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MODELING PARAMETER HETEROGENEITY IN CROSS-COUNTRY REGRESSION MODELS

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Modeling Parameter Heterogeneity in Cross-Country Regression Models*

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Abstract

We employ various local generalizations of the Solow growth model that model parameter heterogeneity using adult literacy rates and life expectancy at birth. The model takes the form of a semiparametric varying coefficient model along the lines of Hastie and Tibshirani (1993). The empirical results show substantial parameter heterogeneity in the cross-country growth process, a finding that is consistent with the presence of multiple steady-state equilibria and the emergence of convergence clubs.

Keywords: Solow growth model, parameter heterogeneity, varying coefficient model, human development

JEL Classifications: C14, C21, C50, N10

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1 Introduction

Empirical growth research has become a dominant field in macroeconomics. However, despite the vast amount of research, there is remarkably little confidence in the results and the implications that have come from conventional empirical methods of growth analysis. A typical example is Pack (1994) (pages 68-69), who describes several problems with cross-country growth regression models:

> *The production function interpretation is further muddled by the assumption that all countries are on the same international production frontier ... regression equations that attempt to sort out the sources of growth also generally ignore interaction effects ... The recent spate of cross-country growth regressions also obscures some of the lessons that have been learned from the analysis of policy in individual countries.*

One of the major reasons for the general mistrust of the conventional approach is the assumption of parameter homogeneity in cross-country growth regression. Indeed, there is a growing number of recent empirical studies that question the assumption of a single linear model that can be applied to all countries. Instead, these studies find evidence that is consistent with multiple steady-state equilibria that classify the countries into different convergence clubs.

One approach to allowing for parameter heterogeneity in cross-country growth regressions is to use threshold regression models or classification algorithms such as a regression tree. In a pioneer paper, Durlauf and Johnson (1995) employ a regression tree approach to uncover multiple regimes in the data. This evidence is formally tested by Hansen (2000) who develops statistical theory for the threshold regression and applies procedures to formally test for the presence of threshold effects and to obtain a confidence set for the threshold parameter. More recently, using a generalized regression tree analysis, Tan (2010) investigates how fundamental determinants, such as institutions, interact to hinder or facilitate development outcomes for different groups of countries.

A conceptually different approach employs semiparametric models based on nonparametric smooth functions to identify general nonlinear growth patterns. Notable examples include Liu and Stengos (1999) and Kalaitzidakis, Mamuneas, Savvides, and Stengos (2001) who employ a partially linear model, Canova (2004) who uses a predictive density approach, Desdoigts (1999) who employs an exploratory projection pursuit (density estimation), and Kourtellos (2002) who uses a projection pursuit regression.

Durlauf, Kourtellos, and Minkin (2001) (DKM) extended this search for nonlinearities in cross-country growth regressions to one for parameter heterogeneity. In particular, DKM employ a varying coefficient approach to estimate a local Solow growth model that allows the parameters for each country to vary as smooth functions of initial income. They find evidence for substantial country-specific heterogeneity that is associated with differences in initial income in the Solow parameters. The varying coefficient approach is also employed in Mamuneas, Savvides, and Stengos (2006) who find nonlinearities in the estimates of the elasticity of human capital with respect to output using annual measures of Total Factor Productivity (TFP) for 51 countries.

There are several interpretations of parameter heterogeneity. First, modern economic growth models provide microfoundations for the presence of multiple steady states and the emergence of convergence clubs. Examples include models with human capital externalities (e.g., Azariadis and Drazen (1990)), imperfections in credit markets and indivisibilities of investment in human capital (e.g., Galor and Zeira (1993)), local technological spillovers (e.g., Durlauf (1993)), Schumpeterian patterns of innovation and technology (e.g., Howitt and Mayer-Foulkes (2005)), institutional barriers (e.g., Acemoglu, Aghion, and Zilibotti (2006)), and differential timing of take-offs (e.g., Galor and Weil (1999), Galor and Weil (2000)). Each of these theories suggests that from the perspective of a local linear approximation of the growth process, different countries will be characterized by different parameters. Second, the assumption of Cobb–Douglas production function as the basis of the derivation of the Solow growth model has been challenged. Duffy and Papageorgiou (2000) and Masanjala and Papageorgiou (2004) find evidence in favor of a constant elasticity of substitution (CES) production function rather than the standard Cobb-Douglas specification. This finding is important given that a Cobb-Douglas production function is a necessary condition for the linearity of the Solow growth model. Third, parameter heterogeneity may be induced by omitted growth determinants. In fact, a range of new growth theories suggest additional covariates beyond those originally proposed by Solow. Durlauf, Johnson, and Temple (2005) identified more than 140 variables used by various researchers, including, but not limited to, market distortions, geographical regions, source endowments, climate, institutions, politics, and war.

In this chapter, we model parameter heterogeneity in the cross-country growth regression using two alternative human development variables that allow us to uncover their complex relationship to economic growth. Methodologically, we employ a local generalization of the Solow growth model along the lines of DKM in the sense that while the Solow model applies to all countries, the parameters of the aggregate production function vary across countries. More precisely, we allow these parameters to vary according to a countrys initial human development level. We study local generalizations of Solow growth models with and without accounting for population growth and saving rates. That is, we study unconditional and conditional local Solow growth models.

The generalization of the Solow growth model takes the form of a semiparametric varying coefficient model along the lines of Hastie and Tibshirani (1993). This model is described as semiparametric because it is a conditional linear model that imposes no assumptions on the functional form of the coefficients, but the shape of the function is estimated by the data. While this restricts the form of parameter heterogeneity, it is an appealing way to generalize the traditional linear Solow model.That is, if we index the countries by an interesting variable, such as the initial conditions of human development, then, near steady state, the Solow model can provide a good approximation. Our approach also allows us to evaluate how the shares of human and physical capital vary with the initial levels of human development.

We measure human development using two key indicators of economic development beyond income: initial levels of adult literacy rates and life expectancy at birth. Literacy rates are a measure of the ability of a country to acquire human capital and may have a large impact on the ability of an economy to generate economic growth. Life expectancy at birth is the most commonly used measure of human capital health. High levels of longevity are critical for a countrys economic and social well-being. Improving health outcomes can have large indirect payoffs because healthy citizens have a positive effect on economic growth. Better health stimulates learning ability, fosters education incentives, and encourages long-term savings (see, e.g., Bloom (1998)).

Our findings suggest that there is substantial heterogeneity across countries. This heterogeneity is reflected in the estimated varying coefficients of the local Solow growth model. The findings also suggest that there is substantial evidence of a latent determinant of negative growth rates or poverty traps. The results suggest the presence of multiple steady-state equilibria in the growth process with respect to initial human capital. This evidence is consistent with the twin peaks found by Quah (1997) in the limit distribution of cross-country per capita income, the presence of multiple regimes found in Durlauf and Johnson (1995) and Masanjala and Papageorgiou (2004) and the global divergence found in Mayer-Foulkes (2006).

Section 2 revisits the standard approach to cross-country growth analysis and proposes a varying coefficient. Section 3 describes the data employed in this chapter. Section 4 presents estimates of the varying coefficients for Solow parameters in unconditional and conditional specifications using human development at the beginning of the period with adult literacy rates and life expectancy at birth as a proxy. Section 5 presents the summary and conclusion.

2 Econometric methodology

The standard approach to cross-country growth analysis as illustrated by Azariadis and Drazen (1991),Barro (1997), Barro and Sala-I-Martin (1995), and Mankiw, Romer, and Weil (1992) and extended by Evans (1998), Islam (1995), and Lee, Pesaran, and Smith (1997) to panel data has focused on the linear regression model. For each country, i, average per capita real GDP growth, g_i, is assumed to obey

$$g_i = \mathbf{X}_i'\gamma + u_i, \tag{2.1}$$

where X_i is a p-dimensional vector of growth determinants and u_i is the regression error.

