

# Fiscal Policy and Asset Prices with Incomplete Markets\*

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# Fiscal Policy and Asset Prices with Incomplete Markets

## **Abstract**

We study the impact of fiscal policy decisions on macroeconomic activity, cross-sectional wealth distribution and asset prices in a unified framework. For a given level of government expenditures, an increase in the ratio of government debt to GDP by (approximately) 7.5% decreases the steady-state capital stock between 1.7% and 2.4%, depending on how the new debt is financed. Government bond yields rise between 23 to 32 basis points and expected stock returns increases between 14 to 32 basis points, thereby marginally decreasing the equity risk premium. We show the importance of treating the capital stock and government bonds as imperfect substitutes, for measuring the impact of fiscal policy shocks. More precisely, household portfolio re-allocation decisions are an important determinant of the impact of these shocks on the real economy and asset prices. We also show that the crowding-out effect driven by the tightening of liquidity constraints is negligible.

JEL Classification: E21, E62, G12.

Key Words: Government Debt, Risk Premium Household Heterogeneity, Overlapping Generations, Incomplete Risk Sharing, Limited Stock Market Participation.

# 1 Introduction

What are the effects of changes in taxation and government debt on macroeconomic activity, wealth inequality and asset prices? We study fiscal policy decisions in a general equilibrium model with incomplete markets, heterogeneous agents and where government debt and capital are imperfect substitutes. Markets are incomplete due to both aggregate uncertainty and idiosyncratic productivity shocks. The idiosyncratic shocks are not perfectly diversifiable due to the presence of borrowing constraints. Our results show that imperfect asset substitution is an extremely important feature of the analysis. Models where the return on capital and the interest rate on government bonds are identical, will either significantly underestimate the former, or overestimate the latter, or both. Typically since this return is calibrated to match the return on capital, those models strongly exaggerate the cost of government debt. This is an important limitation since our results identify the portfolio re-allocation behavior of households (asset substitution) as an important channel for determining the impact of fiscal policy decisions on capital accumulation, aggregate economic activity and asset prices.

Our model presents a unified framework for studying the quantitative impact of fiscal policy on macroeconomic activity, the cross-sectional wealth distribution and asset prices. As a result, our assessment explicitly takes into account the important links between these different elements, and how they might interact in reaction to policy decisions. Before discussing our results, it is important to state that the analysis in this paper is *not* normative. The goal of this paper is to provide a quantitative assessment of the impact of changing government debt and taxes on economic outcomes.

We start by identifying the important economic mechanisms in the context of an infinite-horizon model, where all agents are ex-ante identical but receive different idiosyncratic shocks and face borrowing constraints. Next, we present an overlapping generations model that replicates quantitatively features of the observed cross-sectional dispersion in wealth and consumption in the U.S.. In the overlapping generations model there is less risk sharing, and as a result the equilibrium risk premium is higher. Moreover, this model will also capture another important empirical fact: a significant fraction of households do not participate in the stock market, either directly or through pension funds. Furthermore, non-participation is much more pervasive among poor households,<sup>1</sup> and the model is also calibrated to match this fact.

In our analysis we consider different fiscal policy experiments. Because government expenditures

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<sup>1</sup>For example, in the 2001 SCF the overall participation rate is 45% and it is 88.84% among households with wealth above the median, and only 15.21% for those with wealth below the median.

do not play an explicit role in the model, we first focus on steady-state compensating changes in tax rates and government debt to satisfy the intertemporal government budget constraint. The model includes the three major sources of taxation: labor income taxes, capital income taxes and sales/consumption taxes. For tractability reasons, we do not include a household labor supply decision, and therefore we will refer to taxes on labor income as lump-sum taxes (because they do not affect the labor-leisure choice). We find that, for a given level of government expenditures, an increase in the ratio of government debt to GDP of approximately 7.5% causes a permanent reduction in the capital stock of between 1.7% and 2.4%, depending on how the new debt is financed.<sup>2</sup>

To induce households to hold the extra government debt, the interest rate on government bonds increases by approximately 25 basis points. It is hard to find identical empirical counterparts for these numbers, but they are similar to the values reported by Engen and Hubbard (2004), Laubach (2008), Greenwood and Vayanos (2010a, 2010b) and Krishnamurthy and Vissing-Jorgensen (2010), although the semi-elasticities implied the estimates in Engen and Hubbard (2004) are slightly higher and closer to 60 basis points.<sup>3</sup> Krishnamurthy and Vissing-Jorgensen (2010) focus on how debt to GDP changes affect credit spreads (corporate to treasuries, for example), while Greenwood and Vayanos (2010b) focus on the relative supply of government bonds by maturity and how this variable affects bond returns. Our results can be interpreted as being consistent with the idea that changes in debt to GDP (the supply of government bonds) affect bond yields, and therefore likely to contribute to what Krishnamurthy and Jorgensen (2010) term the “safety premium”. For tractability reasons we do not include defaultable corporate bonds or bonds of different maturities and therefore we cannot exactly replicate the regressions in Krishnamurthy and Vissing-Jorgensen (2010) or Greenwood and Vayanos (2010b), respectively. However, an advantage of our approach is that we can isolate how truly exogenous changes in government debt affect bond yields through the posited structural model. In that respect, our results are more directly comparable to Greenwood and Vayanos (2010a) who use event-study analysis through debt buyback policies to illustrate how changes in the supply for long term bonds affect bond yields. Their results are consistent with what we find through an incomplete markets structural model. We therefore view our analysis as complementary to this empirical literature.

The changes in the cost of capital (expected stock returns) in our economy are of a similar order of magnitude, ranging from 14 to 32 basis points, with the effects being larger when the increase

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<sup>2</sup>We write “approximately 7.5%” since naturally the increase in debt to GDP ratio is endogenous. In the model we calibrate an exogenous increase in government debt, and compute the corresponding endogenous change in debt to GDP, which is almost exactly 7.5%.

<sup>3</sup>In a monetary policy context, Krishnamurthy and Vissing-Jorgensen (2011) argue that the specific impact of a change in interest rates is also strongly dependent on the specific mechanism through which this is implemented.

in government debt is associated with an eventually higher tax rate on capital. In most of our experiments, both the one year mean bond yield and the expected stock return rise, with the bond yield typically rising slightly more than the return on capital. As a result, the mean equity risk premium slightly decreases in most experiments. Despite the small impact of fiscal policy decisions on risk premia, we show that it is very important to study the effects of fiscal policy decisions in a model with non-trivial financial markets. More precisely, we show that, when we account for the fact that capital and government bonds are not perfect substitutes, the quantitative impact of fiscal policy decisions is significantly altered, relative to an otherwise identical model. When the two assets earn different rates of return there is an additional important channel in the model: the *asset substitution channel* resulting from the portfolio re-allocation behavior of households. To illustrate this effect it is easier to consider the case of lump-sum taxes. Lump-sum taxes correspond to negative riskless bond holdings, with the tax payments behaving like fixed coupon payments. In a model where bonds and equity are not perfect substitutes, when lump-sum taxes increase households must compensate for this decrease in their implicit holdings of riskless assets (human wealth) by decreasing equity holdings and replenishing their explicit riskless assets. In equilibrium this results in a lower level of the capital stock. We show that this effect is quantitatively very large. A 20% increase in the ratio of government debt to GDP decreases the capital to GDP ratio by 1.1% if the interest payments are financed by higher lump-sum taxes.<sup>4,5</sup>

Naturally, even if the two assets are perfect substitutes, lump-sum taxes still affect capital accumulation because of the presence of liquidity constraints. However, we solve such an economy and show that this effect is approximately five times smaller than the previously discussed asset substitution channel. Intuitively, the households that are most affected by these constraints own a very small fraction of the capital stock. Therefore, the previous numbers are mostly driven by the portfolio re-allocation channel, and not by the *direct* liquidity constraints channel. It is important to clarify that this result does not negate the importance of borrowing constraints in the model. In fact, without them the equity premium would be much smaller and the risk free rate much larger<sup>6</sup>. Therefore, although their impact alone is very small, the importance of the asset

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<sup>4</sup>Angeletos and Panousi (2008) also obtain a crowding-out effect from lump-sum taxes in a model with incomplete markets and entrepreneurial investment. In their model higher lump sum taxes lower the capital stock through a reduction in risk taking by entrepreneurs (who face undiversifiable, idiosyncratic investment risk).

<sup>5</sup>Elmendorf and Kimball (2000) analyze (in a two period, partial equilibrium model) a different effect from redistributing labor income taxes across time, namely that under certain conditions revenue-neutral deferral of taxes and higher taxation reduce labor income risk and lead to higher investment in the risky asset.

<sup>6</sup>Our model can be interpreted as a large scale model of Constantinides, Donaldson and Mehra (2002) albeit with a production sector. They show how the risk free rate would be larger without the presence of liquidity constraints in the early part of the life cycle.

substitution channel is strongly affected by their existence (or any other mechanism that helps to deliver the equity premium). In the final part of the paper we also consider the impact of temporary government expenditures shocks, financed by changes in tax rates.

Our model is also part of the literature studying fiscal policy decisions in a production economy setting. Baxter and King (1993) and Ludvigson (1996) consider infinite-horizon representative-agent models with and without aggregate uncertainty. Aiyagari (1995), Aiyagari and McGrattan (1998), Floden (2001) and Conesa, Kitao, and Krueger (2007) study economies with heterogeneous agents, idiosyncratic shocks and borrowing constraints, but without aggregate uncertainty. Domeij and Heathcote (2004) study capital gains tax reform with a transition between steady states. All of these models do not capture the asset substitution channel discussed in our paper since, in these economies, government bonds earn the same rate of return as the capital stock. Chari, Christiano and Kehoe (1994) characterize optimal fiscal policy in a model with heterogeneous agents and aggregate uncertainty.<sup>7</sup> However, in their set-up, idiosyncratic risk is perfectly diversifiable, allowing them to determine the optimal allocations by solving the corresponding Ramsey problem. Most of these papers, however, incorporate a labor-leisure decision which is absent in our analysis, but on the other hand, they do not consider limited stock market participation.

The closest paper to ours is probably Heathcote (2005), who also considers an incomplete markets production economy with heterogeneous agents, aggregate uncertainty, and no labor supply decision.<sup>8</sup> As in our model, incomplete markets arise because of idiosyncratic productivity shocks and liquidity constraints. However, in his set-up, aggregate uncertainty is exclusively driven by tax rate shocks and therefore capital and government bonds are perfect substitutes. Our results that government debt has real effects on bond yields can also be interpreted as a clientele effect, similar to the recent work by Guibaud et. al. (2010): the total supply of government debt affects how households view their implicit holdings of (relatively) riskless securities in the form of non-tradeable idiosyncratic labor income. In addition, our work complements the models studying the impact of uncertainty about (future) fiscal policy decisions on asset prices (Croce, Kung and Schmid (2010) and Pastor and Veronesi (2010)). Finally, since ours is not a normative analysis, our results are unrelated to the discussion on the optimal level of capital income taxation or tax smoothing considerations.

The paper is structured as follows. In section 2 we discuss the model with infinitely lived agents and consider cases with and without aggregate uncertainty. Section 3 outlines the OLG model, its

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<sup>7</sup>Shin (2006) considers a similar set-up in an economy without capital.

<sup>8</sup>As a result, he also uses the methodology developed by Krusell and Smith (1998) and den Haan (1997) for solving the model.

calibration and discusses the baseline results. Section 4 studies the impact of fiscal policy decisions for a given level of government expenditures in the OLG model, and section 5 traces out the impulse responses to temporary tax shocks. Section 6 provides the concluding remarks. Technical details of the computational procedure are provided in the appendix.

## 2 Infinite-Horizon Models

Our baseline quantitative model will feature overlapping generations with limited stock market participation and heterogeneous preferences, since that model can account well for life-cycle consumption, saving and portfolio choices, asset prices, macroeconomic variables and cross-sectional distributions of wealth and consumption in the data. Nevertheless, the qualitative predictions are the same, and easier to understand, in simpler, infinite-horizon models without preference heterogeneity and limited participation. Therefore, we first consider a fairly standard growth model with infinitely lived households. In this simpler model households receive wage income, subject to uninsurable idiosyncratic shocks, against which they cannot borrow. Two alternative assets exist for intertemporal consumption smoothing: the risky capital stock (equity) and a (one-period) riskless government bond. Firms are perfectly competitive and combine capital and labor using a constant returns to scale technology to produce a non-durable consumption good. The government taxes wages, capital income and consumption to finance government expenditures and the interest payments on public debt. It is well-known in the literature that such a model will find it hard to match simultaneously important macroeconomic variables and asset returns. However, we emphasize that achieving such an ambitious goal is not the point at this stage of our analysis. Instead, this model simply serves as a starting point for understanding the interaction between household decisions and fiscal policy and builds intuition behind our main results in a relatively transparent way.

We will consider two versions of this infinite horizon model: one where the capital stock and government debt are perfect substitutes due to the absence of aggregate uncertainty (Aiyagari (1994)), and another where capital is riskier than government debt due to the presence of aggregate uncertainty (an extended version of Krusell and Smith (1997)). The latter model nests the former and thus, for brevity, we only describe the model with aggregate uncertainty, noting the relevant differences where appropriate.

