ j = [h, l] \text{ and } j \neq i \qquad (56)
$$

With (18)-(20) substituted in the expressions for L_h^p and L_h^g from (21)-(23) we can derive an expression for $L_h(= L_h^p + L_h^g)$ in terms of only $\tilde{c}_h$, $\tilde{c}_l$ and model parameters so that:

$$\frac{dL_h}{dx_i} = \frac{\partial L_h}{\partial \tilde{c}_h}\frac{d\tilde{c}_h}{dx_i} + \frac{\partial L_h}{\partial \tilde{c}_l}\frac{d\tilde{c}_l}{dx_i} \tag{57}$$

Using (54) we can write:

$$\frac{dL_h}{dx_h} = \frac{d\tilde{c}_h}{dx_h}\left[\frac{\partial L_h}{\partial \tilde{c}_h} + B_l\frac{\partial L_h}{\partial \tilde{c}_l}\right] \tag{58}$$

$$\frac{dL_h}{dx_l} = \frac{d\tilde{c}_l}{dx_l}\left[\frac{\partial L_h}{\partial \tilde{c}_h}B_h + \frac{\partial L_h}{\partial \tilde{c}_l}\right] \tag{59}$$

Next we derive expressions for the terms in the brackets:

$$\left[\frac{\partial L_h}{\partial \tilde{c}_h} + B_l\frac{\partial L_h}{\partial \tilde{c}_l}\right] = \begin{cases} \xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_h)\left[\dfrac{r+\tau-\frac{\partial A_l}{\partial L_l^g}\xi^c(\tilde{c}_l)(1-\Xi^\epsilon(\tilde{\epsilon}_p))}{r+\tau-\frac{\partial A_l}{\partial L_l^g}\frac{\partial L_l^g}{\partial \tilde{c}_l}}\right], & \text{if } \tilde{c}_h < \tilde{c}_l \\ \xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_l)\left[1 - B_l\right] + \int_{\tilde{c}_l}^{\tilde{c}_h}\xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c), & \text{if } \tilde{c}_l < \tilde{c}_h, \end{cases} \tag{60}$$

$$\left[\frac{\partial L_h}{\partial \tilde{c}_l} + B_h\frac{\partial L_h}{\partial \tilde{c}_h}\right] = \begin{cases} -\xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_h)\left[1 - B_h\right] - \int_{\tilde{c}_h}^{\tilde{c}_l}\xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c), & \text{if } \tilde{c}_h < \tilde{c}_l \\ -\xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_l)\left[\dfrac{r+\tau-\frac{\partial A_h}{\partial L_h^g}\xi^c(\tilde{c}_h)\Xi^\epsilon(\tilde{\epsilon}_p)}{r+\tau-\frac{\partial A_h}{\partial L_h^g}\frac{\partial L_h^g}{\partial \tilde{c}_h}}\right], & \text{if } \tilde{c}_l < \tilde{c}_h \end{cases} \tag{61}$$

The terms in (60) are positive while the terms in (61) negative. Using (56) we can therefore conclude that:

$$\frac{dL_i}{dx_i} > 0, \frac{dL_i}{dx_j} < 0, \; x_i = [w_i^g, s_i^g, e_i^g], \; i = [h, l], \; j = [h, l] \text{ and } j \neq i \tag{62}$$

■

A.8 Proof of Proposition 5

Proof. From (46), for $s_i^p = s_i^g$ and $w_i^g = \underline{w}_i^g$ (which means $Q_i = 0$) we obtain that the introduction of a public sector whose entry is costly will cause a change in total employment of workers of type i of

$$e_i - e_i^{ng} = \frac{m(\theta_i)(L_i - L_i^{ng})}{s_i^p + \tau + m(\theta_i)} \tag{63}$$

From the results in Proposition 3 it is easy to verify that, since the share of high-educated labor force decreases (increases) in Case A (case B), while it remains unchanged in case C, then $e_h - e_h^{ng} < 0$ $(e_l - e_l^{ng} > 0)$ in Case A, $e_h - e_h^{ng} > 0$ $(e_l - e_l^{ng} < 0)$ in Case B, and $e_h - e_h^{ng} = 0$ $(e_l - e_l^{ng} = 0)$ in Case C. ■