In the standard Solow model, the determinants consist of the logarithm (log) of population growth rate plus 0.05, which corresponds to the sum of the constant rates of technical change and depreciation; the log of the savings rate for physical capital accumulation; and the log of the real per capita income of the country at the beginning of the period over which growth is measured.[1] The underlying assumption of this regression is that each country is associated with a common Cobb-Douglas aggregate production function.

One way to model parameter heterogeneity in Equation (2.1) is to consider a local generalization, which effectively generalizes the constant coefficient γ to become a smooth function $\gamma(z_i)$ that maps the scalar index z_i into a set of country-specific Solow parameters using the human development index. By local, we refer to the idea that a Solow model applies to each country, but the parameters of the aggregate production function vary according to a slower moving variable, such as countrys initial levels of human development. In other words, although the Solow model can be an inappropriate specification when applied to all countries, it can still be a good approximation locally for an individual country. This generalization yields the varying coefficient model:

$$g_i = \mathbf{X}_i'\gamma(z_i) + u_i, \tag{2.2}$$

where $E(u_i|\mathbf{X}_i) = 0$, $E(u_i^2|\mathbf{X}_i) = \sigma^2(z_i)$, and $\gamma(z_i)$=($\gamma_1(z_i)$,$\gamma_2(z_i)$,. . .,$\gamma_p(z_i))'$.

Two important points need to be highlighted about the relationship of the varying coefficient model in Equation (2.2) with the linear regression and threshold regression or tree regression. First, the varying coefficient model encompasses not only the linear model in Equation (2.1) but also any regression model that augments the latter with z_i in a linear or nonlinear way. Notable examples of nonlinear models that can be viewed as nested models within the varying coefficient model are the semiparametric partially linear model and parametric models with higher-order polynomials or interactions. Second, one important difference of the varying coefficient model vis-a-vis the tree regression and threshold regression models is that the parameter heterogeneity is modeled through smooth functions as opposed to abrupt changes by using indicator functions. In effect, the human development index acts as a threshold variable but in a smooth way. One can argue that given the short span of time in cross-country growth data, smooth functions can be more efficient in identifying nonlinearities in the cross-country growth process.

Following Fan and Zhang (1999), we estimate the varying coefficient model in Equation (2.2) using simple local regression based on a two-stage estimation procedure. The major advantage of a two-stage estimation over a one-stage estimation is that it allows the functional coefficients to possess different degrees of smoothness, which ensures that the optimal rate of convergence for the asymptotic mean-squared error is achieved. The appendix describes the estimation procedure in detail.

[1]Mankiw, Romer, and Weil (1992) extended this model to include the savings rate for physical capital accumulation.

3 Data

This chapter uses a balanced panel dataset for 88 countries (see Table 1). Data are averaged over four 10-year periods between 1960 and 1999.[2] The explanatory variables reflect the standard set variables suggested by the Solow growth theory (see Mankiw, Romer, and Weil (1992)). They include the logarithm of average growth rate of the population plus 0.05 for depreciation, *gpop*; the logarithm of average proportion of real investments, including government, to real GDP, *inv*; and the logarithm of initial per capita income, y_0. The two human development indices that we use are as follows: (a) the logarithm of adult literacy rates defined as the fraction of the population over the age of 15 that is able to read and write in 1960, lit_0 and (b) the logarithm of life expectancy at birth in 1960, $lifee_0$. All explanatory variables, except schooling, were obtained from the Penn World Table 6.1. The two indices are from the World Banks World Report. Table 1 presents the countries along with two human development indices, lit_0 and $lifee_0$.

4 Empirical Results

4.1 Unconditional models

We start by investigating a simplified local generalization of growth regression that assumes that the steady-state value of per capita income is constant across countries. Then, using the human development index, z_i, the varying coefficient model takes the following form,

$$g_i = \gamma_1(z_i) + \gamma_2(z_i)y_{0i} + u_i. \tag{4.3}$$

Figures 1(a)(d) and 2(a)(d) present the results for $z = lit_0$ and $z = lifee_0$, respectively.[3] They present the point estimates and associated 95% pointwise confidence intervals for the varying coefficient functions, conditional variance, and implied convergence rates. Confidence intervals for the implied convergence rates and the implied shares of capital, estimated in the following sections, are computed using the delta method. The superimposed horizontal dashed lines refer to the corresponding least squares estimated (invariant) parameters of a linear Solow growth model.

The results are quite revealing. The varying coefficients are substantially different from the linear Solow growth model, as the least squares estimates cut the confidence intervals

[2] Similar results are obtained for a 20-year period panel data set.

[3] Outliers were omitted from the graphs, as they render the graphs unreadable. For $z = lit_0$, Niger, Burkina Faso, Mali, and Cote dIvoire are omitted, and for $z = lifee_0$, El Salvador and Gambia are omitted. Despite these countries omission from the graph, estimation is based on the full sample. Complete graphs are available upon request.

several times in all of the estimated models. The relationship between the log of adult literacy rates in 1960 and growth is increasing and nonlinear. The estimation reveals a threshold at the level of the 45th percentile corresponding to Nicaragua, below which the relationship between literacy rates in 1960 and growth is negative and above which the relationship is positive. Figure 1(b) shows the estimates for the varying coefficient of the logarithm of initial income, y_0. Unlike the least squares estimate of the constant coefficient for the corresponding linear model, the estimates of $\gamma_2(z_i)$ are mostly negative with a quadratic shape, implying that high and low initial literacy countries have larger estimates than the countries with middle levels of initial literacy rates. These estimates suggest that the differences in per capita incomes are not temporary and that unconditional convergence to a common long-run level is not occurring. Figure 1(c) presents substantial evidence of parameter heterogeneity in the conditional variance, which takes the form of a hump-shaped function.

Although the conditional variance initially appears to increase for the countries with the lowest levels of initial literacy rates, this variance monotonically decreases for countries with higher levels of initial literacy rates than Senegal. We now turn to the case of life expectancy. While Figures 2(a)(d) appear to be qualitatively similar to those obtained using literacy rates, the results based on the initial levels of life expectancy are more volatile. In particular, the estimates of the varying intercept reveal a threshold at the 42nd percentile, corresponding to Lesotho, below which the relationship between initial life expectancy rates and growth is negative, and above which the relationship is positive. The estimates of the varying coefficient of the log of initial income are mostly negative but with substantial variability. For example, the implied convergence rates vary from 0 to approximately 4%. Finally, the conditional variance appears to generally decrease with the levels of initial life expectancy.

In sum, we find that while the average relationship between growth and human development, as measured by the initial literacy rates or life expectancy at birth, appears to be generally increasing, the relationship is positive only for countries above the median index. We also find substantial parameter heterogeneity in the convergence rates when we index them by the initial levels of human development. Taking all of the evidence together, we conclude that initial levels of human development can determine long-run outcomes and that countries with similar initial conditions exhibit similar long-run outcomes. This finding suggests the presence of multiple steady-state equilibria and the emergence of convergence clubs in the growth process consistent with the twin peaks in the cross-country income distribution found by Quah (1997). Finally, we find that the conditional variance appears to be generally decreasing in the levels of initial human development index, which implies the beneficial effect of human development on growth volatility.