## 2.1 Production technology

### 2.1.1 Production function

The technology in the economy is characterized by a standard Cobb-Douglas production function, with total time- $t$  output given by

$$Y_t = Z_t K_t^\alpha L_t^{1-\alpha} \quad (1)$$

where  $K$  is the total capital stock in the economy,  $L$  is the total labor supply, and  $Z$  is a stochastic productivity which follows the process

$$\begin{aligned} Z_t &= G_t U_t \\ G_t &= (1 + g)^t \end{aligned}$$

Secular growth in the economy is determined by the constant  $g$  ( $>0$ ), while the productivity shocks  $U_t$  are stochastic. In the model without aggregate uncertainty we set  $U_t = 1$ .

Firms make decisions after observing aggregate shocks. Therefore, they solve a sequence of static maximization problems with no uncertainty, and factor prices (wages,  $W_t$ , and return on capital,  $R_t^K$ ) are given by their first-order conditions

$$W_t = (1 - \alpha) Z_t (K_t/L_t)^\alpha \quad (2)$$

and

$$R_t^K = \alpha Z_t (L_t/K_t)^{1-\alpha} - \delta_t \quad (3)$$

where  $\delta_t$  is the depreciation rate. The depreciation rate is constant in the model without aggregate uncertainty and is stochastic in the extended model.

### 2.1.2 Stochastic depreciation

Standard frictionless production economies cannot generate sufficient return volatility, since agents can adjust their investment plans to smooth consumption over time (see Jermann (1998) or Boldrin, Christiano and Fisher (2001)). This usually motivates adjustment costs for capital, which create fluctuations in the price of capital and increase return volatility. Since we have incomplete markets, different stockholders have different stochastic discount factors. They will therefore disagree on the solution to the optimal intertemporal decision problem of the firm (see Grossman and Hart (1979)). This is not a concern here because there is no intertemporal dimension to the firm's problem, but introducing adjustment costs would change that.<sup>9</sup> Recent papers with production

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<sup>9</sup>Guvenen (2005) introduces adjustment costs in a model with restricted stock market participation, but in his model there is perfect risk sharing among stockholders. Therefore, there is a unique stochastic discount factor for pricing capital.

economies and incomplete markets have captured the effect of adjustment costs by assuming a stochastic depreciation rate for capital (e.g. Gomes and Michaelides (2008), Storesletten et al. (2007), Krueger and Kubler (2006), and Gottardi and Kubler (2004)). Here we follow the same route and assume that the depreciation rate is given by

$$\delta_t = \delta + s \times \eta_t \quad (4)$$

where  $\eta_t$  is an i.i.d. standard normal and  $s$  is a scalar. Therefore,  $\delta_t$  is a general measure of economic depreciation, combining physical depreciation, adjustment costs, capital utilization and investment-specific productivity shocks.<sup>10</sup> In the baseline case we assume that  $\eta_t$  is uncorrelated with the productivity shock  $U_t$ .

## 2.2 Government debt

The government's budget constraint is

$$B_{t+1} = (1 + R_t^B)B_t + G_t^c - T_t \quad (5)$$

where  $G^c$  is government consumption,  $B$  is public debt,  $R^B$  is the interest rate on government bonds, and  $T$  denotes tax revenues. Tax proceeds arise from proportional taxation on capital (tax rate  $\tau_K$ ), proportional taxation on labor (tax rate  $\tau_L$ ) and a proportional consumption tax (tax rate  $\tau_C$ ). In this type of models government debt can become non-stationary since  $B_{t+1}$  depends on  $B_t$  through a multiplication by a time-varying coefficient that is on average greater than one, since the riskless rate has a positive mean. As a result, if taxes and government consumption are stationary, then government debt becomes non-stationary. Moreover, it is not obvious what normalization may be used to make  $B_t$  stationary. One solution is offered by Heathcote (2005) who makes taxes (and household decisions) depend on government debt: high government debt relative to its long run average implies higher taxation. This requires the addition of one extra state variable in the model, and more importantly it imposes a restriction on the path of tax rates in response to other shocks in the economy.<sup>11</sup> To avoid these complications, and to gain a better understanding of the model's predictions, we instead assume that the government debt is constant over time with government consumption adjusting endogenously to satisfy (5) period-by-period.

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<sup>10</sup>Hercowitz (1986) and Greenwood, Hercowitz and Huffman (1988) use the same approach to model fluctuations in capital utilization, while Greenwood, Hercowitz and Krusell (1997) use it to model investment-specific technological shocks as a reduced form for vintage capital models.

<sup>11</sup>While still feasible in the setting without aggregate productivity or depreciation shocks, the computational burden of the additional state variable required by this method is a serious obstacle when we consider a model with aggregate shocks, either in this section or in the overlapping generations economy later in the paper.

## 2.3 Households and financial markets

### 2.3.1 Preferences

Households have CRRA preferences defined over a single non-durable consumption good. Let  $C_t$  denote consumption in period  $t$ , then preferences are defined by

$$V = E \sum_{t=1}^{\infty} \beta^{t-1} \frac{C_t^{1-\rho}}{1-\rho} \quad (6)$$

where  $\rho$  is the coefficient of relative risk aversion, and  $\beta$  is the discount factor.

### 2.3.2 Labor endowment

Let  $i$  index the households. All households supply labor inelastically, and are subject to idiosyncratic productivity shocks so that individual labor income ( $H_t^i$ ) is

$$H_t^i = W_t L_t^i \quad (7)$$

where  $L_t^i$  is the household's labor endowment (labor supply scaled by productivity), and  $W_t$  is the aggregate wage per unit of productivity. The household's labor productivity is log-normal, and i.i.d. with mean  $-0.5 * \sigma_L^2$  and variance  $\sigma_L^2$ .

### 2.3.3 Wealth accumulation

There are two financial assets: a one-period riskless asset (government bond), and a risky investment opportunity (capital stock). The riskless asset return is  $R_t^B = \frac{1}{P_{t-1}^B} - 1$ , where  $P^B$  denotes the government bond price. The return on the risky asset is denoted by  $R_t^K$ . In the model without aggregate uncertainty the return to capital is constant and equal to the return on the risk-free bond. Total liquid wealth (cash-on-hand,  $X_t^i$ ) can be consumed or invested in the two assets. At each time  $t$ , agents enter the period with wealth, either invested in the bond market,  $B_t^i$ , or in stocks,  $S_t^i$ , and receive  $L_t^i W_t$  as labor income. Thus,

$$(1 + \tau_C)C_t^i + K_{t+1}^i + B_{t+1}^i = X_t^i = K_t^i(1 + (1 - \tau_K)R_t^K) + B_t^i(1 + (1 - \tau_K)R_t^B) + L_t^i(1 - \tau_L)W_t \quad (8)$$

Households cannot borrow against their future labor income ( $B_t^i \geq 0$ ), and cannot short the risky asset ( $K_t^i \geq 0$ ).

In the presence of deterministic growth we need to normalize the non-stationary variables in this economy. This can be achieved by choosing the following normalization  $k_{t+1}^i = \frac{K_{t+1}^i}{G_t^{1-\alpha}}$ ,  $b_{t+1}^i = \frac{B_{t+1}^i}{G_t^{1-\alpha}}$ ,

$c_t^i = \frac{C_{it}}{G_t^{1-\alpha}}$ , and  $x_t^i = \frac{X_{it}}{G_t^{1-\alpha}}$ . Then, defining  $\omega_t = \left(\frac{G_t}{G_{t-1}}\right)^{\frac{1}{1-\alpha}}$  and  $w_t = \frac{W_t}{G_{t-1}^{1-\alpha}}$ , the individual budget constraint (8) becomes, after dividing through by the normalizing factor,

$$(1 + \tau_C)c_{it} + k_{it+1} + b_{it+1} = x_{it} = (1 + R_t^K(1 - \tau_K))\frac{k_{it}}{\omega_t} + (1 + R_t^B(1 - \tau_K))\frac{b_{it}}{\omega_t} + L^i w_t(1 - \tau_L)\frac{1}{\omega_t} \quad (9)$$

Labor taxes are non-distortionary in our model because there is no household labor-leisure decision. As a result we will preferentially refer to them as lump-sum taxes, which is what they effectively are. Naturally, it would also be interesting to include distortionary labor income taxes in the model, however this would require the inclusion of a labor supply decision, a substantial additional complexity in the presence of aggregate uncertainty. In addition, as we discuss below, models with labor taxes and endogenous labor supply face an important calibration problem, unless different complex features of the tax code are carefully modeled, making this an even more formidable computation task. Given the empirical evidence that the labor supply elasticity of prime-age males is very low, we view this as a useful benchmark for more complicated future models that might include those endogenous decisions.

## 2.4 Equilibrium

The equilibrium consists of endogenously determined prices (bond prices, wages, and equity returns), a set of value functions and policy functions, ( $\{V, b, k\}$ ), and rational expectations about the evolution of the endogenously determined variables, such that:

1. Firms maximize profits by equating marginal products of capital and labor to their respective marginal costs (2) and (3).
2. Individuals choose their consumption and asset allocation by maximizing (6).
3. Markets clear and aggregate quantities result from individual decisions. Specifically:

$$k_t = \int_i k_t^i di, \quad b_t = \int_i b_t^i di. \quad (10)$$

The aggregation equation for labor supply is redundant since there is no labor-leisure choice (aggregate labor supply is normalized to one). Once these two equations are satisfied, Walras' law implies that total expenditure (government consumption, investment, and household consumption) must equal total output:

$$c_t^G + k_{t+1} - \frac{(1 - \delta_t)k_t}{\omega_t} + \int_i c_t^i di = U_t k_t^\alpha L_t^{1-\alpha} \frac{(1 + g)}{\omega_t}. \quad (11)$$

4. The government budget [equation (5)] is balanced every period to sustain a given ratio of government debt to GDP. Specifically

$$b_{t+1} = c_t^G + \frac{1}{\omega_t} \times \{(1 + R_t^B)b_t - k_t R_t^K \tau_K - b_t R_t^B \tau_K - w_t \tau_L - \tau_C c_t\} \quad (12)$$

6. Market prices expectations are verified in equilibrium.

Analytical solutions to this problem do not exist and we therefore use a numerical solution method (details are given in Appendix A for the OLG model that nests the two infinite horizon models considered in this section).

### 2.4.1 The dynamic programming problem

In the presence of aggregate uncertainty the model is similar to Krusell and Smith (1997), with the addition of stochastic depreciation. Households are price takers and maximize utility given their expectations about future asset returns and aggregate wages. Under rational expectations, the latter are given by equations (2) and (3): returns and wages are determined by future capital and labor, and by the realizations of aggregate shocks. Labor supply is exogenous, as are the distributions of the aggregate shocks. The capital stock, however, is endogenous. Forming rational expectations of future returns and wages is, therefore, essentially equivalent to forecasting the future mean capital stock. As shown by Krusell and Smith (1998), for this “simple” class of incomplete-markets economies, it is possible to accurately forecast the one-period ahead capital stock using its current value ( $k_t$ ) and the state-contingent realizations of the two aggregate shocks (productivity shock,  $U_t$ , and stochastic depreciation,  $\eta_t$ ):

$$k_{t+1} = \Gamma_K(k_t, U_t, \eta_t) \quad (13)$$

Since government bonds are only riskless over one period, households must forecast future bond prices ( $P_t^B$ ). The forecasting rule for  $P_t^B$  is

$$P_{t+1}^B = \Gamma_P(P_t^B, k_t, U_t, \eta_t) \quad (14)$$

This process introduces four additional state variables in the individual’s maximization problem ( $P_t^B$ ,  $k_t$ ,  $U_t$ , and  $\eta_t$ ).

The individual optimization problem now becomes:

$$V(x_t^i; k_t, U_t, \eta_t, P_t^B) = \underset{\{k_{t+1}^i, b_{t+1}^i\}}{Max} \left\{ \frac{c_t^{i1-\rho}}{1-\rho} + \beta E_t [(\omega_{t+1})^{1-\rho} V(x_{t+1}^i; k_{t+1}, U_{t+1}, \eta_{t+1}, P_{t+1}^B)] \right\} \quad (15)$$

subject to the constraints,

$$\begin{aligned} k_{t+1}^i &\geq 0 \\ b_{t+1}^i &\geq 0 \\ (1 + \tau_C)c_t^i + b_{t+1}^i + k_{t+1}^i &= x_t^i \end{aligned}$$

and with the laws of motion,

$$\begin{aligned} x_{t+1}^i &= \frac{1}{\omega_{t+1}} \times [k_{t+1}^i(1 + (1 - \tau_K)R_{t+1}^K) + b_{t+1}^i(1 + (1 - \tau_K)R_{t+1}^B) + L^i(1 - \tau_L)w_{t+1}] \\ R_{t+1}^K &= R(k_{t+1}, U_{t+1}, \delta_{t+1}) \\ w_{t+1} &= W(k_{t+1}, U_{t+1}) \\ k_{t+1} &= \Gamma_K(k_t, U_t, \eta_t) \\ P_{t+1}^B &= \Gamma_P(k_t, U_t, \eta_t, P_t^B) \end{aligned}$$

## 2.5 Calibration

Decisions are made at an annual frequency. The calibration procedure is described in detail in section 3.2 when considering the OLG model, since that is the one that we ultimately want to consider as our baseline economy. Here we simply pick the same (when applicable) structural parameters as in the OLG baseline model. There is a single group of households with  $\rho = 5$  and deterministic growth is set at 1% ( $G = 1.01$ ).<sup>12</sup> In the model with aggregate uncertainty the parameter  $s$  (the stochastic depreciation volatility) determines the return of equity volatility and is set at 15%, while the aggregate productivity shock follows a two-state Markov Chain with a standard deviation of 2.5%, and with the transition probability of changing the state set to 0.4. Capital's output share ( $\alpha$ ) is set to 34%, and the average annual depreciation rate ( $\delta$ ) is 8%. The capital income tax rate is set at 40%, the labor income tax rate to 10% and the consumption tax rate at 13%. The aggregate supply of bonds is equal to 36% of GDP, as in the data.