A.9 Proof of Proposition 6

Proof. Using the labor force shares in (21)-(23) and the result in (54) we can write

$$\frac{dL_i^p}{dx_i} = \frac{\partial L_i^p}{\partial \tilde{c}_i}\frac{\partial \tilde{c}_i}{\partial x_i} + \frac{\partial L_i^p}{\partial \tilde{c}_j}\frac{\partial \tilde{c}_j}{\partial x_i} \tag{64}$$

$$\frac{dL_j^p}{dx_i} = \frac{\partial L_j^p}{\partial \tilde{c}_i}\frac{\partial \tilde{c}_i}{\partial x_i} + \frac{\partial L_j^p}{\partial \tilde{c}_j}\frac{\partial \tilde{c}_j}{\partial x_i} \tag{65}$$

$$\frac{dL_i^g}{dx_i} = \frac{\partial \tilde{c}_i}{\partial x_i}\left[\frac{\partial L_i^g}{\partial \tilde{c}_i} + B_j\frac{\partial L_i^g}{\partial \tilde{c}_j}\right] \tag{66}$$

$$\frac{dL_j^g}{dx_i} = \frac{\partial \tilde{c}_i}{\partial x_i}\left[\frac{\partial L_j^g}{\partial \tilde{c}_i} + B_j\frac{\partial L_j^g}{\partial \tilde{c}_j}\right] \tag{67}$$

where $i = [h, l]$, $j = [h, l]$ and $i \neq j$. Using (21)-(23) one can easily verify that $\frac{\partial L_i^p}{\partial \tilde{c}_i} < 0$, $\frac{\partial L_j^p}{\partial \tilde{c}_j} < 0$, $\frac{\partial L_i^p}{\partial \tilde{c}_j} \leq 0$, $\frac{\partial L_j^p}{\partial \tilde{c}_i} \leq 0$, while, as shown above (see 56), for $x_i = [w_i^g, s_i^g, e_i^g]$, $\frac{d\tilde{c}_i}{dx_i} > 0$ and $\frac{d\tilde{c}_j}{dx_i} > 0$. It follows that

$$\frac{dL_i^p}{dx_i} < 0$$

$$\frac{dL_j^p}{dx_i} < 0 \tag{68}$$

Further, $\frac{\partial L_i^g}{\partial \tilde{c}_i} > -\frac{\partial L_i^g}{\partial \tilde{c}_j} > 0$, $\frac{\partial L_j^g}{\partial \tilde{c}_j} > -\frac{\partial L_j^g}{\partial \tilde{c}_i} > 0$, meaning that $1 > B_j$ and the term in the bracket of (66) is positive. Using the expression for B_j in (55) we obtain

$$\left[\frac{\partial L_j^g}{\partial \tilde{c}_i} + B_j\frac{\partial L_j^g}{\partial \tilde{c}_j}\right] = \frac{\partial L_j^g}{\partial \tilde{c}_i}\left[\frac{r+\tau}{r+\tau-\frac{\partial A_j}{\partial L_j^g}\frac{\partial L_j^g}{\partial \tilde{c}_j}}\right] < 0 \tag{69}$$

It follows that

$$\frac{dL_i^g}{dx_i} > 0$$

$$\frac{dL_j^g}{dx_i} < 0 \tag{70}$$

Since market tightness in the private sector is independent of public sector policies (see Lemma 1), from (43) we can write:

$$\frac{de_i}{dw_i^g} = \frac{m(\theta_i)}{s_i^p + \tau + m(\theta_i)}\frac{dL_i^p}{dw_i^g} < 0 \tag{71}$$

$$\frac{de_j}{dw_i^g} = \frac{m(\theta_j)}{s_j^p + \tau + m(\theta_j)}\frac{dL_j^p}{dw_i^g} < 0 \tag{72}$$

■

A.10 Proof of Proposition 7

Proof. Let $L_i^{p,ng}$ denote the size of the labor force of type i when there is no government sector, and thus all labor forced is attached to the private sector. When no public-sector exists, the cutoff education cost is