4.2 Conditional models on population growth and investments

This section estimates a local generalization of the basic Solow growth model which is based on a two-factor Cobb–Douglas production function with physical capital and labor as inputs. In this case, the varying coefficient model in Equation (4.3) is augmented by the variables of the population growth rates and the saving rate of physical capital and takes the form of Equation (4.4):

$$g_i = \gamma_1(z_i) + \gamma_2(z_i)gpop_i + \gamma_3(z_i)inv_i + \gamma_4(z_i)y_{0i} + u_i. \tag{4.4}$$

Figures 3(a)(g) and 4(a)(g) present the results for the two development indices lit_0 and $lifee_0$, respectively. Let us first discuss the results based on adult literacy rates, $z = lit_0$. First, for the varying coefficients associated with the intercept, population growth, and physical capital, the estimates exhibit substantial parameter heterogeneity for countries with literacy rates in 1960 lower than the 18th percentile, corresponding to Senegal. Second, the estimates of the varying intercept show that the relationship between literacy rates in 1960 and growth is negative for countries with literacy rates lower than the rate that corresponds to Senegal. Similar to unconditional models, this pattern suggests that negative growth rates may be the result of a latent determinant of countries with low literacy rates. However, this threshold appears to be lower than in the case of the unconditional models. It is estimated to be around the 18th percentile rather than around the 45th percentile. Third, for countries with literacy rates lower than the 12th percentile, corresponding to Nepal, the estimates of the varying coefficient of population growth rates are positive. This finding suggests the presence of possible scale effects for the poorest countries. Fourth, the estimates for the varying coefficients of physical capital and initial income do not exhibit any sort of monotonicity. The highest values of the varying coefficients of physical capital are associated with countries that have higher literacy rates. For the majority of countries with initial literacy rates higher than the 16th percentile, corresponding to Togo, the estimate for the varying coefficient of physical capital is larger than that predicted by the linear Solow model. For the varying coefficient of initial income, the estimates are negative and mostly significant. However, for countries with point estimates smaller than the 29th percentile, corresponding to Malawi, the estimates are insignificant. This finding may imply the absence of conditional convergence for this group. Alternatively, the non-monotonicity of the estimates may suggest the presence of multiple steady states. For instance, the majority of countries with lower initial literacy rates experience lower convergence rates, and countries with high initial literacy rates have higher convergence rates. It is worth noting that the estimates for the majority of countries are larger than the ones predicted by the linear Solow growth model.

Fifth, the implied shares of physical capital are generally quite large and display a hump for a range of countries with lower literacy rates, between the 9th percentile, corresponding to Benin, and the 32nd percentile, corresponding to Ghana. Countries within the above range exhibit higher shares of physical capital than countries with higher literacy rates. Notably, the estimate of the implied share of physical capital for Burundi is as high as 0.91. However, for countries with literacy rates higher than Ghana, the shares are much lower and rather stable, with an approximate value of about 0.65.

In the case of life expectancy at birth ($z = lifee_0$), the results in Figure 4(a)(g) show a stronger parameter heterogeneity. First, the estimates of the varying intercept exhibit an increasing pattern with negative estimates below the 49th percentile, corresponding to Peru. Second, although the evidence is weaker, there is also a group of countries with positive estimates for the varying coefficient of the population growth rates. More precisely, for countries with literacy rates lower than the 10th percentile, which corresponds to Benin, the estimates are positive. Third, for the majority of countries, the implied convergence rates are larger than the ones predicted by the linear Solow growth model. Interestingly, countries with the lowest rates of life expectancy enjoy convergence rates as high as countries with the highest rates of life expectancy. For instance, Mozambique has the same convergence rate as that of Switzerland. Fourth, the implied shares of physical capital is also hump shaped as that of lit0. In particular, countries with rates of life expectancy between the 10th and 27th percentiles, corresponding to Nepal and India, respectively, have higher shares than the other countries in the sample. Interestingly, many countries in this interval enjoy shares very close to one. Moreover, the only major difference between the two human development indices is that the evidence for parameter heterogeneity in the varying coefficients of physical capital and initial income appears to be stronger for the estimates based on lifee0 than those based on lit_0.

Furthermore, we impose the theoretical restriction that the coefficients on *inv* and *gpop* sum up to zero; see Mankiw, Romer, and Weil (1992). Under this restriction, we estimate the following varying coefficient model:

$$g_i = \gamma_1(z_i) + \gamma_2(z_i)(inv_i - gpop_i) + \gamma_3(z_i)y_{0i} + u_i. \tag{4.5}$$

Figures 5(a)(f) and 6(a)(f) present the results for adult literacy rate ($z = lit_0$) and life expectancy at birth ($z = lifee_0$), respectively. The results are similar to those obtained in the unrestricted models for both indices. The only notable exception is that many countries that have life expectancies between the 16th and 22nd percentiles, corresponding to Cote dIvoire and Madagascar, respectively, exhibit negative convergence rates and have shares of physical capital that are greater than one.

In sum, the empirical results show that the Solow growth model exhibits strong evidence of parameter heterogeneity. More precisely, the coefficients of the Solow regression and the corresponding implied parameters of the Solow model (convergence rates and shares of physical and human capital) vary substantially with initial levels of literacy rates and life expectancy. This evidence is consistent with the presence of multiple regimes found in Durlauf and Johnson (1995) and Masanjala and Papageorgiou (2004) that are associated with adult literacy rates and the global divergence found in Mayer-Foulkes (2006) that is associated with life expectancy. Furthermore, the average relationship between growth and human development appears to be nonlinear and generally increasing. This relationship suggests the presence of a latent determinant of negative growth rates or poverty trap that is omitted from the Solow model. Kourtellos (2003) provided evidence that this latent variable is not associated with the omitted variable of human capital accumulation. In particular, Kourtellos extended the basic local Solow growth model in Equation (4.4) to include human capital accumulation along the lines of Mankiw, Romer, and Weil (1992) to find similar results.

5 Conclusion and directions for future research

This chapter studies local generalizations of the Solow model that take the form of varying coefficient models. In particular, using two measures of initial human development, initial literacy rates and initial life expectancy rates, we investigate parameter heterogeneity and study the complex relationship of human development and economic growth in the context of unconditional and conditional local Solow growth specifications.

We find that both development indices provide strong evidence of parameter heterogeneity. In particular, we find that the parameters of the local Solow growth model vary substantially with the initial levels of literacy rates and especially life expectancy in unconditional and conditional specifications. Furthermore, we find that there may be a latent determinant of negative growth rates or poverty trap that is omitted from the Solow model. Overall, our findings are suggestive of multiple steady states and richer growth dynamics than neoclassical theories, and hence, empirical studies that do not account for parameter heterogeneity are likely to produce a misleading inference.