One main difference between the OLG and the infinite horizon models is the idiosyncratic labor income process. In this version of the model, all shocks are transitory. We make this choice to be able to understand the predictions of the model in a relatively simple setting. In the OLG economy, we introduce separate permanent and transitory shocks, a deterministic hump in labor income and a social security system. Deaton (1991) and Carroll (1992) estimate volatilities of 8% and 10% for permanent and transitory shocks, respectively. Heaton and Lucas (1996) estimate an AR(1)

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<sup>12</sup>The discount factor and the volatility of the idiosyncratic shocks are the only parameters in this calibration that are different from the OLG model.

process with a conditional volatility of 25%, and a persistence parameter of 0.53. Naturally there is no direct match with our set-up with purely i.i.d. shocks but given our aim to keep the analysis in this section as parsimonious as possible we set  $\sigma_L$  equal to 30%.

## 2.6 Model Without Aggregate Uncertainty

The model in this section is very close to the one studied in Aiyagari (1994 and 1995).

### 2.6.1 Benchmark results

In the absence of aggregate uncertainty (no depreciation shocks and no productivity shocks), the return from holding government bonds or stocks is the same ( $R^K = R^B$ ). The normalized individual optimization problem is then:

$$V(x_t^i; R^K) = \underset{\{k_{t+1}^i\}}{\text{Max}} \left\{ \frac{(c_t^i)^{1-\rho}}{1-\rho} + \beta E_t [(\omega_{t+1})^{1-\rho} V(x_{t+1}^i; R^K)] \right\} \quad (16)$$

subject to the constraints and laws of motions given above. Market clearing then implies that individual savings (capital and bond holdings) have to add up to the total capital stock and total government debt in the economy, since debt and capital are perfect substitutes.

The solution to this problem is well understood since the seminal contribution by Aiyagari (1994). At this stage, our interest is in understanding the mechanisms behind the effects of fiscal policy decisions. The baseline results are reported in table 1 (column ‘‘Model I’’). Since there is no aggregate uncertainty, all securities earn the same rate of return, which therefore represents both the return on capital and the interest rate on government bonds. As a result, this economy will either significantly underestimate the former, or overestimate the latter, or both. In this case we have an equilibrium gross real rate of return of 6.59% which, most notably, strongly exaggerates the cost of government debt.

Having established a benchmark case, we next proceed to our comparative statics. More precisely, here we consider tax rate changes accompanied by offsetting changes in level of government debt, so that the long-run level of government expenditures remains unchanged. Later on, in the OLG economy, we will consider additional policy experiments.

### 2.6.2 Impact of changes in tax rates

Since in our model labor income taxes are effectively lump-sum taxes, it is easier to study them first. In a complete markets representative agent model, changing lump sum taxes does not affect the firm’s or the household’s first-order conditions. Therefore, the equilibrium rate of return, aggregate

capital and aggregate investment do not change. As a result, aggregate output also remains constant and, since  $G$  is being held fixed, total private consumption is also unchanged. Households buy the additional government debt and the higher taxes are exactly offset by the additional interest income (since, from the government's budget constraint,  $\Delta T = r\Delta B$  in the aggregate). Therefore, both household consumption and household wealth remain the same.

In Table 3 (columns 2 and 3) we show that this is not the case in our economy. When we increase the lump-sum tax rate by 2.5%, the capital stock decreases by 1.28%, while the return on capital increases by 12 basis points. Changing  $\tau_L$  has real effects in this economy because of liquidity constraints. In the presence of liquidity constraints and uninsurable idiosyncratic risk, consumption does not fall one for one with lower disposable income due to precautionary savings. Thus, liquidity constraints induce a distortionary effect of lump-sum taxes and this effect is stronger when households face higher income risk: although we do not report those results in the tables, we find that the crowding-out effect increases with the amount of idiosyncratic uncertainty existing in the model.

Capital taxation naturally has distortionary effects, even in an otherwise frictionless model, since it changes relative prices and thus the first-order conditions. Table 3 (columns 4 and 5) shows that, in our economy, a 2.5% increase in the capital tax rate decreases capital accumulation by 3.09%. The return on capital increases by 31 basis points and, since crowding out affects output more than consumption,  $C/Y$  increases by 0.42%. These results will serve as a benchmark for comparison with our next economy.

## 2.7 Model With Aggregate Uncertainty / Imperfect Substitutability

We now introduce aggregate uncertainty in the previous model through aggregate productivity and depreciation shocks. As a result, the returns on government bonds and capital are no longer identical. At the micro level, this implies that households now have a portfolio decision, in addition to their savings decision, and they need to form expectations about the evolution of the aggregate capital stock. At the macro level we can now try to match the rate of return on capital, without imposing a counterfactually high rate of return on government bonds. The parameter values are identical to the ones used in the previous model except for a lower discount factor generating the same  $K/Y$  in both economies.

### 2.7.1 Benchmark results

Table 1 (in column “Model II”) reports the equilibrium macroeconomic quantities obtained in this economy, which are extremely close to the ones obtained in the previous model. Table 2 reports the equilibrium returns. We now have different rates of return for capital (8.79%) and for government bonds (4.89%). Since firms in the model are not levered, our return on capital corresponds to the return of unlevered equity in the data. Therefore, we adjust the moments of our return series by the average leverage of US corporations to make them comparable with the CRSP data.<sup>13</sup> This economy still undershoots the former and overestimates the latter, relative to their empirical counterparts. This merely reflects the inability of these models to match the historical equity premium. This is one of the reasons why will consider a more realistic model later on. Nevertheless, we will show in this section that even a 3.9% equity premium, is enough to have a significant *quantitative* impact on the results.

### 2.7.2 Impact of changing the lump-sum tax rate

Table 4 (columns 2 and 3) compares the baseline economy (with  $\tau_L = 10\%$ ), with an otherwise identical economy where the lump-sum tax rate has been increased by 2.5%. As before, a higher lump-sum tax rate can finance a higher steady-state level of government debt. In the model without aggregate uncertainty the presence of borrowing constraints induces a small decrease in the capital stock (1.28%) and a modest change in the equilibrium rate of return (12 basis points). In contrast, when we introduce aggregate uncertainty, we obtain a much bigger response: the capital stock decreases by 8.74% and the return on capital increases by 92 basis points. Therefore, the economic impact of taxes is approximately 7 – 8 times higher than in the one-asset model.

Why do the results change so dramatically in the presence of aggregate uncertainty? There is a second channel operating here, in addition to the borrowing constraint channel. Since capital and bonds are no longer perfect substitutes, households also make a portfolio allocation decision.

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<sup>13</sup> Since we are implicitly assuming risk-free corporate debt, expected levered returns are computed using the simple Modigliani-Miller formula:

$$r^{equity} \equiv r_K^{levered} = r_K^{unlevered} + \frac{D}{E} (r_K^{unlevered} - r_f)$$

Along the same lines, the Sharpe ratio on levered equity must be identical to the Sharpe ratio on unlevered equity, allowing us to compute the standard deviation on the levered claim from:

$$\sigma_K^{Levered} = \sigma_K^{unlevered} \frac{r_K^{levered} - r_f}{r_K^{unlevered} - r_f}$$

Lump-sum taxes are essentially equivalent to a negative position in riskless bonds: a non-contingent future payment. Therefore, a higher lump-sum tax rate increases the value of this implicit negative bond position, and consequently households respond to this by shifting their portfolio allocations more towards bonds. Therefore, investment and capital decrease by more than in the economy without aggregate uncertainty. Comparing the results in Tables 3 and 4 we see that this effect can be very large.

Since we now consider an economy with aggregate uncertainty, we can also study the impact of fiscal policy decisions on the different rates of return and on the risk premium. As aggregate savings decrease, both rates of return must increase (just as in the single-asset economy). Interestingly, the return on capital increases by less than the risk-free rate (0.92% versus 1.55%), and as a result the equity premium is lower in the new equilibrium. Although, as previously discussed, the relative demand for stocks has decreased, the supply of government bonds has increased by 134%, and this effect clearly dominates: with a higher proportion of government debt to risky capital in the economy, consumption smoothing can more easily be achieved and thus a lower equity premium is generated.

### 2.7.3 Impact of changing the capital income tax rate

We next consider a 2.5% increase in the capital income tax rate with the results shown in Table 4 (columns 5 and 6). Comparing with the previous results (Table 3, columns 5 and 6) we find that the crowding out effect is higher when capital and government bonds are not perfect substitutes. The capital stock decreases by 5.72% versus 3.09% in the previous economy.<sup>14</sup> With the higher tax rate, after-tax returns decrease for given pre-tax returns, inducing investors to lower both their supply of capital and their demand for government bonds. As a result both (pre-tax) rates of return must increase in equilibrium to clear the financial markets. However, the return on capital increases less than the risk-free rate (0.56% versus 0.77%) leading to a lower equity premium for two reasons. First, the firm's demand for capital is downward sloping, while the supply of bonds is perfectly inelastic. Second, and more importantly, the supply of government bonds has increased by 33% thus requiring a significant change in the riskless rate to clear the market. Therefore, in the new equilibrium the equity premium is lower.

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<sup>14</sup>The crowding-out effect is smaller than in the lump-sum tax experiment simply because, in our set-up, lump-sum taxes apply to a much larger tax base (labor income). Later on we will report comparable results, when instead of considering equal changes in the tax rate, we consider equal changes in total taxation. Naturally, we will then find that capital income taxes are more distortionary.

### 2.7.4 Summary

We have shown that, when taking into account the fact that capital and government bonds are not perfect substitutes, the *quantitative* impact of fiscal policy decisions is significantly altered, relative to an otherwise identical model with borrowing constraints but no aggregate uncertainty. Lump-sum taxes have a significant crowding-out effect, while the crowding-out effect of capital income taxes is also higher. The economic magnitudes of these results are very substantial, even though the model only generates a 3.9% equity premium. Intuitively, as the two assets become even less close substitutes, i.e. as the equity premium increases towards the historical average, we expect these results to be even stronger.

Finally, the model with aggregate uncertainty also allows us to study the impact of fiscal policy decisions on asset prices and returns. The changes in the cost of government debt are substantial. The impact on equity returns and risk premia is smaller, but still economically significant. Increasing tax rates (lump-sum or on capital) will increase both the riskless rate and the return on capital, as there are less savings in the economy. Since the supply of government bonds is fixed, the riskless rate must adjust by more, and thus the equity premium falls.

Having identified the main economic mechanisms that are present in our analysis, and having measured their relative contributions in a relatively simple model, we now proceed to build a more complex model that will deliver more accurate quantitative predictions.

## 3 OLG Model

In this section we build an overlapping generations model that will improve our ability to match important macroeconomic moments and aggregate returns. Specifically, in the time series dimension, we focus on matching the unconditional shares of consumption, government and investment expenditures in output, the volatility of consumption growth, and unconditional asset pricing moments (the mean return and volatility of the interest rate on government debt, the market return and the equity premium). In the cross section, we focus on matching consumption and wealth inequality, both in the aggregate and over the life cycle. We then use this model as a laboratory to conduct our fiscal policy experiments.

We now incorporate the additional features that we think are necessary to make the model more consistent with the key empirical observations that we want to match. These extensions are essentially at the household level, where we now have finite-horizons, a retirement period, and limited stock market participation. In addition, we now consider Epstein-Zin preferences which will

allow us to obtain a better calibration of the model and, combined with preference heterogeneity, will be important in matching the wealth distributions and asset allocations, conditional on stock market participation. The production and government sector are the same as in the model with aggregate shocks considered in the previous section, except for the introduction of a social security system. The model is solved at an annual frequency as before, and below we describe the elements which are incremental, or changed, from the earlier setup.

### 3.1 Households

We now consider households with a finite horizon, (a life-cycle model), and Epstein-Zin preferences (Epstein-Zin (1989)), so that the household's objective function is now

$$V_t = \left\{ (1 - \beta)C_t^{1-1/\psi} + \beta \left( E_t(V_{t+1}^{1-\rho}) \right)^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}}$$

The household's life cycle is divided in two periods: working life and retirement. During working life, all households supply labor inelastically as before.