$\tilde{\epsilon}_p$, $L_h^{p,ng} = \Xi^\epsilon(\tilde{\epsilon}_p)$ and $L_l^{p,ng} = 1 - \Xi^\epsilon(\tilde{\epsilon}_p)$ so that:

$$\frac{L_h^{p,ng}}{L_l^{p,ng}} = \frac{\Xi^\epsilon(\tilde{\epsilon}_p)}{1 - \Xi^\epsilon(\tilde{\epsilon}_p)}$$

Using (21)-(23), we get:

$$\frac{L_h^p}{L_l^p} - \frac{L_h^{p,ng}}{L_l^{p,ng}} = \begin{cases} \frac{\int_{\tilde{c}_h}^{\tilde{c}_l} \Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c)}{(1-\Xi^c(\tilde{c}_l))(1-\Xi^\epsilon(\tilde{\epsilon}_p))} > 0, & \text{if } \tilde{c}_h < \tilde{c}_l \\ -\frac{\Xi^\epsilon(\tilde{\epsilon}_p)}{1-\Xi^\epsilon(\tilde{\epsilon}_p)}\left[1 - \frac{1}{1+\frac{\int_{\tilde{c}_l}^{\tilde{c}_h}(1-\Xi^\epsilon(\tilde{\epsilon}_m(c)))d\Xi^c(c)}{(1-\Xi^c(\tilde{c}_h))(1-\Xi^\epsilon(\tilde{\epsilon}_p))}}\right] < 0, & \text{if } \tilde{c}_l = \tilde{c}_h \\ 0, & \text{if } \tilde{c}_l < \tilde{c}_h \end{cases} \tag{73}$$

From (43) and (45) we obtain

$$\frac{e_h^p}{e_l^p} - \frac{e_h^{p,ng}}{e_l^{p,ng}} = \frac{\frac{m(\theta_h)}{s_h^p+\tau+m(\theta_h)}}{\frac{m(\theta_l)}{s_l^p+\tau+m(\theta_l)}}\left[\frac{L_h^p}{L_l^p} - \frac{L_h^{p,ng}}{L_l^{p,ng}}\right]$$

where as above the superscript "ng" is used to denote the case of no government sector. Hence, from (73), it follows that

$$\frac{e_h^p}{e_l^p} - \frac{e_h^{p,ng}}{e_l^{p,ng}} = \begin{cases} > 0, & \text{if } \tilde{c}_h < \tilde{c}_l \\ < 0, & \text{if } \tilde{c}_l = \tilde{c}_h \\ 0, & \text{if } \tilde{c}_l < \tilde{c}_h \end{cases}$$

■

B Random search

B.1 Setup

In this appendix we give the full set of equations of the model with random search and characterize it's steady-state equilibrium. Further, we provide proofs of Propositions 8 and 9.

The values of being employed in either the private or the public sector for workers (equations 2 and 4), and the values of private-sector filled jobs and vacancies (equations 5 and 6) remain as in the Benchmark model. As discussed in the text, only the value of unemployment changes. It is now given by equation (35). The Nash bargaining wage of the private sector changes accordingly and is as given in (36).

Both government and private firms that seek to hire workers meet with them at rate $q(\theta_i) = \frac{m(\theta_i)}{\theta_i}$, where $\theta_i = \frac{v_i^p+v_i^g}{u_i}$. The number of vacancies in the private sector is determined endogenously by free entry that drives the value of a private-sector vacancy to zero, $V_i^p = 0$. The government needs to post enough vacancies to ensure that the total number of matches, $q(\theta_i)v_i^g$, equals the number of workers that it needs to hire. Hence, the government posts v_i^g vacancies to ensure $q(\theta_i)v_i^g = (s_i^g + \tau)\, e_i^g$.