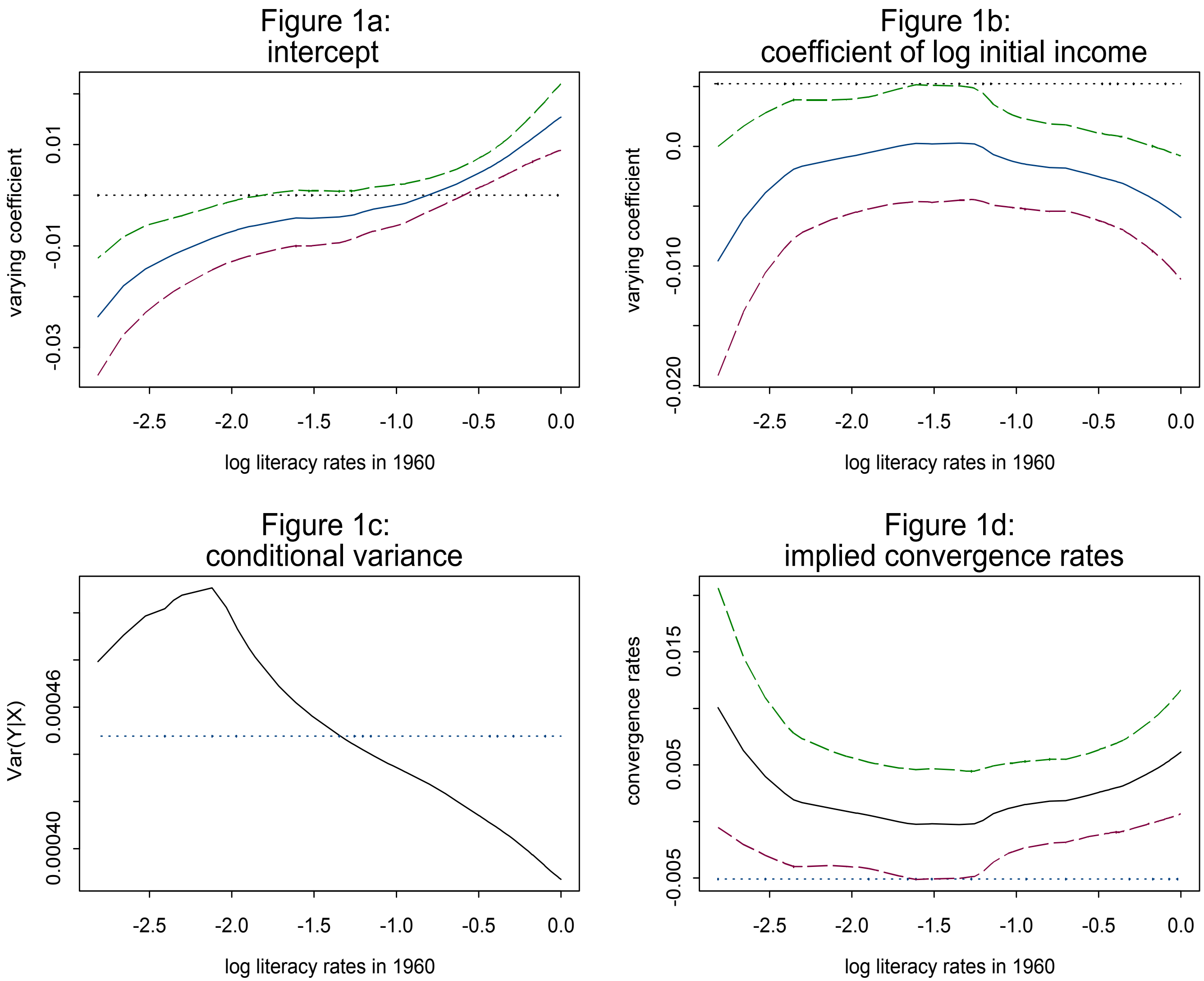
Finally, we point out that this chapter does not purport to make strong structural claims per se; rather, it shows that structural claims in the literature are exaggerated due to the failure of the linear model to account for parameter heterogeneity. That being said, future research should attempt to unify the new set of statistical or reduced form findings with growth theories to provide testable econometric models that can be used for policy analysis.

A first step toward that direction is to deal with model uncertainty. As Brock and Durlauf (2001), among others, have argued, the inherent openendedness of new growth theories presents unique challenges to researchers in exploring their quantitative consequences on growth. The statement that a particular theory of growth is empirically relevant does not logically preclude other theories of growth from also being relevant and therefore the inclusion or exclusion of growth variables may significantly alter previous conclusions.

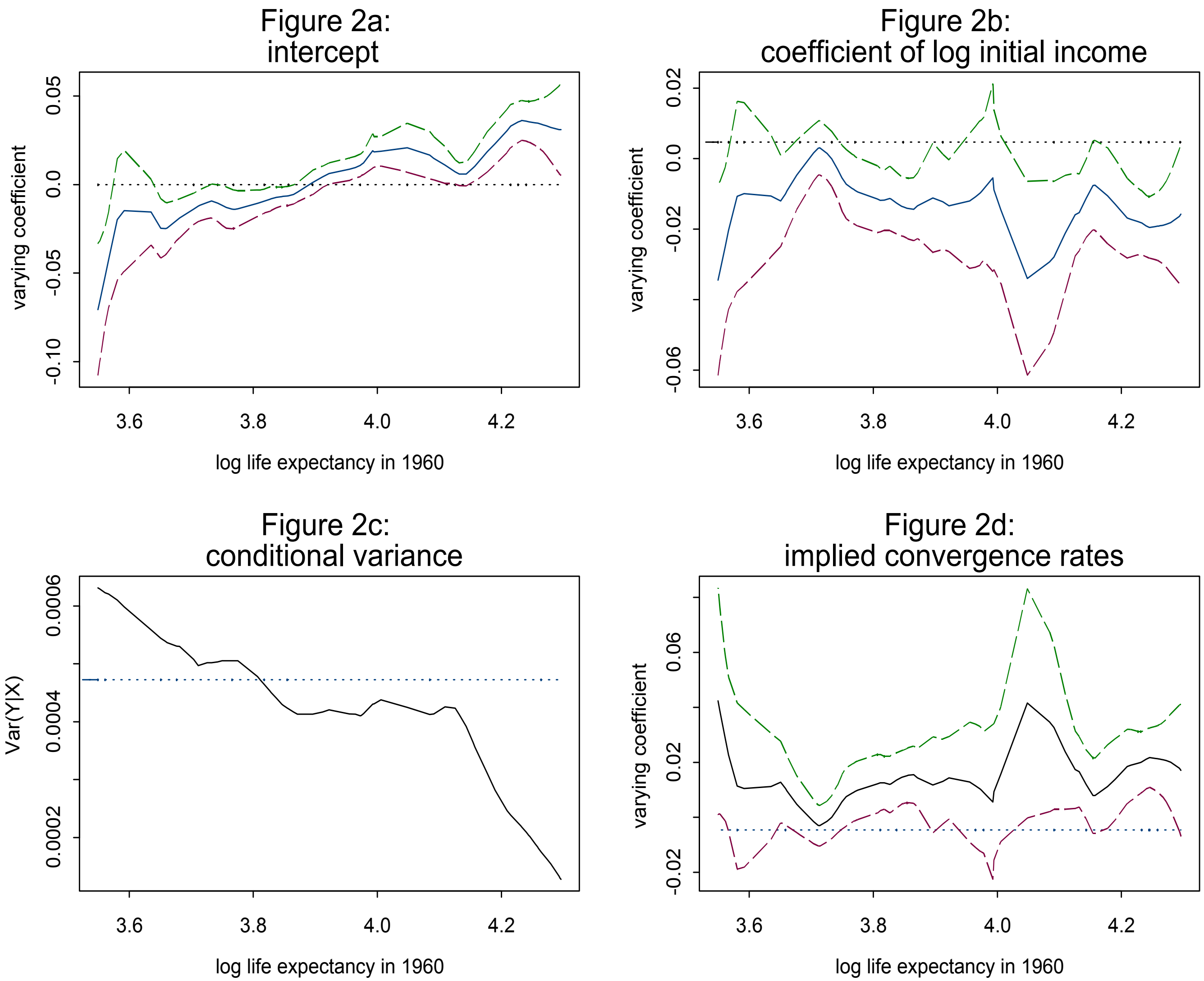
One appealing approach to deal with the problem of model uncertainty is to employ a Bayesian model averaging (BMA) by constructing estimates conditional on a model space with elements that span an appropriate range of determinants suggested by a large body of work. A number of recent papers have documented the advantages of using BMA in constructing robust estimates primarily in the context of the linear model (see, e.g., Brock and Durlauf (2001); Fernandez, Ley, and Steel (2001); Sala-I-Martin, X. and Doppelhofer, G. and Miller, R. (2004); Durlauf, Kourtellos, and Tan (2008); Masanjala and Papageorgiou (2008); Ciccone and Jarocinski (2010)). However, model averaging methods have yet to account for nonlinearities and parameter heterogeneity in a systematic way that deals with the problem of model uncertainty as a whole. Some initial attempts in this direction have been made by Brock and Durlauf (2001), Kourtellos, Tan, and Zhang (2007), and Cuaresma and Doppelhofer (2007). We expect this avenue of research to provide fruitful results.