#### 3.1.1 Labor income process and retirement transfers

We let  $i$  index individual households as before, but we now add an index  $a$  for household age/cohort. The stochastic process for individual labor income ( $H_{at}^i$ ) is again given by:

$$H_{at}^i = W_t L_a^i, \quad (17)$$

but  $L_a^i$  (the household's labor productivity) is now a function of age. This productivity is specified to match the standard stochastic earnings profile in life-cycle models. More precisely, labor income productivity combines both permanent ( $P_a^i$ ) and transitory ( $\varepsilon^i$ ) shocks with a deterministic age-specific profile:

$$L_a^i = P_a^i \varepsilon_i \quad (18)$$

$$P_a^i = \exp(f(a)) P_{a-1}^i \xi^i, \quad (19)$$

where  $f(a)$  is a deterministic function of age, capturing the typical hump-shape profile in life-cycle earnings. We assume that  $\ln \varepsilon^i$ , and  $\ln \xi^i$  are each independent and identically distributed with mean  $\{-.5 * \sigma_\varepsilon^2, -.5 * \sigma_\xi^2\}$ , and variances  $\sigma_\varepsilon^2$  and  $\sigma_\xi^2$ , respectively. Retirement is exogenous and deterministic. All households retire at age 65 ( $a^R = 46$ ) and retirement earnings are given by:  $\lambda P_{a^R}^i W_t$ , where  $\lambda$  is the (exogenous) replacement ratio. The retirement income is funded by a proportional social security tax  $\tau_s$  discussed later. Including a social security system is important

to provide the model with a realistic labor income process. If we were to ignore social security transfers we would significantly increase households' income risk and wealth accumulation.

### 3.1.2 Wealth accumulation.

Total liquid wealth (cash-on-hand) can be consumed or invested in the two assets. At each age ( $a$ ), households enter the period with wealth invested in the bond market,  $B_{at}^i$ , and (potentially) in stocks,  $K_{at}^i$ , and receive  $L_a^i W_t$  as labor income. Cash-on-hand at time  $t$  is given by:

$$X_{at}^i = K_{at}^i(1 + (1 - \tau_K)R_t^K) + B_{at}^i(1 + (1 - \tau_K)R_t^B) + L_a^i(1 - \tau_s - \tau_L)W_t \quad (20)$$

before retirement ( $a < a^R$ ), and by:

$$X_{at}^i = K_{at}^i(1 + (1 - \tau_K)R_t^K) + B_{at}^i(1 + (1 - \tau_K)R_t^B) + \lambda P_{at^R}^i(1 - \tau_s - \tau_L)W_t \quad (21)$$

during retirement ( $a \geq a^R$ ).

The new normalization includes the permanent component of labor income during working life so that  $k_{a,t+1}^i = \frac{K_{a,t+1}^i}{P_a^i G_t^{1-\alpha}}$  and  $b_{at+1}^i = \frac{B_{a,t+1}^i}{P_a^i G_t^{1-\alpha}}$  but  $c_{at}^i = \frac{C_{a,t}^i}{P_a^i G_t^{1-\alpha}}$ ,  $x_{at}^i = \frac{X_{a,t}^i}{P_a^i G_t^{1-\alpha}}$ . The individual budget constraint can then be written as

$$(1 + \tau_C)c_{at}^i + k_{a,t+1}^i + b_{at+1}^i = x_{at}^i = (1 + R_t^K(1 - \tau_K))\frac{k_{at}^i}{\omega_t \omega_a} + (1 + R_t^B(1 - \tau_K))\frac{b_{at}^i}{\omega_t \omega_a} + \varepsilon_t^i w_t (1 - \tau_s - \tau_L)\frac{1}{\omega_t}$$

where  $\omega_a = \exp(f(a))\xi^i$ . After retirement, the equation looks the same except for the retirement benefit:

$$x_{at}^i = (1 + R_t^K(1 - \tau_K))\frac{k_{at}^i}{\omega_t \omega_a} + (1 + R_t^B(1 - \tau_K))\frac{b_{at}^i}{\omega_t \omega_a} + w_t \lambda (1 - \tau_L - \tau_s)\frac{1}{\omega_t}$$

where  $\omega_a = 1$ .

## 3.2 Calibration

The household earnings processes and social security are calibrated from evidence based on micro-economic data (PSID), while the other parameters are used to match several empirical moments. The government sector variables are calibrated to match the ratios of government bonds, government expenditures and tax revenues to GDP. The technological parameters and preference parameters are chosen to try to replicate, as close as possible, multiple different moments such as the consumption and investment shares of GDP, consumption volatility, wealth distribution, limited participation, and the mean and volatility of returns.

### 3.2.1 Labor income and social security

Agents begin working life at age 20, retire at age 65, and can live up to 90 years. The parameters for the household earnings processes are taken from the previous studies using the PSID. The variances of the idiosyncratic shocks are taken from Carroll (1992): 10 percent per year for  $\sigma_\epsilon$  and 8 percent per year for  $\sigma_\xi$ . The parameter values for the deterministic labor income profile, reflecting the hump shape of earnings over the life-cycle, are taken from Cocco, Gomes and Maenhout (2005).

For tractability we assume that the social security budget is balanced in all periods. Given a value for the replacement ratio of working life earnings ( $\lambda$ ), the social security tax rate ( $\tau_s$ ) is determined endogenously. This tax rate ensures that social security taxes are equal to total retirement benefits, taking into account the demographic weights. Consistent with the empirical evidence with regards to median replacement rates from the U.S. social security system, we use a 40% replacement rate (as in Cagetti and De Nardi (2006)), which implies an endogenous social security tax ( $\tau_s$ ) of approximately 17.5% to maintain social security balance period by period.

### 3.2.2 Technology

Capital's share of output ( $\alpha$ ) is set to 34%, and the average annual depreciation rate ( $\delta$ ) is 8% to match the investment to output ratio. To match stock market return volatility we set the standard deviation of the stochastic depreciation shock at 15%. The aggregate productivity shock follows a two-state Markov Chain and its unconditional standard deviation (2.5%) is picked to generate a 4.2% standard deviation in aggregate output (matching the annual U.S. GDP volatility since 1930). The transition probability of changing state is set to 0.4 to match the duration of business cycles.

### 3.2.3 Government sector

The aggregate supply of bonds is set to 36% of GDP, which is the average value of U.S. Treasury securities held by the U.S. public, as reported by the Congressional Budget Office (from 1960 to 2009). The ratio of total outstanding debt to GDP is higher, but the difference is due to the significant amount of US government bonds that is being held abroad. Including these in the model would lead to an extremely incorrect calibration of either total wealth or the capital stock in our economy. Of course excluding them also has a cost, since we are ignoring the interest payments on these bonds in the government's budget constraint. However, we can simply interpret these as an additional exogenous source of government expenditures. Using the average historical values for both the cost of debt and total debt outstanding, this corresponds to an additional 0.6% of GDP, which has a fairly negligible impact on our baseline calibration.

We also want to match the share of government expenditures in GDP, which is an endogenous quantity in the model. This is achieved through an appropriate calibration of the tax rates. Even ignoring this extra constraint, the calibration of each tax rate already requires a compromise between matching two different features of the data: the tax rate itself or the corresponding share of tax revenues in GDP. We compute the tax shares using data from the Bureau of Economic Analysis from 1929 until 2009.<sup>15</sup> For capital income taxes we set tax rate to 40%, following Trabandt and Uhlig (2006), Carey and Rabesona (2002) and Mendoza, Razin and Tesar (1994) and, as shown in table 5, the implied share of capital income revenues over GDP in the model is 5.41%, which is extremely close to the value in the data.

With respect to the tax rate on labor income, the calibration decision is clear: since we do not have a labor supply decision in the model, then these are effectively lump-sum taxes, and therefore we want to match the revenue share, as opposed to the tax rate. It turns out that this is actually an advantage of our model. As shown in table 5, a flat tax rate of 10% generates tax revenues which are in line with the empirical numbers.<sup>16</sup> However, in reality the marginal tax rate on labor income is much higher than 10%. This shows that, with a linear tax schedule, researchers face an important trade-off. They can either match the marginal tax rate and dramatically over-estimate the importance of labor tax revenues in the data, or match the revenues themselves and significantly under-estimate the distortion at the margin.<sup>17</sup> In models with exogenous labor supply, such as ours, this is not an issue. As previously discussed, the choice is very clear: match the revenue share. However, in models with an endogenous labor supply this represents a serious concern, unless we also carefully incorporate different sources of non-linearity in the tax system (as in Castaneda et al. (03)), which represents a significant additional computational challenge.

It is important to point out that this tension is not very specific to our model and it is essentially driven by the assumption of Cobb-Douglas production technology. With such a technology

$$\frac{WL}{Y} = 1 - \alpha \implies \tau_L \frac{WL}{Y} = \tau_L(1 - \alpha)$$

and, in a model without retirement, the left-hand-side denotes the share of labor income revenues

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<sup>15</sup>The BEA data does not provide a disaggregation of total personal income taxes, and therefore we combine it with data from the IRS to compute the relative percentages of labor income and capital income taxation in this category.

<sup>16</sup>As we can see from the table, the ratio of labor tax revenues to GDP has increased over time. Although in most of our calibration we have considered long time-series as much as possible, we also want the fiscal policy conditions in our baseline economy to be fairly close to the current values, so that our results are directly applicable to the current US economy. Therefore, here we put also emphasis on matching the 2007 (pre-crisis) value (9.38%) and not only the 1929-2009 average (6.80%).

<sup>17</sup>This simply reflects the multiple sources of deductions and exemptions that are not being modeled with a linear tax schedule.

in GDP.<sup>18</sup> This will hold regardless of most other features of the model (namely whether we have endogenous labor supply or not). Therefore, any tax rate higher than 10% will over-estimate total labor income revenues. Considering a different production technology, or adding labor market frictions is unlikely to resolve this problem as long as the model is forced to match the labor income share of GDP.

This still leaves us with one parameter left to calibrate: the tax rate on consumption. As previously discussed we want the model to match the share of government expenditures in GDP, so this is actually not a free parameter. We set  $\tau_C = 13\%$  to match  $G/Y$  given the other tax rates and the calibration of  $B/Y$ . It turns out that this number delivers total tax revenues which, as a share of GDP, are fairly close to their empirical counterpart.

### 3.2.4 Preference heterogeneity and limited participation

We consider two groups ( $A$  and  $B$ ) of households in the model: stock market participants and non-participants. In the recent data, the two groups are almost identical in size (55% and 45% respectively, using the data from the 2001 SCF).<sup>19</sup> However, they have very different wealth accumulation profiles: the participation rate is 88.84% among households with wealth above the median, and only 15.21% for those with wealth below the median. In the model we treat limited participation as exogenous for tractability reasons (as in Basak and Cuoco (1998)), but make sure that the wealth accumulation differences are consistent with the data.<sup>20</sup> We use ex-ante preference heterogeneity in the discount factor and the elasticity of intertemporal substitution to endogenously generate different wealth accumulation profiles, and we assume stockholders make up 50% of the population, consistent with the empirical magnitudes in the U.S. economy.

We rely primarily on discount factor and EIS heterogeneity to generate different wealth profiles. Type- $A$  (non-stockholders) have a very low discount factor ( $\beta = 0.7$ ) and never accumulate much wealth over the life cycle, while type- $B$  (stockholders) have a higher discount factor ( $\beta = 0.99$ ) chosen to match the historical risk free rate.<sup>21</sup> There is strong evidence that stockholders have

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<sup>18</sup>In a model with retirement, such as ours, the comparison is even worse.

<sup>19</sup>These numbers take into account households that participate in the stock market indirectly through pension funds.

<sup>20</sup>Given the low wealth accumulation of non-stockholders, a small one-time entry cost would suffice to endogeneize the non-participation decision. For example, Alan (2006) estimates a structural participation model and finds that a one-time entry cost equal to approximately 2-3% of average annual income explains limited stock market participation. Gomes and Michaelides (2008) show that a one-time cost of 5% of average annual income or lower would deter participation for the poorer households. We leave such an entry cost out of the model to reduce the computational burden.

<sup>21</sup>We emphasize that the quantitative results are almost identical regardless of the method we use to generate “poor” non-stockholders. What really matters is that we replicate poor households within the model. The same

a higher EIS than non-stockholders (see, for example, Vissing-Jorgensen (2002)). Therefore, we assume that non-stockholders have a lower EIS in the model as well. We pick  $\psi^A = 0.45$  to match the wealth accumulation of this group, in combination with the discount factor. The value of the EIS stockholders is chosen to match, as close as possible, two different moments: the volatility of consumption growth for this group, and the volatility of the riskless rate. This gives us  $\psi^B = 0.7$  and, as we will see later, a good calibration of both of these moments. Finally, both types have the same risk aversion coefficient ( $\rho = 5$ ). The risk aversion coefficient is picked to generate the highest possible equity premium in the range of plausible coefficients and we view  $\rho = 5$  as a sensible upper bound.