Setting $V_i^p = 0$ and using the Nash bargaining conditions in (8), we can write the surplus of a private-sector match with a type i worker as

$$S_i^p = \frac{y_i - b_i - D_i(w_i^g - b_i)}{r + \tau + s_i^p + (1 - D_i)\beta m(\theta_i)\nu_i^p} \tag{74}$$

and the zero-profit condition that determines job creation in the private sector becomes:

$$\frac{\kappa_i}{q(\theta_i)} = \frac{(1-\beta)(y_i - b_i - D_i(w_i^g - b_i))}{r + \tau + s_i^p + (1 - D_i)\beta m(\theta_i)\nu_i^p} \tag{75}$$

We can write the threshold level of education cost:

$$\tilde{\epsilon} = \frac{1}{r+\tau}\left[b_h - b_l + D_h(w_h^g - b_h) - D_l(w_l^g - b_l) + (1-D_h)\frac{\beta\nu_h^p\kappa_h\theta_h}{(1-\beta)} - (1-D_l)\frac{\beta\nu_l^p\kappa_l\theta_l}{(1-\beta)}\right] \tag{76}$$

where it may be recalled that $D_i = \frac{(1-\nu_i^p)m(\theta_i)}{r+\tau+s_i^g+(1-\nu_i^p)m(\theta_i)}$.

As in the benchmark model we treat public sector employment as an exogenous policy variable. There are e_i^g workers of each skill type employed in the public sector. The number of workers employed in the private sector is endogenous and depends on job creation in the private sector as well as conditions in the public sector. The labor force consists of those employed in the public sector, those employed in the private sector (e_i^p), and the unemployed (u_i). Hence, $u_i = L_i - e_i^g - e_i^p$. By equating the flows in, $m(\theta_i)\nu_i^p u_i$, to the flows out of the state where a worker is employed in the private sector, $e_i^p(s_i^p + \tau)$ we obtain:

$$e_i^p = \frac{m(\theta_i)\nu_i^p\left[L_i - e_i^g\right]}{m(\theta_i)\nu_i^p + \tau + s_i^p} \tag{77}$$

$$u_i = \frac{(\tau + s_i^p)\left[L_i - e_i^g\right]}{m(\theta_i)\nu_i^p + \tau + s_i^p} \tag{78}$$

Given $\theta_i = \frac{v_i^p + v_i^g}{u_i}$ and $q(\theta_i)v_i^g = (s_i^g + \tau)\,e_i^g$, we can use (78) to write:

$$\nu_i^p = \frac{s_i^p + \tau}{m(\theta_i)}\left[\frac{m(\theta_i)\left[L_i - e_i^g\right] - (s_i^g + \tau)e_i^g}{(s_i^p + \tau)\left[L_i - e_i^g\right] + e_i^g(s_i^g + \tau)}\right] \tag{79}$$

Using (77) and (79) we can write the total employment of type-i workers, $e_i = e_i^p + e_i^g$ as:

$$e_i = \frac{m(\theta_i)L_i + e_i^g(s_i^p - s_i^g)}{s_i^p + \tau + m(\theta_i)} \tag{80}$$

B.2 Definition of Equilibrium

A steady state equilibrium consists of a cut-off $\tilde{\epsilon}$, tightness $\{\theta_h, \theta_l\}$, and unemployed $\{u_h, u_l\}$, such that, given some exogenous government policies $\{w_h^g, w_l^g, e_h^g, e_l^g\}$, the following apply.

1. Private-sector firms satisfy the free-entry condition (75) $i = [h, l]$.
2. Private-sector wages are the outcome of Nash Bargaining (36) $i = [h, l]$.
3. Newborns decide optimally their investments in education and population shares are determined by $L_h = \Xi^\epsilon(\tilde{\epsilon})$ and $L_l = 1 - \Xi^\epsilon(\tilde{\epsilon})$.
4. Flows between private employment and unemployment are constant

$$(s_h^p + \tau)e_h^p = m(\theta_h)\nu^h u_h^p \tag{81}$$

$$(s_l^p + \tau)e_l^p = m(\theta_l)\nu^l u_l^p \tag{82}$$

5. Population add up constraints are satisfied:

$$L_h = e_h^p + e_h^g + u_h$$
$$L_l = e_l^p + e_l^g + u_l$$
$$L_h + L_l = 1$$

B.3 Proof of Existence and Uniqueness

Proof. To prove the existence and uniqueness of a steady state equilibrium under random search we show below that the two free-entry conditions in (75) cross only once in the $[\theta_h, \theta_l]$ plane, giving a unique set of equilibrium values for θ_h and θ_l. The equilibrium value of the cut-off education cost can then be determined by substituting the equilibrium values of the theta's in equation (76). Then we can determine $L_h = 1 - \Xi(\tilde{\epsilon})$, $L_l = 1 - \Xi(\tilde{\epsilon})$, which in turn, together with the equilibrium values of theta's, can be substituted in equations (36), (77), (78) to determine wages, employment in the private sector and unemployment.