Table 1: List of Countries and the logarithms of literacy rates and life expectancy in 1960

Code	Country	lit0	lifee0	Code	Country	lit0	lifee0
ARG	Argentina	-0.09	4.18	JOR	Jordan	-1.14	3.85
AUS	Australia	-0.02	4.26	JPN	Japan	-0.02	4.22
AUT	Austria	-0.02	4.23	KEN	Kenya	-1.61	3.81
BDI	Burundi	-1.97	3.73	KOR	Korea, Rep.	-0.35	3.99
BEL	Belgium	-0.02	4.24	LKA	Sri Lanka	-0.29	4.13
BEN	Benin	-2.53	3.66	LUX	Luxembourg	-0.02	-
BFA	Burkina Faso	-4.20	3.59	LSO	Lesotho	-	3.86
BGD	Bangladesh	-1.53	3.78	MAR	Morocco	-1.97	3.85
BOL	Bolivia	-0.95	3.76	MDG	Madagascar	-1.02	3.71
BRA	Brazil	-0.49	4.01	MEX	Mexico	-0.43	4.05
BRB	Barbados	-0.06	-	MLI	Mali	-3.51	3.58
CAN	Canada	-0.07	4.26	MOZ	Mozambique	-2.53	3.56
CHE	Switzerland	-0.01	4.27	MUS	Mauritius	-0.50	4.08
CHL	Chile	-0.17	4.05	MWI	Malawi	-1.51	3.63
CHN	China	-	3.59	MYS	Malaysia	-0.63	3.99
CIV	Cote d'Ivoire	-3.00	3.68	NER	Niger	-4.61	3.57
CMR	Cameroon	-1.66	3.77	NGA	Nigeria	-1.90	3.68
COG	Congo, Rep.	-1.86	3.86	NIC	Nicaragua	-0.70	3.86
COL	Colombia	-0.46	3.97	NLD	Netherlands	-0.02	4.29
CRI	Costa Rica	-0.17	4.13	NOR	Norway	-0.01	4.30
DNK	Denmark	-0.01	4.28	NPL	Nepal	-2.41	3.65
DOM	Dominican Republic	-0.44	3.96	NZL	New Zealand	-0.02	4.26
DZA	Algeria	-2.30	3.86	PAK	Pakistan	-1.90	3.77
ECU	Ecuador	-0.39	3.98	PAN	Panama	-0.31	4.11
EGY	Egypt, Arab Rep.	-1.35	3.84	PER	Peru	-0.49	3.87
ESP	Spain	-0.14	4.23	PHL	Philippines	-0.33	3.97
ETH	Ethiopia	-2.81	3.74	PRT	Portugal	-0.48	4.15
FIN	Finland	-0.01	4.23	PRY	Paraguay	-0.29	4.16
FRA	France	-0.02	4.25	SEN	Senegal	-2.03	3.66
GAB	Gabon	-2.12	3.71	SLV	El Salvador	-2.66	3.45
GBR	United Kingdom	-0.04	4.26	SWE	Sweden	-0.01	4.29
GHA	Ghana	-1.31	3.81	SYR	Syrian Arab Republic	-1.20	3.91
GMB	Gambia, The	-2.81	3.48	TCD	Chad	-	3.55
GNB	Guinea-Bissau	-2.66	3.55	TGO	Togo	-2.30	3.68
GRC	Greece	-0.21	4.23	THA	Thailand	-0.39	3.96
GTM	Guatemala	-1.16	3.83	TTO	Trinidad and Tobago	-0.07	4.16
HKG	Hong Kong, China	-0.35	4.19	TUR	Turkey	-0.97	3.92
HND	Honduras	-0.80	3.85	TZA	Tanzania	-2.35	3.70
IDN	Indonesia	-0.94	3.73	UGA	Uganda	-1.05	3.77
IND	India	-1.27	3.75	URY	Uruguay	-0.10	4.21
IRL	Ireland	-0.04	4.24	USA	United States	-0.02	4.25
IRN	Iran, Islamic Rep.	-1.71	3.83	VEN	Venezuela	-0.46	4.09
ISL	Iceland	0.00	-	ZAF	South Africa	-0.56	3.90
ISR	Israel	-0.17	4.27	ZMB	Zambia	-1.26	3.73
ITA	Italy	-0.09	4.24	ZWE	Zimbabwe	-0.37	3.82
JAM	Jamaica	-0.20	4.14				



Figures 1a - 1d show the varying coefficients of the unconditional local Solow model using $z = lit_0$



Figures 2a - 2d show the varying coefficients of the unconditional growth regression model using $z = lifee_0$