### 3.3 Equilibrium

The equilibrium is characterized by a set endogenously determined prices (bond prices, wages, and equity returns), a set of cohort specific value functions, policy functions,  $\{V_a, b_a, k_a\}_{a=1}^A$ , and rational expectations about the evolution of the endogenously determined variables, such that firms and consumers make optimal decisions and markets clear. Since most of the equations are equivalent to the ones presented for the previous economy (section 2.6), we leave the precise definition, along with details of the numerical solution method, to Appendix A.

### 3.4 Baseline results

#### 3.4.1 Macroeconomic variables and asset prices

Table 6 reports the main macroeconomic quantities. The shares of consumption, investment and government expenditures and debt relative to GDP match their empirical counterparts quite accurately (panel A). The empirical moments are taken from the National Accounts reported by Bureau of Economic Analysis, from 1929 until 2009. Following Castaneda et al. (2003) we classify 75% of durable consumption expenditures as investment and 25% as consumption. Panel B shows that the model matches extremely well the volatilities of aggregate consumption growth. Panel B also shows that consumption growth of stockholders is more volatile than the consumption growth of non-stockholders, consistent with the empirical evidence in Malloy, Moskowitz and Vissing-Jorgensen (2005).

Table 7 reports the main asset pricing moments implied by the model, along with their empirical

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*quantitative* results would be obtained under alternative specifications, as long as these two groups are calibrated to match the same heterogeneity in wealth accumulation. For example, Gomes and Michaelides (2008) consider heterogeneity in risk aversion and EIS, with  $\beta = 0.99$  for both groups, among other combinations.

U.S. counterparts. The consumption and output series are taken from the NIPA tables, published by the Bureau of Economic Analysis. The returns series are taken from CRSP. The equity return is the real return on the CRSP value-weighted index (including dividends), and the rate of return on government bonds is the real return on 1-year government bonds.<sup>22</sup> As previously discussed, since firms in the model are not levered, our return on capital corresponds to the return of unlevered equity in the data. Therefore, we adjust the moments of our return series by the average leverage of US corporations to make them comparable with the CRSP data, using the formulas in fn. 13. The risk free rate is matched very closely (1.72%) and the equity premium is now also closer to the data (4.61%). The return standard deviations (18.98% and 1.58%) are very similar to their empirical counterparts.

### 3.4.2 Consumption and wealth inequality

Table 8 reports the shares of wealth held by different percentiles of the wealth distribution in the model and in the 2001 SCF data.<sup>23</sup> We also report wealth distributions conditional on stockholding status since, as previously argued, matching the relative wealth of stockholders and non-stockholders is important for consistency. In the data, stockholders are defined as households owning stocks directly or through mutual funds either in taxable accounts or in pension plans. Overall, the model captures relatively well the wealth distribution. In particular, it replicates the fact that wealth below the median is negligible, while households in the top quintile hold 68% of total assets in our economy versus 83% in the data. The model also matches well the wealth distribution of non-stockholders. For stockholders, the wealth distribution is not as skewed as in the data, since our economy does not capture the rich entrepreneurs that dominate the top end of the distribution. Therefore, to match the capital stock, the model overshoots wealth accumulation in the intermediate percentiles (50-80). Finally, the model's results can also be recast in terms of aggregate gini coefficients. Aggregate wealth inequality in the data is 0.8, while consumption is much more evenly distributed, with a gini

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<sup>22</sup>We consider 1-year bonds because we have a yearly model and, in the model, government bonds are risk free over 1 period. In the data, the average maturity for government debt has changed over time, but it is close to 5 years. The rates of return on this debt however, also include a potentially non-trivial risk premium. Nevertheless, if we use the price series for 5-year government bonds, we would actually obtain a very similar average return (1.66% versus 1.23%), and the main difference would be the standard deviation: 6.10% versus 3.83%.

<sup>23</sup>In the SCF, wealth is defined as liquid assets net of all non-real estate loans plus real estate equity. Liquid wealth is made up of all types of transaction accounts, certificates of deposit, total directly-held mutual funds, stocks, bonds, total quasi-liquid financial assets, savings bonds, the cash value of whole life insurance, other managed assets (trusts, annuities and managed investment accounts) and other financial assets. Home equity is defined as the value of the home less the amount still owed on the first and 2nd/3rd mortgages and the amount owed on home equity lines of credit. Debts include all uncollateralized loans (credit cards, consumer installment loans) and loans against pensions.

coefficient of 0.25.<sup>24</sup> These numbers compare very well with those in the model, which are 0.7 and 0.29, respectively.

### 3.4.3 Life cycle profiles

The combination of idiosyncratic shocks, preference heterogeneity and differences in stock market participation status induces significant cross-sectional heterogeneity in wealth accumulation and consumption over the life cycle. Figure 1 plots the gini coefficient for consumption, conditional on age. Consistent with the empirical evidence in Deaton and Paxson (1994), and more recently in Krueger and Perri (2006), consumption inequality tends to increase with age, as households are hit by different labor income shocks and also start saving for retirement. Total consumption inequality is much more pronounced during retirement because a significant fraction of the population (mostly non-stockholders) saves very little wealth during working years, due to their high discount rate, and thus have to rapidly scale down consumption towards their pension income.

Figure 2 plots the same graph for wealth inequality over the life cycle. Overall, there is substantial wealth inequality in the economy reflecting the differential savings behavior across the two different groups. Initially wealth inequality is reduced a bit as stockholders start saving aggressively. Wealth inequality then rises from age 25 onwards as the stockholders accumulate substantial amounts of wealth. Close to age 65 there is a significant decrease in inequality as non-stockholders finally decide to save something for retirement. Since they do not actually save much, this wealth is quickly consumed and thus the aggregate gini coefficient rapidly increases again.

## 4 Permanent fiscal policy shocks

We first consider the effects of fiscal policy decisions for a constant level of government expenditures, as in our previous experiments. More precisely, we will consider permanent changes in tax rates accompanied by offsetting changes in government debt. Given that government expenditures do not play a role in our economy, it is not particularly interesting to compare steady-states with different levels of  $G$ .<sup>25</sup> In the next section we will consider temporary fiscal policy shocks, and there we will study expenditure shocks.

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<sup>24</sup>The wealth gini coefficient is computed from the 2001 Survey of Consumer Finances, while the consumption gini coefficient is taken from Krueger and Perri (2006).

<sup>25</sup>Those results are, nevertheless, available upon request.

## 4.1 Changes in capital income taxes

We start by analyzing the impact of changes in the capital income tax rate. More precisely, in Table 9 we consider a 2.5% increase in  $\tau_K$  for a given level of government expenditures. To understand these results it is important to remember that we are comparing two economies in their respective steady-states. In our model tax revenues are either used to finance government expenditures and/or interest payments on debt. If we had considered a temporary one-year tax rate increase then, for a constant level of government expenditures, the *current* stock of government bonds would decrease. However, in steady-state, a higher tax rate implies higher government interest payments, to satisfy the long-run budget constraint.

A higher capital income tax rate crowds out investment. In the new equilibrium the capital stock falls by 3.85%. As a result, consumption also falls but less than output, as households now save a smaller fraction of their income, thus the consumption share of GDP increases by 0.50%. As consumers/investors reduce their savings, both in the stock market and in government bonds, asset returns must increase to clear the financial markets. The equity premium falls since the riskless rate increases by more than the equity return, due to the significant increase in the supply of government bonds (32%), and since the two assets are not perfect substitutes. More precisely the mean equity return increases by 32 basis points while the riskless rate increases by 41 basis points.<sup>26</sup> However, the effect on the equity premium is quantitatively very small (a 10 basis points decrease).

Finally, consumption volatility also falls (by 0.21%). This is partially due to the reduction in both aggregate savings and (net-of-taxes) return volatility, but not only. There is also a significant portfolio rebalancing effect from shareholders: since the capital stock has fallen while the level of government bonds has increased, household portfolios are now relatively more invested in riskless bonds. This explains why the consumption volatility of stockholders falls by 0.39%, as opposed to 0.08% for non-stockholders, and it is consistent with the reduction in the equity premium.

## 4.2 Changes in public debt accompanied by alternative tax rate changes

Now we consider changes in the level of public debt accompanied either changes in the capital income tax rate or the lump-sum tax rate.<sup>27</sup> More precisely, in Table 10, we increase the steady-state level of government debt by 20%, and compare the results from financing this higher level of interest expenses with either higher lump-sum taxes (columns 2 and 3) or higher capital income

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<sup>26</sup>Naturally the difference is even more significant if measured relative to the base: the return on capital has increased by 6.0%, while the riskless rate has gone up by 24.1%.

<sup>27</sup>Results from changing the consumption tax rate are available upon request. Given the set-up of our model, these results are very similar to the ones obtained when we increase the lump-sum tax rate.

taxes (columns 4 and 5). This increase in government debt leads to an endogenous increase in the debt-to-GDP ratio of approximately 7.5%. Since lump-sum taxes apply to a larger tax base (total labor income instead of financial income), the capital income tax rate must increase by more (1.54%) than the labor income tax rate (0.24%) to finance the same additional level of government debt.

As seen in columns 2 and 3, when we increase  $\tau_L$ , the capital stock falls by 1.68% relative to the benchmark. As discussed earlier, lump-sum taxes are effectively negative positions in riskless bonds, and therefore changes in the tax rate induce a reallocation of households' financial portfolios: households compensate for the higher taxes by increasing (decreasing) their bond (equity) holdings, to keep their total risk exposure unchanged. Alternatively, when the additional debt repayments are being financed by capital income taxes, the crowding out effect is naturally larger, with the capital stock falling by 2.35%. By comparison, the crowding-out effect of lump-sum taxes (1.68%) is more than 2/3 of the crowding out effect of distortionary capital income taxes (2.35%). Since lump-sum taxes would have no impact on real quantities in a frictionless economy, this again highlights the importance of carefully capturing these frictions to obtain a correct assessment of the quantitative implications of fiscal policy decisions. Higher capital income taxes decrease the volatility of aggregate consumption, and in particular the volatility of stockholder's consumption. This is expected since the volatility of their after-tax wealth is now lower. Given that our lump-sum taxes are proportional to labor income, they also decrease the consumption volatility, but by much less, and the effect is very similar for stockholders and non-stockholders.

The impact on rates of return is relatively small in both cases. When the additional government debt is being financed by lump-sum taxes the riskless rate increases by 23 basis points, while the return on capital increases by 14 basis points. When the extra financing is coming from distortionary capital income taxes then the percentage increase in the riskless rate is only marginally higher (26 basis points). In both cases we observe a marginal decrease in the risk premium: 9 and 7 basis points, respectively.

In summary, the response of the riskless rate is not significantly affected by the tax rate chosen to finance the additional debt repayments: the semi-elasticity ranges between 1.2 to 1.3 basis points in response to a 1% change (corresponding to elasticities of 0.60% to 0.66%). On the other hand, the response of the cost of capital is naturally higher when increase capital income taxes: the return elasticity is 0.17%, as opposed to 0.13% with lump-sum taxes.

### 4.3 Changes public debt accompanied by alternative tax rate changes and different return volatility

As previously discussed, in the model equity return volatility is effectively an exogenous variable, determined by the volatility of the stochastic depreciation shocks. This is a potential concern when considering policy experiments such as the ones discussed in this paper: if any of these changes in government policy affects stock return volatility our model will fail to capture this effect. To the extent that this will have an impact on all other economic quantities, we might be mis-measuring the effects of fiscal policy decisions.

To address this concern we now repeat the analysis in the previous subsection, a 20% increase in government bonds, financed by a compensating change in capital income taxes, under two alternative scenarios: in one case we assume that the volatility of the returns (stochastic depreciation shocks to be more precise) has increased by 2% following the increase in government debt and taxes, while in the other we assume that this volatility has decreased by 2%. Based on the historical evidence, and on our previous results regarding the impact of fiscal policy decisions on expected returns, we view these as upper bounds on the potential movements in return volatility resulting from these decisions. Therefore, we believe that these experiments provide a conservative confidence bound for our results.

Table 11 reports the results from these two new experiments and compares them with the ones obtained in the previous case, i.e. with constant equity return volatility. If the volatility increases (decreases) by 2% the reduction in the aggregate capital stock is now 2.28% (2.66%) instead of 2.35%. This happens mostly because with a higher (lower) return volatility the required compensating change in the capital income tax is now also higher (lower).<sup>28</sup> The same result, and relative magnitudes, apply to the other macroeconomic variables: investment, consumption and output. Naturally, if equity return volatility is lower then the volatility of aggregate consumption falls by more, while in the alternative scenario, it can actually increase. Overall, we conclude that the macro-economic results are not significantly affected by a potential impact on the volatility of capital returns, unless we believe that this volatility can change quite a lot as a result of these fiscal policy decisions.

As expected, the largest differences are on the average rates of return. If return volatility is decreased after the fiscal policy decision then households are more willing to invest in equities and as a result the return on capital doesn't have to fall as much (in theory it could even decrease), and

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<sup>28</sup>Naturally, for the same change in tax rate, the crowding-out effect will be higher as the volatility of returns increases, but this would not be the correct comparison: the level of government expenditures would be different in the two cases, since the change in tax rate would no longer be exactly off-setting the increase in government debt.

the cost of government debt must increase by much more. On the other hand, if return volatility increases, then we have the reverse: the cost of capital must increase even more and the cost of debt doesn't have to increase as much, and in fact, under our calibration it actually decreases.