The two job creation conditions in (75), and the cut-off education cost in (77) can be written as:

$$\frac{\kappa_i}{q(\theta_i)} = \frac{(y_i - b_i - OO_i)}{r + \tau + s_i^p} \tag{83}$$

$$\tilde{\epsilon} = \frac{1}{r + \tau} [b_h - b_l + OO_h - OO_l] \tag{84}$$

where

$$OO_i = D_i(w_i^g - b_i) + (1 - D_i)\frac{\beta}{1 - \beta}\nu_i^p \kappa_i \theta_i \tag{85}$$

is the expression for the outside option of workers of skill type $i = [h, l]$ and A_i is as defined in (40).

In what follows let $j = [h, l]$, $i = [h, l]$ and $j \neq i$. Taking the derivative with respect to θ_j of (84) and of (85), after we substitute in for ν_i^p using (79), we obtain:

$$\frac{d\tilde{\epsilon}}{d\theta_j} = \frac{1}{r + \tau}\left[\frac{dOO_h}{d\theta_j} - \frac{dOO_l}{d\theta_j}\right] \tag{86}$$

$$\frac{dOO_j}{d\theta_j} = -K_j \frac{dL_j}{d\tilde{\epsilon}}\frac{d\tilde{\epsilon}}{d\theta_j} + \Sigma_j \tag{87}$$

$$\frac{dOO_i}{d\theta_j} = -Z_i \frac{dL_i}{d\tilde{\epsilon}}\frac{d\tilde{\epsilon}}{d\theta_j} \tag{88}$$

where

$$K_j = \frac{(1 - D_j)m(\theta_j)\left(E_j^g - E_j^p\right)(s_j^p + \tau)(1 - \nu_j^p)}{(s_j^p + \tau)\left[L_j - e_j^p\right] + e_j^g(s_j^g + \tau)} \tag{89}$$

$$\Sigma_j = (1 - D_j)q(\theta_j)\left[(1 - \nu_j^p)\eta\left(\frac{E_j^g m(\theta_j) + E_j^P(s_j^p + \tau)}{s_j^p + \tau + m(\theta_j)} - U_j\right) + \nu_j^p\left(E_j^p - U_j\right)\right] \tag{90}$$

$$Z_i = \frac{(1 - D_i)m(\theta_i)\left(E_i^g - U_i\right)(s_i^p + \tau)(1 - \nu_i^p)}{(s_i^p + \tau)\left[L_i - e_i^p\right] + e_i^g(s_i^g + \tau)} \tag{91}$$

Using these expressions we can solve for $\frac{dOO_j}{d\theta_j}$ and $\frac{dOO_i}{d\theta_j}$.

$$\frac{dOO_h}{d\theta_h} = \frac{\Sigma_h}{1 + \frac{K_h \frac{dL_h}{d\tilde{\epsilon}}\frac{1}{r+\tau}}{1 - Z_l \frac{dL_l}{d\tilde{\epsilon}}\frac{1}{r+\tau}}} > 0 \tag{92}$$

$$\frac{dOO_l}{d\theta_l} = \frac{\Sigma_l}{1 - \frac{K_l \frac{dL_l}{d\tilde{\epsilon}}\frac{1}{r+\tau}}{1 + Z_h \frac{dL_h}{d\tilde{\epsilon}}\frac{1}{r+\tau}}} > 0 \tag{93}$$

$$\frac{dOO_l}{d\theta_h} = \left[\frac{-Z_l \frac{dL_l}{d\tilde{\epsilon}}\frac{1}{r+\tau}}{1 - Z_l \frac{dL_l}{d\tilde{\epsilon}}\frac{1}{r+\tau}}\right]\frac{dOO_h}{d\theta_h} > 0 \tag{94}$$

$$\frac{dOO_h}{d\theta_l} = \left[\frac{Z_h \frac{dL_h}{d\tilde{\epsilon}}\frac{1}{r+\tau}}{1 + Z_h \frac{dL_h}{d\tilde{\epsilon}}\frac{1}{r+\tau}}\right]\frac{dOO_l}{d\theta_l} > 0 \tag{95}$$