Figure 3a:
intercept

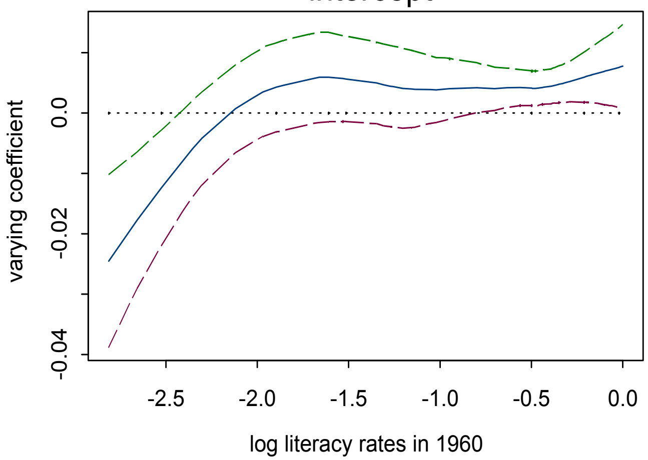


Figure 3b:
coefficient of log population growth rates

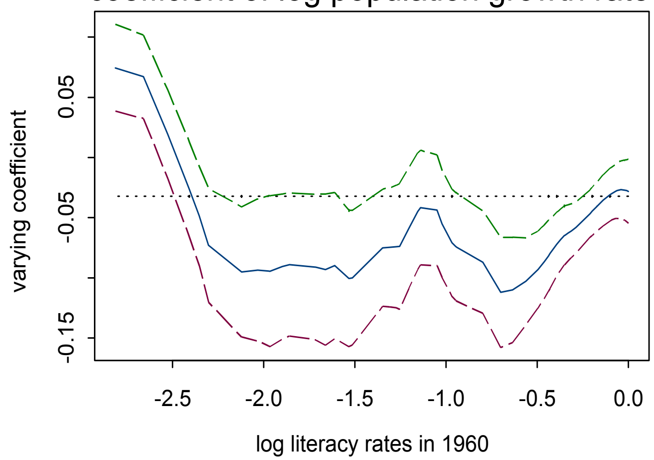


Figure 3c:
coefficient of log investments

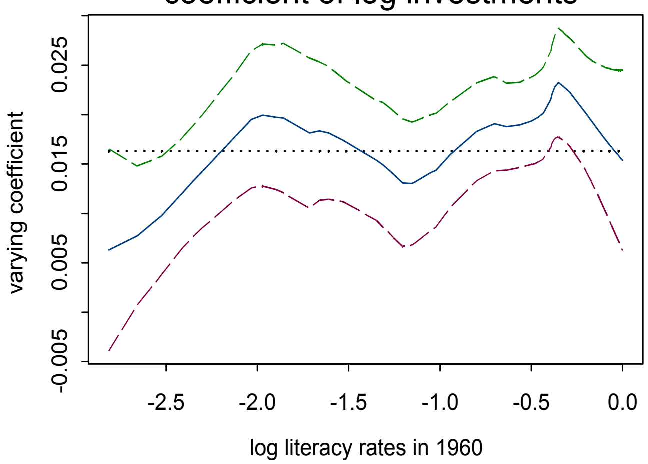
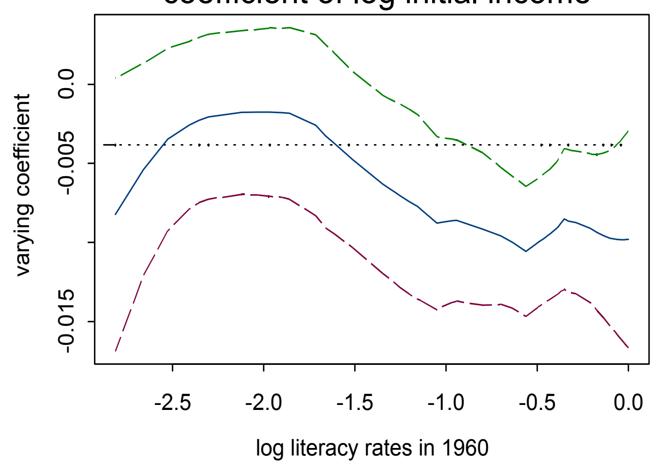
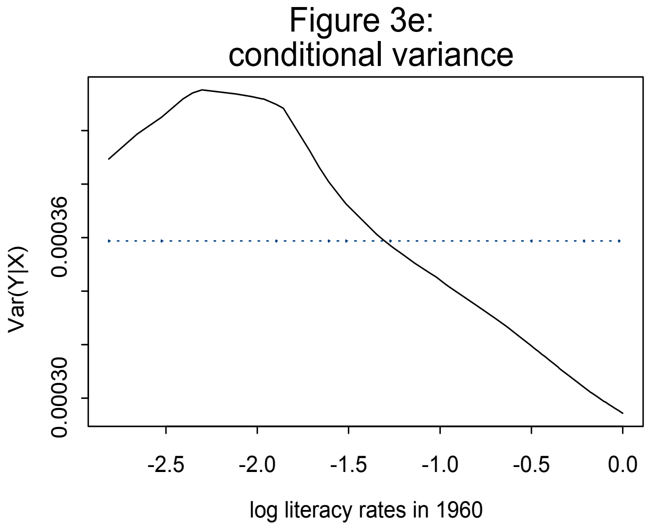
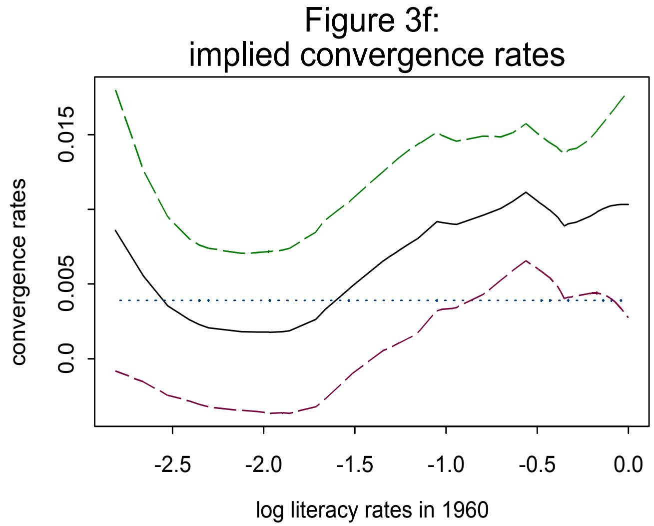
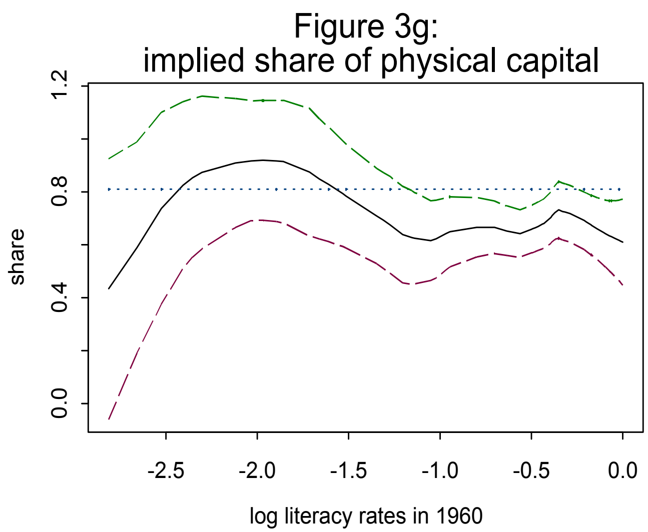


Figure 3d:
coefficient of log initial income









Figures 3a - 3g show the varying coefficients of the local Solow model using $z = lit_0$