## 5 Temporary fiscal policy shocks

In this section we extend the model to allow for temporary shocks to the government's intertemporal budget constraint. These innovations will capture government expenditure shocks that will be financed by variations in the capital income tax rate. For convenience we actually model these as tax shocks directly (matched by an equivalent change in government expenditure), but naturally these two are equivalent representations of the phenomena.

### 5.1 Model set-up

To investigate these responses we use a VAR analysis and extend the previous model by introducing variation in the capital income tax around the mean values investigated in the baseline model above. More precisely, we now assume that capital income tax rates follow a Markov process with two values (high and low),  $\{\tau_K^H, \tau_K^L\}$ , where  $\tau_K^H > \tau_K^L$ . We consider different cases for the probability of remaining in the current state ( $\pi^\tau$ ) and set the standard deviation of the tax shock ( $\sigma_\tau$ ) to 2.5%.<sup>29</sup>

We solve the model under this new set-up, adding the new shock as an additional conditioning variable in all regressions, and verifying that the expectations fixed point continue to hold, with the same convergence criteria. We can now use the results of the model to compute the responses to any given sequence of tax shocks. Since we are solving for an economy with aggregate uncertainty, our steady-state is stochastic. Therefore we cannot simply compute responses from a given steady-state level. Even setting all variables equal to their unconditional means would be incorrect, since this is a highly non-linear model: setting the consumption of all agents equal to the average consumption would not deliver the average aggregate capital stock, or the average equilibrium bond price, for example.

Therefore, we must compute the average response across a large range of alternative starting points, where all those starting points are indeed consistent with the actual stochastic equilibrium that we have just solved for. More precisely, we consider 1000 different points in our simulation as alternative initial conditions. For each of these we then simulate further under two hypothetical

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<sup>29</sup>This exact value is not particularly important here since we are computing response functions and elasticities. Therefore, we choose a small number so that the unconditional moments implied by the model remain very close to the ones reported in the previous sections.

paths: with and without tax shocks. The tax shocks scenarios correspond to an initial one-standard deviation increase in the capital income tax rate, used to finance additional government expenditures, and below we will consider shocks with different durations. Both of the subsequent paths are characterized by the absence of shocks to the other two exogenous aggregate variables (productivity and depreciation), which are kept equal to their unconditional means. The differential behavior of the economy under these two alternative series of 1000 draws gives us the average response to the fiscal policy shock.

## 5.2 I.i.d. tax shocks

We first consider the case of i.i.d tax shocks ( $\pi^\tau = 0.5$ ), and simulate a path where the tax rate shock reverts back to the mean after just one year. Figure 3 shows the responses of aggregate consumption (total and for each subgroup), output, capital, wages, riskless rate and return on capital. The return responses are measured in basis points, while all other variables are reported in percentage changes “from the steady-state”, i.e. relative to the zero-shock case.

The higher capital income tax rate leads to an immediate reduction in the consumption of stockholders since their disposable income is now lower. The risk-free rate also responds within the same period since aggregate savings are now reduced. The lower savings lead to a decrease in next-period’s capital stock which in turn induces an additional reduction in the consumption of stockholders. As the capital stock falls return on capital must increase and the wage rate must decrease. Lower wages now lead to a reduction in non-stockholders’ consumption as well, which combining with the additional fall in stockholders’ consumption (described above), implies a second year percentage drop in total aggregate consumption that is almost double of its first year value. Since the tax rate as now reverted back to its average value, in the subsequent periods we simply observe a steady convergence of all of these variables to their zero-shock values. Naturally, all values are relatively small since we are considering a small shock: a one-year increase of the capital income tax rate by 2.5 percentage points, reverting back to its previous value immediately after. In the next subsection we consider a more persistence shock.

## 5.3 Persistent tax shocks

We now consider an economy with more persistent tax shocks, more precisely with  $\pi^\tau = 0.9$ , corresponding to a half-life of 5 years. In figure 4 we plot the responses of aggregate variables to a 5-year increase in the capital income tax rate (again by 2.5%), and compare it with the previous case of the one-year tax shock. Naturally now we observe strong hump-shape responses for all variables,

as the impact of the tax rate change continues to accumulate over the first five years. At the peak, capital stock has now decreased by almost 2.5%, and aggregate consumption by almost 0.25%. These are quite significant numbers considering that the capital income tax rate only increased by 2.5%, and all other tax rates (consumption and labor income) remain unchanged. The cost of government debt goes up by 33 basis points, while the (pre-tax) cost of capital increases by 44 basis points. This is in line with our previous results: the impact on the cost of debt is quite significant (33 basis points relative to an average level of 172 basis points), while the change in the return on capital is more moderate (44 basis points relative to an average level of 533 basis points).

## 6 Conclusion

We analyze the implications of fiscal policy changes in a heterogeneous agent model with incomplete markets, and where the stock market and government debt are not perfect substitutes. The model is calibrated to fit the main macroeconomic and asset pricing moments, and to generate wealth and consumption heterogeneity consistent with the data. We quantify the impact of changes in tax rates and government debt on the macro-economy and on rates of returns. We find that a permanent 20% increase in government debt is associated with a 1.7% to 2.4% decrease in the steady-state level of capital stock, depending on the exact tax rate used to finance the interest payments in this new steady state. We identify household portfolio rebalancing decisions as a quantitatively important channel (*the asset substitution channel*) for determining the macro-economic impact of fiscal policy measures. Through this channel, lump sum taxes can have real effects by changing the riskiness of total household wealth by reducing the implicit holdings of riskless bonds in the form of human capital. Bond yields rise between around 0.23%-0.32% and expected stock returns between 0.14%-0.32%, thereby slightly decreasing the equity risk premium.

Our results are qualitatively (and sometimes quantitatively) consistent with the recent empirical evidence in Laubach (2008), Greenwood and Vayanos (2010a, 2010b) and Krishnamurthy and Jorgensen (2010) on the effects of government debt on bond yields. Unfortunately, the complexity of the model does not allow us to include defaultable corporate debt to study the effects of debt to GDP changes on credit spreads, as is done in some of this recent empirical literature. Perhaps other solution techniques (like the ones proposed in Dumas and Lyasoff (2010)) can be combined with the methodology used in this paper to study extensions that can be used to investigate the effect of taxes and government debt on credit spreads.

## Appendix A Solving the OLG model

### A.1 Equilibrium Definition

1. Firms maximize profits by equating marginal products of capital and labor to their respective marginal costs.

2. Individuals choose their consumption and asset allocation by solving Equation (15).

3. Markets clear and aggregate quantities result from individual decisions. Specifically:

$$k_t = \int_i \int_a P_{a-1}^i k_{at}^i da di, \quad b_t = \int_i \int_a P_{a-1}^i b_{at}^i da di. \quad (\text{A.1})$$

4. Aggregate labor supply is normalized to one.

5. Once (3) and (4) are satisfied, Walras' law implies that total expenditure (government consumption, investment, and household consumption) must equal total output:

$$c_t^G + k_{t+1} - \frac{(1 - \delta_t)k_t}{\omega_t} + \int_i \int_a P_a^i c_{at}^i da di = U_t k_t^\alpha L_t^{1-\alpha} \frac{(1 + g)}{\omega_t}. \quad (\text{A.2})$$

6. The social security system is balanced at all times:

$$\int_i \int_{a \in I_W} \tau_s L_a^i w_t da di = \int_i \int_{a \in I_R} [\lambda \exp(f(a^R)) w_t P_{aR}^i] da di, \quad (\text{A.3})$$

where the left-hand side is integrated over all workers ( $a \in I_W$ ), while the right-hand side is integrated over retirees ( $a \in I_R$ ). This equation determines  $\tau_s$  for a given value of  $\lambda$ .

7. The government budget [equation (5)] is balanced every period to sustain a given ratio of government debt to GDP. Specifically

$$b_{t+1} = \frac{(1 + R_t^B)b_t}{\omega_t} + c_t^G - \frac{k_t R_t^K \tau_K}{\omega_t} - \frac{b_t R_t^B \tau_K}{\omega_t} - \frac{w_t(1 - \tau_s)\tau_L}{\omega_t}$$

8. Market prices expectations are verified in equilibrium.

### A.2 Solution method outline

The solution method builds on den Haan (1994, 1997), Krusell and Smith (1997, 1998), Storesletten et al. (2007) and Gomes and Michaelides (2008). We start by presenting the outer loop of the code and discuss the details afterwards.

The numerical sequence works as follows:

- i. Specify a set of forecasting equations ( $\Gamma_K$  and  $\Gamma_P$ ).
- ii. Solve the household's decision problem, taking prices as given, and using the forecasting equations to form expectations (details in A.3).
- iii. Given the policy functions, simulate the model (10100 periods) while computing the market clearing variables at each period (details in A.4).
- iv. Use the last 10000 periods to update the coefficients in the forecasting equations (details in A.5).
- v. Repeat steps 1, 2, 3, 4, with the new forecasting equations until convergence. We have two convergence criteria:
  - Stable coefficients in the forecasting equations.
  - Forecasting equations with regression  $R^2$  above 99.9%.

## A.3 Solving the household's decision problem

### A.3.1 Normalization

We first simplify the solution by exploiting the scale-independence of the maximization problem and rewriting all individual variables as ratios to the permanent component of labor income ( $P_a^i$ ) and of the deterministic growth ( $G_t^{\frac{1}{1-\alpha}}$ ). Likewise all aggregate variables (the wage and capital) are normalized by  $G_t^{\frac{1}{1-\alpha}}$  thus inducing stationarity in the model. Using lower case letters to denote the normalized variables we have, for instance

$$\begin{aligned}
 x_{at}^i &\equiv \frac{X_{at}^i}{P_a^i G_t^{\frac{1}{1-\alpha}}} \\
 k_{t+1} &\equiv \frac{K_{t+1}}{G_t^{\frac{1}{1-\alpha}}}, \\
 w_t &\equiv \frac{W_t}{G_{t-1}^{\frac{1}{1-\alpha}}}
 \end{aligned}$$

The equations of motion and the value function can then be rewritten as normalized variables, allowing us to reduce the number of state variables. The normalized individual cash on hand state variable follows

$$x_{ait} = \frac{k_{ait}(1 + R_t^K(1 - \tau_K))}{\omega_t \omega_a} + \frac{b_{ait}(1 + R_t^B(1 - \tau_K))}{\omega_t \omega_a} + w_t \lambda (1 - \tau_L)(1 - \tau_s) / \omega_t$$

where  $\omega_a = \frac{P_a^{i+1}}{P_a^i}$  and  $\omega_t = (1+g)^{\frac{1}{1-\alpha}}$ , and the value function becomes  $V_a(x_{at}^i; k_t, U_t, \eta_t, P_t^B)$ .

i. The rates of return on the factors of production can be written as

$$R_t^K = MP_K = \alpha Z_t \left( \frac{k_t G_{t-1}^{\frac{1}{1-\alpha}}}{L_t} \right)^{\alpha-1} - \delta_t = \alpha U_t k_t^{\alpha-1} L_t^{1-\alpha} (1+g) - \delta_t$$

and

$$W_t = (1-\alpha) Z_t (K_t/L_t)^\alpha = (1-\alpha) G_t U_t \left( \frac{k_t G_{t-1}^{\frac{1}{1-\alpha}}}{L_t} \right)^\alpha = (1+g) G_{t-1}^{\frac{1}{1-\alpha}} (1-\alpha) U_t \left( \frac{k_t}{L_t} \right)^\alpha$$

so that  $w_t = \frac{W_t}{G_{t-1}^{\frac{1}{1-\alpha}}} = (1-\alpha) U_t \left( \frac{k_t}{L_t} \right)^\alpha (1+g)$ .

### A.3.2 Discretization of the state space

Age ( $a$ ) is a discrete state variable taking 71 possible values. We discretize the cash-on-hand dimension ( $x_t^i$ ) using 51 points, with denser grids closer to zero to take into account the higher curvature of the value function in this region. The discrete aggregate state variables (the depreciation shock ( $\eta_t$ ) and the aggregate productivity shock ( $U_t$ )) each take only two possible values. With respect to the other two continuous aggregate state variables, we use an adaptable grid that takes into account the availability of high or low capital in the economy and allows higher accuracy with a fewer number of grid points. The grid is based on the idea that the expected conditional equity premium has to be positive and therefore the price of the bond is an increasing function of the available capital stock. This adaptive grid (as opposed to a fixed, rectangular grid) allows greater accuracy since it neglects points in the state space that, according to the economics of the problem, will never be visited conditional on being at a particular level of a capital stock at a given point in time. This is a guess and verify method and the simulated bond prices are confirmed ex post (after convergence) to lie within the specified range. Typically, the R-squared statistic from the bond regression is below 99.9% when the price of the bond hits the edges of this grid during the simulation. We use 15 points to discretize  $k_t$ , and 15 points to discretize  $P_t^B$ .