Since $L_h = \Xi(\tilde{\epsilon})$ and $L_l = 1 - \Xi(\tilde{\epsilon})$ then, evidently, $\frac{dL_l}{d\tilde{\epsilon}} < 0$ and $\frac{dL_h}{d\tilde{\epsilon}} > 0$. Further, as can be easily verified from (90) and (91), $\Sigma_h > 0$, $\Sigma_l > 0$, $Z_h > 0$, $Z_l > 0$, while, as can be seen from (89), sufficient condition to ensure $K_j \geq 0$, $j = [h, l]$ is $E_j^g \geq E_j^p$. Hence, if $E_j^g \geq E_j^p$, then $\frac{dOO_j}{d\theta_j} > 0$ and $\frac{dOO_j}{d\theta_i} > 0$,

$i = [h, l]$, $j = [h, l]$, $j \neq i$.

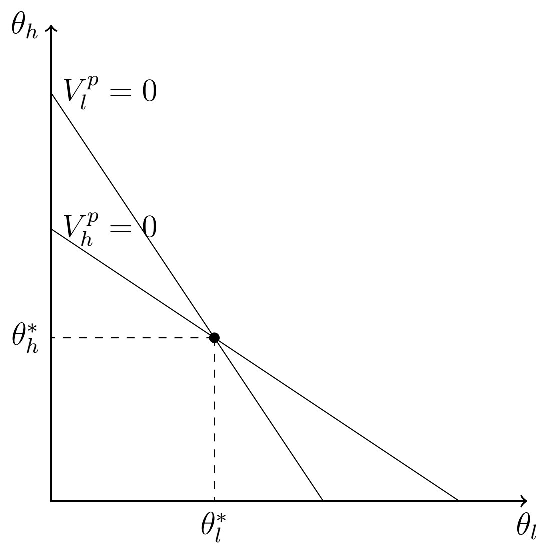
By total differentiation of (83) we can derive the slopes of the two job creation conditions in the $[\theta_h, \theta_l]$ plane:

$$\text{high-education market: } \left.\frac{d\theta_h}{d\theta_l}\right|_{V_h^p=0} = \frac{-\frac{dOO_h}{d\theta_l}}{-\frac{q'(\theta_h)\kappa_h}{(q(\theta_h))^2}(r+\tau+s_h^p) + \frac{dOO_h}{d\theta_h}} < 0 \quad (96)$$

$$\text{Low-education market: } \left.\frac{d\theta_h}{d\theta_l}\right|_{V_l^p=0} = \frac{-\frac{q'(\theta_l)\kappa_l}{(q(\theta_l))^2}(r+\tau+s_l^p) + \frac{dOO_l}{d\theta_l}}{-\frac{dOO_l}{d\theta_h}} < 0 \quad (97)$$

Both slopes are negative, since $q'(\theta_i) < 0$, $\frac{dOO_i}{d\theta_i} > 0$, $\frac{dOO_i}{d\theta_j} > 0$ but, as can be easily verified from (94) and (95), $\frac{dOO_h}{d\theta_l} < \frac{dOO_l}{d\theta_l}$ and $\frac{dOO_l}{d\theta_h} < \frac{dOO_h}{d\theta_h}$, which ensures $\left.\frac{d\theta_h}{d\theta_l}\right|_{V_h^p=0} > \left.\frac{d\theta_h}{d\theta_l}\right|_{V_l^p=0}$ and the two job creation conditions cross once once in the $[\theta_h, \theta_l]$ plane, as shown in Figure 5. This completes the proof of existence and uniqueness.