Figure 4a:
intercept

Figure 4b:
coefficient of log population growth rates

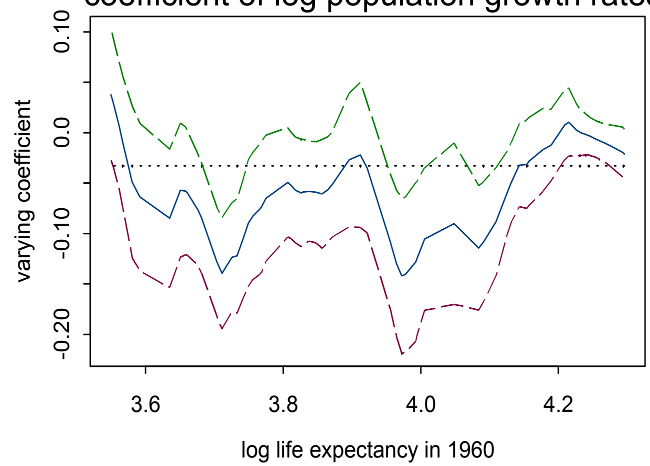


Figure 4c:
coefficient of log investments

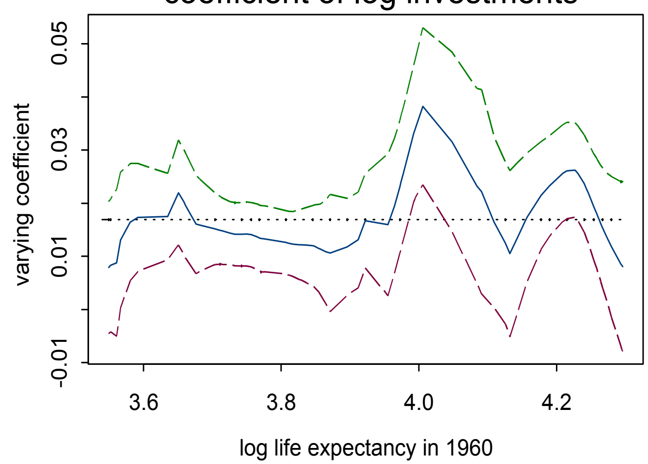


Figure 4d:
coefficient of log initial income

Figure 4e:
conditional variance

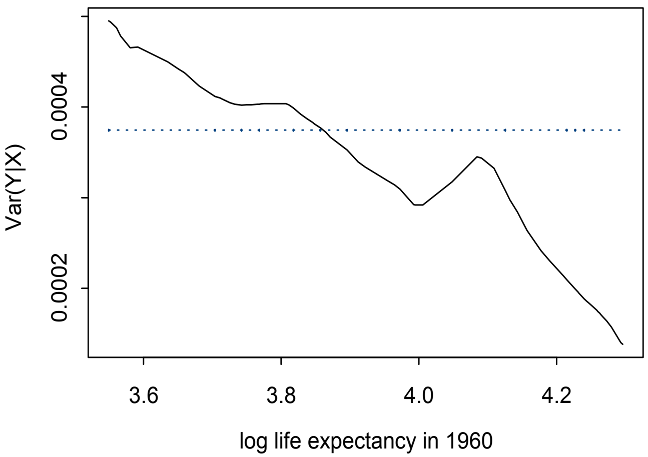


Figure 4f:
implied convergence rates

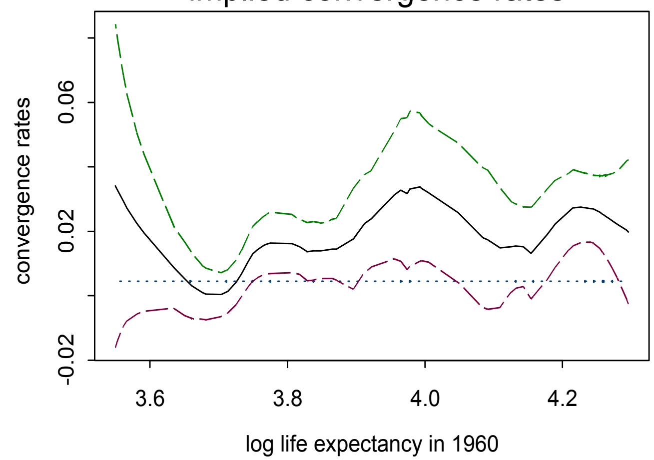
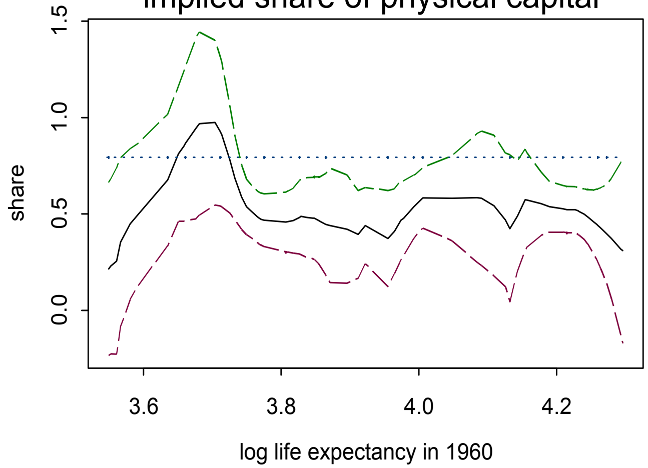
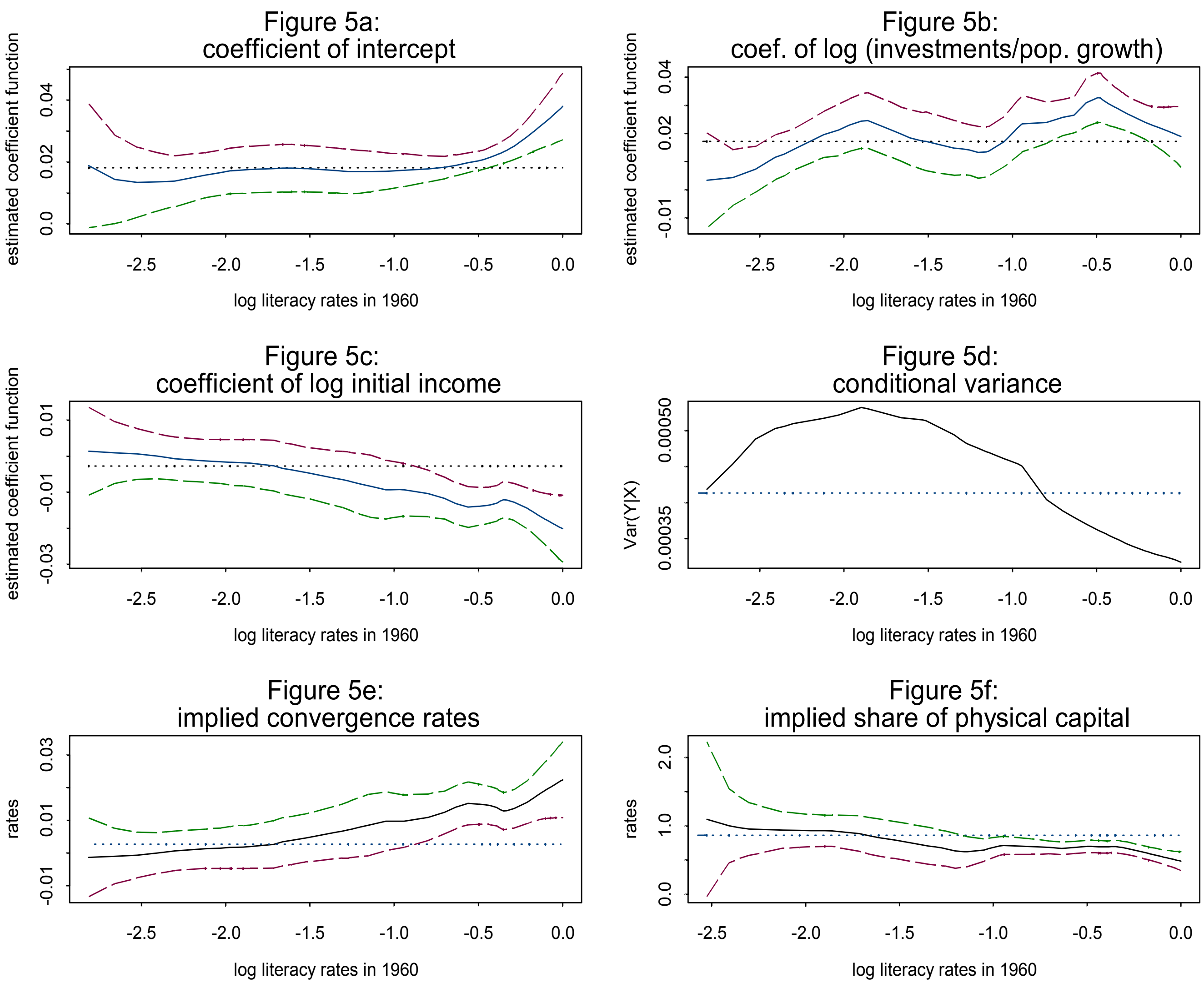


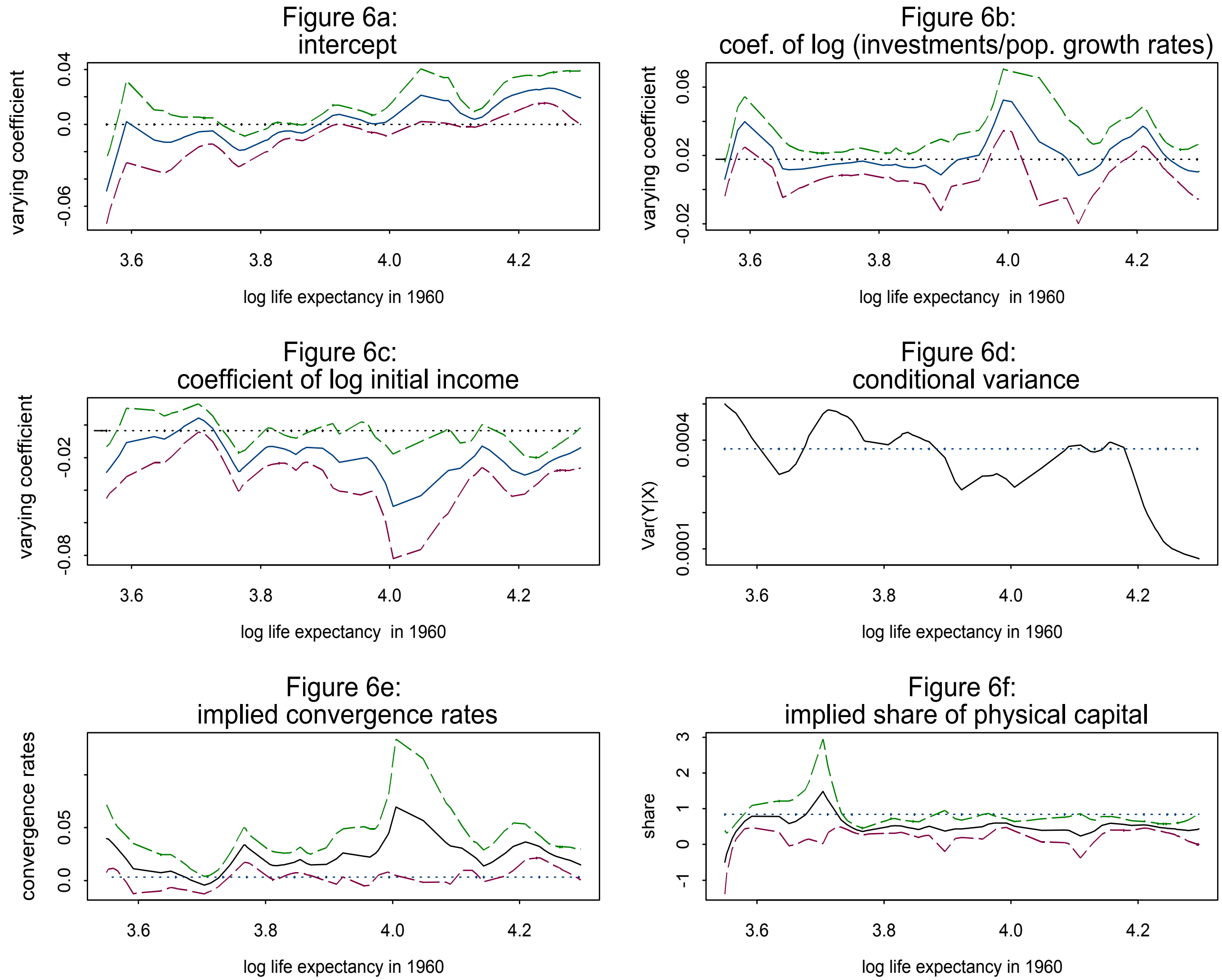
Figure 4g:
implied share of physical capital