The grid range for the continuous state variables is verified ex-post by comparing with the values obtained in the simulations, and with the results obtained when this range is increased. A smaller number of grid points for  $k_t$  and for  $P_t^B$  would not affect the policy functions directly. It would, however, affect the R-squared of the forecasting equations and the convergence of their respective coefficients.

### A.3.3 Maximization

We solve the maximization problem for each agent type using backward induction. For every age  $a$  prior to  $A$ , and for each point in the state space, we optimize using grid search. We need to compute the value associated with each set of controls (consumption, and share of wealth invested in stocks). From the Bellman equation,

$$V_a(x_{at}^i; k_t, U_t, \eta_t, P_t^B) = \underset{\{k_{a+1,t+1}^i, b_{a+1,t+1}^i\}_{a=1}^A}{Max} \left\{ (1 - \beta)(c_{at}^i)^{1-1/\psi} \right. \quad (\text{A.4}) \\ \left. + \beta \left( E_t \left[ (\omega_{a+1}\omega_{t+1})^{1-\rho} p_a V_{a+1}^{1-\rho} (x_{a+1,t+1}^i; k_{t+1}, U_{t+1}, \eta_{t+1}, P_{t+1}^B) \right] \right)^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}}$$

these values are given as a weighted sum of current utility  $((c_{at}^i)^{1-1/\psi})$  and the expected continuation value  $(E_a V_{a+1}(\cdot))$ , which we can compute once we have obtained  $V_{a+1}$ . In the last period the policy functions are trivial and the value function corresponds to the indirect utility function. This gives us the terminal condition for our backward induction procedure. Once we have computed the value of all the alternatives we pick the maximum, thus obtaining the policy rules for the current period. Substituting these decision rules in the Bellman equation we obtain this period's value function  $(V_a(\cdot))$ , which is then used to solve the previous period's maximization problem. This process is iterated until  $a = 1$ .

We use the forecasting equations  $(\Gamma_K$  and  $\Gamma_P)$  to form expectations of the aggregate variables, and we perform all numerical integrations using Gaussian quadrature to approximate the distributions of the innovations to the labor income process  $(\varepsilon^i$  and  $\xi^i)$  and the aggregate shocks  $(\eta_t$  and  $U_t)$ . For points which do not lie on state space grid, we evaluate the value function using a cubic spline interpolation along the cash-on-hand dimension, and a bi-linear interpolation along the other two continuous state variables  $(k_t$  and  $P_t^B)$ . Bi-linear interpolation works well along these two dimensions because households are price takers, and therefore these state variables are not affected by the control variables.

## A.4 Simulating the model and clearing markets

### A.4.1 Simulation

We use the policy functions for the two agent types ( $A$  and  $B$ ) to simulate the behavior of 500 agents of each type in each of the 71 cohorts over 10500 periods. The realizations of the aggregate random variables (stochastic depreciation  $\eta_t$  and aggregate productivity  $U_t$ ) are drawn from their

original two-point distributions, while the idiosyncratic productivity shocks ( $\varepsilon^i$  and  $\xi^i$ ) are drawn from the corresponding log-normal distributions. All other random variables are endogenous to the model. The realizations of the exogenous random variables are held constant within the outer loop, i.e. across iterations, so as not to affect the convergence criteria.

#### A.4.2 Market clearing

For every time period we simulate the households' behavior for every possible bond price (i.e. every point in the grid for  $P_t^B$ ). We then aggregate the individual bond demands and use a linear interpolation to determine the market clearing bond price. All household equilibrium allocations (consumption and asset holdings) are then obtained from a linear interpolation with the same coefficients, while the aggregate variables (capital and output) are computed by aggregating these market clearing allocations. This then determines the state variables for simulating the next period's decisions.

### A.5 Updating the forecasting equations

Using the simulated time-series (after discarding the first 500 observations) we estimate the following OLS regressions, for every pair of productivity shock ( $U_{t+1}$ ) and depreciation shock ( $\eta_{t+1}$ ) realizations,

$$\ln(k_{t+1}) = q_{10} + q_{11} \ln(k_t) \tag{A.5}$$

and

$$\ln(P_{t+1}^B) = q_{20} + q_{21} \ln(k_t) + q_{22} \ln(P_t^B) \tag{A.6}$$

This gives us 8 equations and 8 sets of coefficients that forecast *state-contingent* capital ( $k_{t+1}$ ) and bond prices ( $P_{t+1}^B$ ). We iterate the code until we have converged on the coefficients and on the R-squared of these regressions. For the first set of equations (A.5) we obtain R-squared values around 99.99%. For the second set of equations (A.6), the R-squared values are in the 90% – 95% range when we only use  $\ln(k_t)$  as a regressor, increase to about 99.9% when we add  $\ln(P_t^B)$ .

## References

- Aiyagari, Rao S., and Ellen R. McGrattan, 1998, The optimum quantity of debt, *Journal of Monetary Economics* 42, 447–69.
- Aiyagari, S. Rao, 1994, Uninsured idiosyncratic risk and aggregate saving, *The Quarterly Journal of Economics* 109, 659–84.
- , 1995, Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting, *Journal of Political Economy* 103, 1158–1175.
- Alan, Sule, 2006, Entry costs and stock market participation over the life cycle, *The Review of Economic Dynamics* 9, 588–611.
- Basak, Suleyman, and Domenico Cuoco, 1998, An equilibrium model with restricted stock market participation, *The Review of Financial Studies* 11, 309–341.
- Baxter, Marianne, and Robert G. King, 1993, Fiscal policy in general equilibrium, *American Economic Review* 83, 315–334.
- Boldrin, Michele, Larry Christiano, and Jonas Fisher, 2001, Habit persistence, asset returns and the business cycle, *American Economic Review* 91, 149–166.
- Cagetti, Marco, and Mariacristina De Nardi, 2006, Entrepreneurship, frictions, and wealth, *Journal of Political Economy* 114, 835–70.
- Carey, David, and Josette Rabesona, 2002, Tax ratios on labour and capital income and on consumption, *OECD Economic Studies* 35.
- Carroll, Christopher D., 1992, The buffer-stock theory of savings: Some macroeconomic evidence, *Brooking Papers on Economic Activity* 2, 61–156.
- Chari, V.V., Lawrence Christiano, and Patrick Kehoe, 1994, Optimal fiscal policy in a business cycle model, *Journal of Political Economy* 102, 617–652.
- Cocco, João F., Francisco J. Gomes, and Pascal J. Maenhout, 2005, Consumption and portfolio choice over the life-cycle, *The Review of Financial Studies* 18, 491–534.
- Conesa, Juan Carlos, Sagiri Kitao, and Dirk Krueger, 2007, Taxing capital? Not a bad idea after all!, *working Paper, Universitat Autònoma de Barcelona*.

- Constantinides, George M., John B. Donaldson, and Rajnish Mehra, 2002, Junior can't borrow: A new perspective of the equity premium puzzle, *The Quarterly Journal of Economics* 117, 269–96.
- Croce, Mariano M., Howard Kung, Thien T. Nguyen, and Lukas Schmid, 2011, Fiscal policies and asset prices, *University of North Carolina, working paper*.
- Deaton, Angus, 1991, Saving and liquidity constraints, *Econometrica* 59(5), 1221–48.
- , and Christina Paxson, 1994, Intertemporal choice and inequality, *Journal of Political Economy* 102, 437–67.
- Domeij, David, and Jonathan Heathcote, 2004, On the distributional effects of reducing capital taxes, *International Economic Review* 45, 523–54.
- Dumas, Bernard, and Andrew Lyasoff, 2010, Incomplete-market equilibria solved recursively on an event tree, *working paper INSEAD*.
- Elmendorf, Douglas W., and Miles S. Kimball, 2000, Taxation of labor income and the demand for risky assets, *International Economic Review* 41, 801–832.
- Engen, Eric M., and R. Glenn Hubbard, 2004, Federal government debt and interest rates, in *NBER Macroeconomics Annual*, vol. 19 pp. 83–138. MIT Press.
- Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.
- Floden, Martin, 2001, The effectiveness of government debt and transfers as insurance, *Journal of Monetary Economics* 48, 81–108.
- Gomes, Francisco, and Alexander Michaelides, 2008, Asset pricing with limited risk sharing and heterogeneous agents, *The Review of Financial Studies* 21, 415–448.
- Gottardi, Piero, and Felix Kubler, 2004, Ex ante optimality and social security, *Working Paper, University of Venice and Stanford University*.
- Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman, 1988, Investment, capacity utilization and the business cycle, *American Economic Review* 78, 402–418.
- Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell, 1997, Long-run implications of investment-specific technological change, *American Economic Review* 87, 342–362.

- Greenwood, Robin, and Dimitri Vayanos, 2010, Price pressure in the government bond market, *American Economic Review* 100, 585–590.
- Grossman, Sanford, and Oliver Hart, 1979, A theory of competitive equilibrium in stock market economies, *Econometrica* 47, 293–329.
- Guibaud, Stephane, Yves Nosbusch, and Dimitri Vayanos, 2010, Bond market clienteles, the yield curve and the optimal maturity structure of government debt, *working paper*.
- Guvenen, Fatih, 2005, A parsimonious macroeconomic model for asset pricing: Habit formation or cross-sectional heterogeneity?, *UT Austin, Department of Economics working paper*.
- Haan, Wouter Den, 1994, Heterogeneity, aggregate uncertainty and the short-term interest rate, *Journal of Business and Economic Statistics* 14, 399–411.
- , 1997, Solving dynamic models with aggregate shocks and heterogenous agents, *Macroeconomic Dynamics* 1, 355–386.
- Heathcote, Jonathan, 2005, Fiscal policy with heterogeneous agents and incomplete markets, *The Review of Economic Studies* 72, 161–88.
- Hercowitz, Zvi, 1986, The real interest rate and aggregate supply, *Journal of Monetary Economics* 18, 121–145.
- Jermann, Urban, 1998, Asset pricing in production economies, *Journal of Monetary Economics* 41, 257–275.
- Krishnamurthi, Arvind, and Annette Vissing-Jørgensen, 2010, The aggregate demand for treasury debt, *working paper, Northwestern University*.
- , 2011, The effects of quantitative easing on interest rates, *working paper, Northwestern University*.
- Krueger, Dirk, and Felix Kubler, 2006, Pareto-improving social security reform when financial markets are incomplete, *American Economic Review* 96, 737–755.
- Krueger, Dirk, and Fabrizio Perri, 2006, Does income inequality lead to consumption inequality? Evidence and theory, *The Review of Economic Studies* 73 (1), 163–193.

- Krusell, Per, and Anthony A. Smith Jr., 1997, Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns, *Macroeconomic Dynamics* 1, 387–422.
- , 1998, Income and wealth heterogeneity in the macroeconomy, *Journal of Political Economy* 106, 867–96.
- Laubach, Thomas, 2008, New evidence on the interest rate effects of budget deficits and debt, *Journal of the European Economic Association* 7, 858–85.
- Ludvigson, Sidney, 1996, The macroeconomic effects of government debt in a stochastic growth model, *Journal of Monetary Economics* 38, 25–45.
- Malloy, Christopher, Tobias Moskowitz, and Annette Vissing-Jørgensen, 2009, Long-run stockholder consumption risk and asset returns, *Journal of Finance* 64, 2427–79.
- Mendoza, Enrique G., Assaf Razin, and Linda L. Tesar, 1994, Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption, *Journal of Monetary Economics* 34, 297–323.
- Pastor, Lubos, and Pietro Veronesi, 2010, Uncertainty about government policy and stock prices, *University of Chicago working paper*.
- Shin, Yongseok, 2006, Ramsey meets bewley: Optimal government financing with incomplete markets, *working paper, University of Wisconsin*.
- Storesletten, Kjetil, Chris Telmer, and Amir Yaron, 2007, Asset pricing with idiosyncratic risk and overlapping generations, *The Review of Economic Dynamics* 10, 519–48.
- Trabandt, Mathias, and Harald Uhlig, 2007, How far are we from the slippery slope? The laffer curve revisited, *Working Paper, Humboldt University Berlin*.
- Vissing-Jørgensen, Annette, 2002, Limited asset market participation and the elasticity of intertemporal substitution, *Journal of Political Economy* 110, 825–53.

Table 1: Panel A reports values from the two models with infinitely lived agents and aggregate U.S. data from the BEA National Accounts (1929-2007). Following Castaneda et al. (2003) we classify 75 percent of durable consumption expenditures as investment and the remainder 25 percent as consumption. Model I (II) is without (with) aggregate shocks and are recalibrated through the discount factor and idiosyncratic uncertainty to have the same K/Y ratio keeping all other parameters the same. Government debt is the U.S. federal debt held by the public between 1952 and 2002. Output excludes net exports. Panel B reports the standard deviation of consumption and output (NIPA tables from the BEA, 1929-2007).

Panel A: Share of Output (percent)			
	Model I	Model II	Data
Consumption	60.8	58.7	59.4
Investment	19.8	20.9	20.2
Government	19.4	20.4	20.4
Government Debt	35.6	35.9	35.8
Panel B: Standard deviation of growth rates (percent)			
	Model I	Model II	Data
Aggregate Output	0.0	4.2	3.8
Aggregate Consumption	0.0	2.8	3.3

Table 2: Asset returns from the data (CRSP) and models with infinitely lived agents. Model I (II) is without (with) aggregate shocks and are recalibrated through the discount factor and idiosyncratic uncertainty to have the same K/Y ratio keeping all other parameters the same.