Figure 5: Steady State Equilibrium under Random Search



■

B.4 Proof of Proposition 8

Proof. It can be shown that an increase in either w_i^g or e_i^g will lower the surplus of private-sector jobs (right-hand-side of 75) of both skill types, thereby lowering job creation in both sectors. The $V_l^p = 0$ and $V_h^p = 0$ loci will shift inwards as illustrated in Figure 6. Both θ_h^* and θ_l^* will decrease.

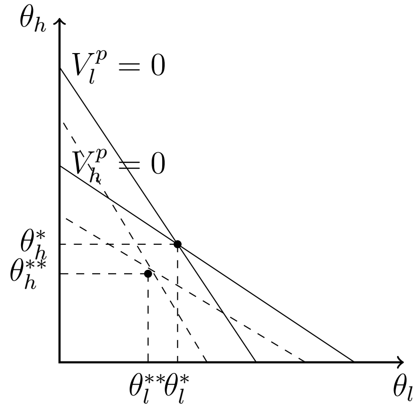
Let $x_i = [w_i^g, e_i^g]$. In what follows we show that $\frac{dOO_i}{dx_i} > 0$ and $\frac{dOO_j}{dx_i} > 0$ (for $j \neq i$), which as can be inferred from (83) imply the shifts depicted in Figure 6.

From (85) we get:

$$\frac{dOO_i}{dx_i} = -K_i \frac{dL_i}{dx_i} + \Lambda_i \quad (98)$$

$$\frac{dOO_j}{dx_i} = -K_j \frac{dL_j}{dx_i} \;\; j \neq i \quad (99)$$

Figure 6: Effects of more generous government policies under random search



where K_i and K_j, as defined in (89), are both greater than zero when $E_i^g - E_i^P > 0$ and $E_j^g - E_j^P > 0$, while

$$\Lambda_i = \begin{cases} \left[K_i + \frac{(1-D_i)(E_i^g - E_i^p)(s_i^g+\tau)(s_i^p+\tau+m(\theta_i)\nu_i^p)}{(s_i^p+\tau)\left[L_i - e_i^p\right] + e_i^g(s_i^g+\tau)}\right] > 0 \text{ if } x_i = e_i^g \\ D_i > 0 \text{ if } x_i = w_i^g \end{cases}$$

Since $L_h = \Xi^\epsilon(\tilde{\epsilon})$ and $L_l = 1 - L_h = 1 - \Xi^\epsilon(\tilde{\epsilon})$, using (84) we can write:

$$\begin{aligned} \frac{dL_l}{dx_i} &= -\frac{\xi(\tilde{\epsilon})}{r+\tau}\left[\frac{dOO_h}{dx_i} - \frac{dOO_l}{dx_i}\right] \\ \frac{dL_h}{dx_i} &= \frac{\xi(\tilde{\epsilon})}{r+\tau}\left[\frac{dOO_h}{dx_i} - \frac{dOO_l}{dx_i}\right] \end{aligned} \tag{100}$$

By combining (98)-(99) and (100) we obtain that for $i = [h,l]$ and $j = [h,l]$:

$$\begin{aligned} \frac{dOO_j}{dx_i} &= \left(\frac{K_j\xi(\tilde{\epsilon})}{r+\tau+K_j\xi(\tilde{\epsilon})}\right)\frac{dOO_i}{dx_i} \text{ if } j \neq i \\ \frac{dOO_i}{dx_i} &= \frac{\Lambda_i(r+\tau)}{r+\tau+K_i\xi(\tilde{\epsilon})\left(1 - \frac{K_j\xi(\tilde{\epsilon})}{r+\tau+K_j\xi(\tilde{\epsilon})}\right)} \end{aligned} \tag{101}$$

Given $\Lambda_i > 0$, $K_i > 0$ and $K_j > 0$ it can be easily verified from (101) that $\frac{dOO_i}{dx_i} > 0$, $\frac{dOO_j}{dx_i} > 0$. ■