Figures 4a - 4g show the varying coefficients of the local Solow model using $z = lifee_0$



Figures 5a - 5f show the varying coefficients of the restricted local Solow model using $z = lit_0$



Figures 6a - 6f show the varying coefficients of the restricted local Solow model using $z = lifee_0$

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Appendix

Following Fan and Zhang (1999) this paper adopts a two-stage estimation procedure based on a simple local regression. The major advantage of a two-stage estimation over a one-stage estimation is that it allows the functional coefficients to possess different degrees of smoothness that ensure that the optimal rate of convergence for the asymptotic mean-squared error is achieved.

A one stage estimation solves a simple weighted local least squares problem. More precisely, for each given point z_0, the functions $\gamma_j(z)$, $j = 1, \ldots p$, are approximated by local linear polynomials

$$\gamma_j(z) \approx c_{j0} + c_{j1}(z - z_0) \tag{A1}$$

for sample points z in a neighborhood of z_0. This approximation results in the following weighted local least squares problem:

$$\underset{\{(c_{j0},c_{j1})\}}{min} \sum_{i=1}^{N} \left[g_i - \sum_{j=1}^{p} \left[c_{j0} + c_{j1}(z - z_0)\right] X_{ij} \right]^2 K_h(z_i - z_0) \tag{A2}$$

where $K_h(\cdot) = \frac{1}{h} K\left(\frac{\cdot}{h}\right)$ and $K(\cdot)$ is the Epanechnikov kernel.

Let $\mathbf{g} = (g_1, \ldots, g_N)'$, $\mathbf{W} = diag\left(\frac{1}{h}K(\frac{z_1 - z_0}{h}), \ldots, \frac{1}{h}K(\frac{z_N - z_0}{h})\right)$, and

$$\mathbf{X} = \begin{pmatrix} X_{11} & (z_1 - z_0)X_{11} & \cdots & X_{1p} & (z_1 - z_0)X_{1p} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ X_{N1} & (z_N - z_0)X_{N1} & \cdots & X_{Np} & (z_N - z_0)X_{Np} \end{pmatrix} \tag{A3}$$

The solution of the problem (A2) is then given by

$$\widehat{\gamma}_j(z) = e'_{2j-1,2p}(\mathbf{X'WX})^{-1}\mathbf{X'Wg} \tag{A4}$$

where $e_{k,m}$ denote the unit vector of length m with 1 at the k_{th} position.

The conditional variance is estimated by a normalized weighted residual sum of squares

$$\widehat{\sigma}^2(z) = \frac{\sum_{i=1}^{N} (g_i - \widehat{g}_i)^2 K_h(z_i - z)}{tr\{\mathbf{W} - \mathbf{WX}(\mathbf{X'WX})^{-1}\mathbf{X'W}\}} \tag{A5}$$

where

$$\widehat{\mathbf{g}} = (\widehat{g}_1, \ldots, \widehat{g}_N)' = \mathbf{X}(\mathbf{X'WX})^{-1}\mathbf{X'Wg} \tag{A6}$$

In the first stage of a two-stage procedure, we obtain initial estimates $\widehat{\gamma}_{j,0}(z)$ using $h = h_0$, for $j = 1, 2, .., p$; see equation (A4). In the second stage, a two-stage estimate

$\widehat{\gamma}_{j,2}(z)$ is obtained by replacing the unknown varying coefficient $\gamma_k(z)$, for $k \neq j$ into the local least-squares (A2) by their initial estimates $\widehat{\gamma}_k(z)$, for $k \neq j$. Then a local least-squares regression is fitted again by minimizing[4]

$$\sum_{i=1}^{N} \left[g_i - \sum_{k \neq j}^{p} \widehat{\gamma}_{k,0}(z) X_{ik} - [c_{j0} + c_{j1}(z - z_0)] X_j \right]^2 K_{h_{j,2}}(z_i - z_0) \tag{A7}$$

This paper employs the cross-validation to select both the initial and the two-step bandwidths. However, the initial bandwidth h_0 is chosen so that the estimate is undersmoothed. In particular, the optimal rates of convergence for estimating the two-stage coefficient is achieved when the optimal $h_{j,2}$ is of the order $O(N^{-1/9})$ and the initial bandwidth[5] is between $O(N^{-1/3})$ and $O(N^{-2/9})$. In practice, we choose the initial bandwidth to ensure that the bias of the initial estimator is small and that makes the two-step estimator not sensitive to the choice of the initial bandwidth; see Fan and Zhang (1999).

By defining

$$\mathbf{A}_j = e'_{2j-1,2p} \left(\mathbf{X}'_j \mathbf{W}_j \mathbf{X}_j \right)^{-1} \left(\mathbf{X}'_j \mathbf{W}_j \mathbf{B}_j \right) \tag{A8}$$

the two-step estimator can be written in the familiar form of

$$\widehat{\gamma}_{j,2} = \mathbf{A}_j \mathbf{g} \tag{A9}$$

where $\mathbf{X}_j$ denotes the matrix $\mathbf{X}$ with only those columns that refer to the variable j, $\mathbf{W}_j$ is the diagonal weight matrix $\mathbf{W}$ with $h = h_{j,2}$, and $\mathbf{B}_j$ is the $N \times N$ matrix of some complicated weights

$$\mathbf{B}_j = I_N - \sum_{k \neq j}^{p} \begin{pmatrix} X_{1k} e'_{2j-1,2p} (\mathbf{X}'_{(1)} \mathbf{W}_{(1)} \mathbf{X}_{(1)})^{-1} \mathbf{X}'_{(1)} \mathbf{W}_{(1)} \\ \vdots \\ X_{1k} e'_{2j-1,2p} (\mathbf{X}'_{(N)} \mathbf{W}_{(N)} \mathbf{X}_{(N)})^{-1} \mathbf{X}'_{(N)} \mathbf{W}_{(N)} \end{pmatrix}$$

where $\mathbf{X}_{(i)}$ and $\mathbf{W}_{(i)}$ are the matrices $\mathbf{X}$ and $\mathbf{W}$ with $z_0 = z_i$, respectively. The asymptotic confidence intervals for the two-stage estimator $\widehat{\gamma}_{j,2}(z)$ are based on the asymptotic approximation of the variance given by

$$\mathbf{A}_j \mathbf{A}'_j \widehat{\sigma}^2(z) \tag{A10}$$

where $\widehat{\sigma}^2(z)$ is the estimate of the corresponding conditional variance.

[4]In theory a local cubic fit should be used in the second step. In practice, however, it is not substantially different from the local linear fit to justify the extra computational burden.

[5]In practice, we use $h_0 = 0.5\widehat{h}$, where $\widehat{h}$ is the optimal h for one-stage estimation.