B.5 Proof of Proposition 9

As above (see section B.4) let $x_i = [w_i^g, e_i^g]$. By substituting for $\frac{dOO_i}{dx_i}$ and $\frac{dOO_j}{dx_i}$ into (100), using (101), we get that for $i = [h,l]$, $j = [h,l]$ and $i \neq j$:

$$\begin{aligned} \frac{dL_i}{dx_i} &= \left[\frac{\xi(\tilde{\epsilon})\frac{dOO_i}{dx_i}}{r+\tau+K_j\xi(\tilde{\epsilon})}\right] > 0 \\ \frac{dL_j}{dx_i} &= -\left[\frac{\xi(\tilde{\epsilon})\frac{dOO_j}{dx_j}}{r+\tau+K_i\xi(\tilde{\epsilon})}\right] < 0 \end{aligned} \tag{102}$$

C Endogenizing public-sector employment and wages

We can provide microeconomic foundations for the public-sector employment and wage policies that are taken as exogenous in the baseline model. Consider a government that is limited in its amount of spending to $\bar{\omega}$, exogenous, that arises from budgetary constraints. The government has an objective function with two components. The first, is the production of government services, g that use a Cobb-Douglas production function in skilled and unskilled public employment, with weight γ on the skilled workers, $g = (e_h^g)^\gamma (e_l^g)^{1-\gamma}$. The second, is the preferences of a union represented by $\varsigma\mu(a_h) + (1-\varsigma)\mu(a_l)$. Here ς represents the weight of skilled workers in the union's preferences and a_h and a_l are the extra payment to public-sector workers on top of the minimum required wage for the existence of the public sector ($w_h^g = \underline{w}_h^g + a_h$ and $w_l^g = \underline{w}_l^g + a_l$). The union knows what this minimum required wage is and tries to push the wages above. $\mu(a_i)$ is a function expressing the utility of the extra payment to type i workers, which for convenience we assume it is $\log(a_i)$.

The government's problem can be written as:

$$\max_{e_h^g, e_l^g, a_h, a_l} (e_h^g)^\gamma (e_l^g)^{1-\gamma} + \varphi(\varsigma \log(a_h) + (1-\varsigma)\log(a_l))$$

$$s.t.$$

$$(\underline{w}_h^g + a_h)e_h^g + (\underline{w}_l^g + a_l)e_l^g = \bar{\omega}.$$

where φ is the weight of the unions in the government's maximization problem. The first-order conditions determining employment and wages of skilled and unskilled workers are given by:

$$e_g^g = \bar{\omega}\left(\frac{\gamma}{w_h^g}\right), \tag{103}$$

$$e_l^g = \bar{\omega}\left(\frac{1-\gamma}{w_l^g}\right), \tag{104}$$

$$\frac{\varphi\varsigma}{a_h} = \frac{g e_h^g}{\bar{\omega}} \tag{105}$$

$$\frac{\varphi(1-\varsigma)}{a_l} = \frac{g e_l^g}{\bar{\omega}} \tag{106}$$

The first two conditions pin down the employment level of the government. Given technology and a certain wage, the government spends a constant fraction γ of its budget on skilled workers and $1-\gamma$ on unskilled workers. In this setting, the reason why the government hires more skilled workers is due to technology – skilled workers are more important inputs in the production functions. We can rearrange the last two conditions to pin down government wages:

$$w_h^g = \frac{g\gamma}{g\gamma - \varphi\varsigma}\underline{w}_h^g \tag{107}$$

$$w_l^g = \frac{g(1-\gamma)}{g(1-\gamma) - \varphi(1-\varsigma)}\underline{w}_l^g \tag{108}$$

If the weight of the unions in the government objective function tends to zero, the government would set the wages equal to the minimum required for the government to hire its workers. However, if this weight is higher, the government raises public wages. Whether it raises more the skilled or unskilled wages, depends on the relative weight on the union preference. Note, however, that government wages and employment are independent of productivity; they only reflect budgetary constraints, union preferences and technology.

Summarizing, we can endogenize the four policy parameters, that now depend on four exogenous parameters reflecting technology (γ), budgetary pressures ($\bar{\omega}$), union power (φ) and union relative preferences (ς). This is one model of government behaviour, but there could certainly be others. We think however, when studying the effects of public-sector employment and wages, it is a clearer exercise to take them as exogenous.