Variable	Moment	Model I	Model II	Data
$r_f$	Mean	6.59	4.89	1.23
	Std. Dev.	0.0	2.50	3.83
$r_m$	Mean	6.59	8.01	7.77
	Std. Dev.	0.0	15.32	20.11
$r_m - r_f$	Mean	0.0	3.12	6.54

Table 3: Comparative statics for changes in taxes in the model with infinitely lived agents without aggregate shocks. The Table shows long-run averages of the variables in the baseline model and in the identical models but with permanently higher taxes. Changes are reported in percentages relative to the baseline case (except return changes which are in percentage points).

	Baseline $\tau_L = 10\%$ $\tau_K = 40\%$	Higher $\tau_L$ $\tau_L = 12.5\%$	Change (%)	Higher $\tau_K$ $\tau_K = 42.5\%$	Change (%)
$K$	3.62	3.57	-1.28	3.51	-3.09
$B/Y$	0.36	0.85	137.4	0.46	28.64
$K/Y$	2.34	2.32	-0.85	2.29	-2.05
$C/Y$	0.61	0.61	0.23	0.61	0.42
$I/Y$	0.20	0.20	-0.85	0.19	-2.05
$G/Y$	0.19	0.19	-0.24	0.19	1.50
$R^K = R^B$ , (% <i>pts</i> )	6.59	6.72	0.12	6.90	0.31

Table 4: Comparative statics for changes in taxes in the model with infinitely lived agents with aggregate shocks. The Table shows long-run averages of the variables in the baseline model and in the identical models but with permanently higher taxes. Changes are reported in percentages relative to the baseline case (except return changes which are in percentage points).

	Baseline $\tau_L = 10\%$ $\tau_K = 40\%$	Higher $\tau_L$ $\tau_L = 12.5\%$	Change (%)	Higher $\tau_K$ $\tau_K = 42.5\%$	Change (%)
$K$	3.46	3.15	-8.74	3.26	-5.72
$B/Y$	0.359	0.840	133.63	0.478	32.90
$K/Y$	2.256	2.126	-5.74	2.172	-3.74
$C/Y$	0.587	0.593	1.06	0.591	0.66
$I/Y$	0.209	0.197	-5.89	0.201	-3.83
$G/Y$	0.204	0.210	2.98	0.208	2.01
$R^B$ (% <i>pts</i> )	4.89	6.44	1.55	5.66	0.77
$R^K$ (% <i>pts</i> )	8.01	8.93	0.92	8.57	0.56
$R^K - R_B$ (% <i>pts</i> )	3.12	2.49	-0.63	2.91	-0.21

Table 5: Table reports tax revenue (in percent of output) by source from the baseline model and aggregate U.S. data from the BEA National accounts.

	Model	Data (1929-2006)	Data (2006)
Capital	5.41	5.26	5.78
Labor	8.00	6.80	8.71
Consumption	7.64	8.53	7.77

Table 6: Panel A reports values from the baseline model and aggregate U.S. data from the BEA National Accounts (1929-2007). Following Castaneda et al. (2003) we classify 75 percent of durable consumption expenditures as investment and the remaining 25 percent as consumption. Government debt is the U.S. federal debt held by the public between 1952 and 2002. Output excludes net exports. Panel B reports the standard deviation of consumption and output (NIPA tables from the BEA, 1929-2007). Panel B also reports the standard deviation of stockholders' and non-stockholders' consumption growth rates from the baseline model and from the data. We use the values of consumption growth volatilities reported by Malloy, Moskowitz and Vissing-Jørgensen (2009) of 1.4% and 3.6% for non-stockholders and stockholders (from CEX data) and scale them by the ratio of the aggregate consumption volatilities from the CEX sample (1.7%) and the longer aggregate sample from 1929 to 2007 (3.3%).

Panel A: Share of Output (percent)		
	Model	Data
Consumption	58.6	59.4
Investment	20.9	20.2
Government	20.5	20.4
Government Debt	35.6	35.8
Panel B: Standard deviation of growth rates (percent)		
	Model	Data
Aggregate Output	4.2	3.8
Aggregate Consumption	3.4	3.3
Stockholders Consumption	5.0	6.9
Non-Stockholders Consumption	3.4	2.7

Table 7: Unconditional asset pricing moments from the data (CRSP) and baseline model.

Variable	Moment	Model	Data
$r_f$	Mean	1.72	1.23
	Std. Dev.	1.51	3.83
$r_m$	Mean	5.33	7.77
	Std. Dev.	15.18	20.11
$r_m - r_f$	Mean	3.61	6.54

Table 8: Wealth Distribution. The table reports the percentage of each group’s total wealth held within a given percentile range. Data source: 2001 Survey of Consumer Finances. Wealth is the net worth of households as defined in the text and stockholders are defined as households who own stocks directly or through mutual funds either in taxable accounts or in tax-deferred pension plans. Figures from model are averages over the last 200 simulations.

Percentile	Non-stockholders		Stockholders		All	
	Model	Data	Model	Data	Model	Data
0-20	0.00	-1.72	3.30	0.40	0.00	-0.310
20-50	0.55	1.57	17.97	5.59	0.13	2.80
50-80	6.99	18.18	36.67	16.62	31.49	14.57
80-100	92.47	80.24	42.07	77.78	68.39	82.64

Table 9: Tax rates comparative statics for fixed level of government expenditures. The Table shows long-run averages of the variables in the baseline model and in the model with permanently changed tax rates. Changes are reported in percentages relative to the baseline case (except return and growth rate volatility changes which are in percentage points).  $\sigma(\Delta LnC^A)$ ,  $\sigma(\Delta LnC^{NS})$ ,  $\sigma(\Delta LnC^S)$  and  $\sigma(\Delta LnY)$  denote, respectively, the volatilities of log growth for aggregate, non-stockholders and stockholders consumption and aggregate output.

	Baseline $\tau_L = 10\%$ $\tau_K = 40\%$	Higher $\tau_K$ $\tau_K = 42.5\%$ (2)	Change (%) (3)
$\sigma(\Delta LnC^A)(\%pts)$	3.38	3.17	-0.21
$\sigma(\Delta LnC^{NS})(\%pts)$	3.42	3.34	-0.08
$\sigma(\Delta LnC^S)(\%pts)$	5.01	4.62	-0.39
$\sigma(\Delta LnY)(\%pts)$	4.18	4.09	-0.08
$K$	4.67	4.49	-3.85
$K/Y$	2.680	2.612	-2.53
$C/Y$	0.586	0.589	0.50
$I/Y$	0.209	0.203	-2.69
$G/Y$	0.205	0.207	1.31
$B/Y$	0.356	0.477	33.93
$r_m$ ( $\%pts$ )	5.33	5.65	0.32
$r_f$ ( $\%pts$ )	1.72	2.13	0.41
$r_m - r_f$ ( $\%pts$ )	3.61	3.51	-0.10

Table 10: Tax-financed debt increase. The table shows long-run averages of the variables in the baseline model and in the model with permanently higher government debt. Changes are reported in percentages relative to the baseline case (except return and growth rate volatility changes which are in percentage points).  $\sigma(\Delta \log C^A)$ ,  $\sigma(\Delta \log C^{NS})$ ,  $\sigma(\Delta \log C^S)$  denote, respectively, the volatilities of log growth for aggregate, non-stockholders and stockholders consumption.

Variable	Baseline	Debt financed by lump-sum tax	Change (%)	Debt financed by capital tax	Change (%)
	(1)	(2)	(3)	(4)	(5)
$\sigma(\Delta \log C^A)$ (%pts)	3.38	3.34	-0.04	3.24	-0.14
$\sigma(\Delta \log C^{NS})$ (%pts)	3.42	3.39	-0.03	3.37	-0.05
$\sigma(\Delta \log C^S)$ (%pts)	5.01	4.92	-0.08	4.74	-0.27
$\sigma(\Delta \log Y)$ (%pts)	4.18	4.18	0.00	4.13	-0.05
$K$	4.67	4.59	-1.68	4.56	-2.35
$K/Y$	2.680	2.650	-1.11	2.639	-1.54
$C/Y$	0.586	0.588	0.20	0.588	0.30
$I/Y$	0.209	0.207	-1.12	0.205	-1.64
$G/Y$	0.205	0.206	0.58	0.206	0.81
$B/Y$	0.356	0.430	20.69	0.430	20.95
$r_m$ (%pts)	5.33	5.47	0.14	5.52	0.19
$r_f$ (%pts)	1.72	1.95	0.23	1.98	0.26
$r_m - r_f$ (%pts)	3.61	3.52	-0.09	3.54	-0.07

Table 11: Tax-financed debt increase with changing return volatility. The table shows long-run averages of the variables in the baseline model and in the model with permanently higher government debt, and compensating higher capital income taxation, for different scenarios about the impact of stock return volatility (unchanged, as in the previous table, and plus or minus 2 percentage points). Changes are reported in percentages relative to the baseline case (except return and growth rate volatility changes which are in percentage points).  $\sigma(\Delta \log C^A)$ ,  $\sigma(\Delta \log C^{NS})$ ,  $\sigma(\Delta \log C^S)$  denote, respectively, the volatilities of log growth for aggregate, non-stockholders and stockholders consumption.

Variable	s=0.13	Change	s=0.15	Change	s=0.17	Change
	(1)	(%) (2)	(3)	(%) (4)	(5)	(%) (6)
$\sigma(\Delta \log C^A)$ (%pts)	2.88	-0.51	3.24	-0.14	3.67	0.29
$\sigma(\Delta \log C^{NS})$ (%pts)	3.19	-0.23	3.37	-0.05	3.57	0.15
$\sigma(\Delta \log C^S)$ (%pts)	4.11	-0.90	4.74	-0.27	5.47	0.46
$\sigma(\Delta \log Y)$ (%pts)	3.86	-0.31	4.13	-0.05	4.42	0.25
$K$	4.55	-2.66	4.56	-2.35	4.56	-2.28
$K/Y$	2.645	-1.63	2.639	-1.54	2.636	-1.63
$C/Y$	0.589	0.37	0.588	0.30	0.588	0.25
$I/Y$	0.205	-1.69	0.205	-1.64	0.205	-1.66
$G/Y$	0.206	0.67	0.206	0.81	0.207	0.99
$B/Y$	0.430	20.94	0.430	20.95	0.431	21.09
$r_m$ (%pts)	5.42	0.09	5.52	0.19	5.65	0.33
$r_f$ (%pts)	2.36	0.64	1.98	0.26	1.57	-0.14
$r_m - r_f$ (%pts)	3.06	-0.55	3.54	-0.07	4.08	0.47

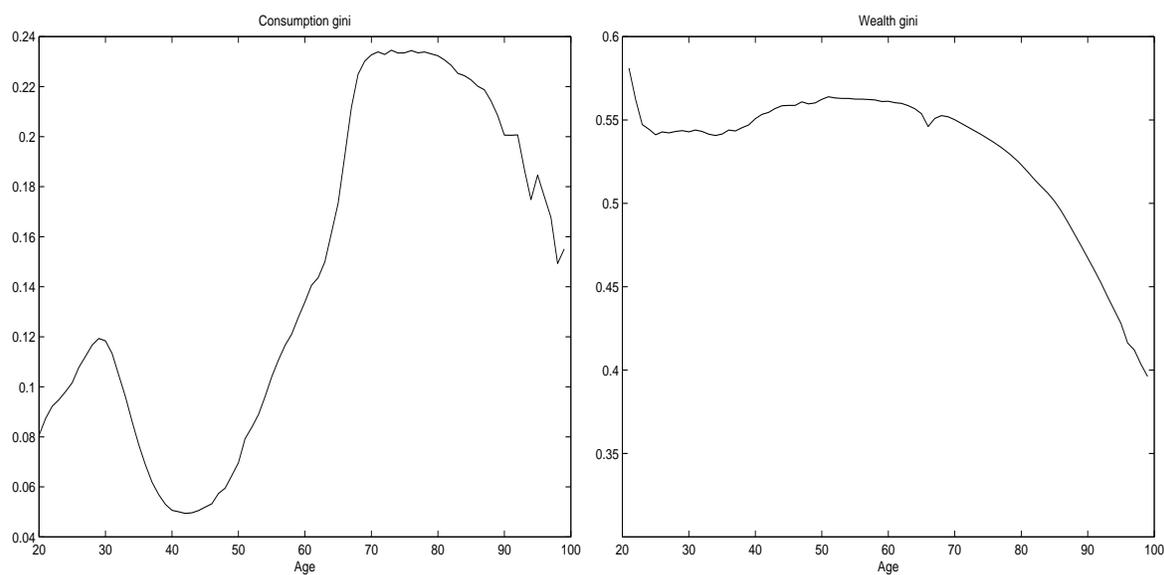


Figure 1: Consumption and wealth inequality over the life-cycle. The figure shows the cross-sectional gini coefficients for consumption (left panel) and wealth (right panel).

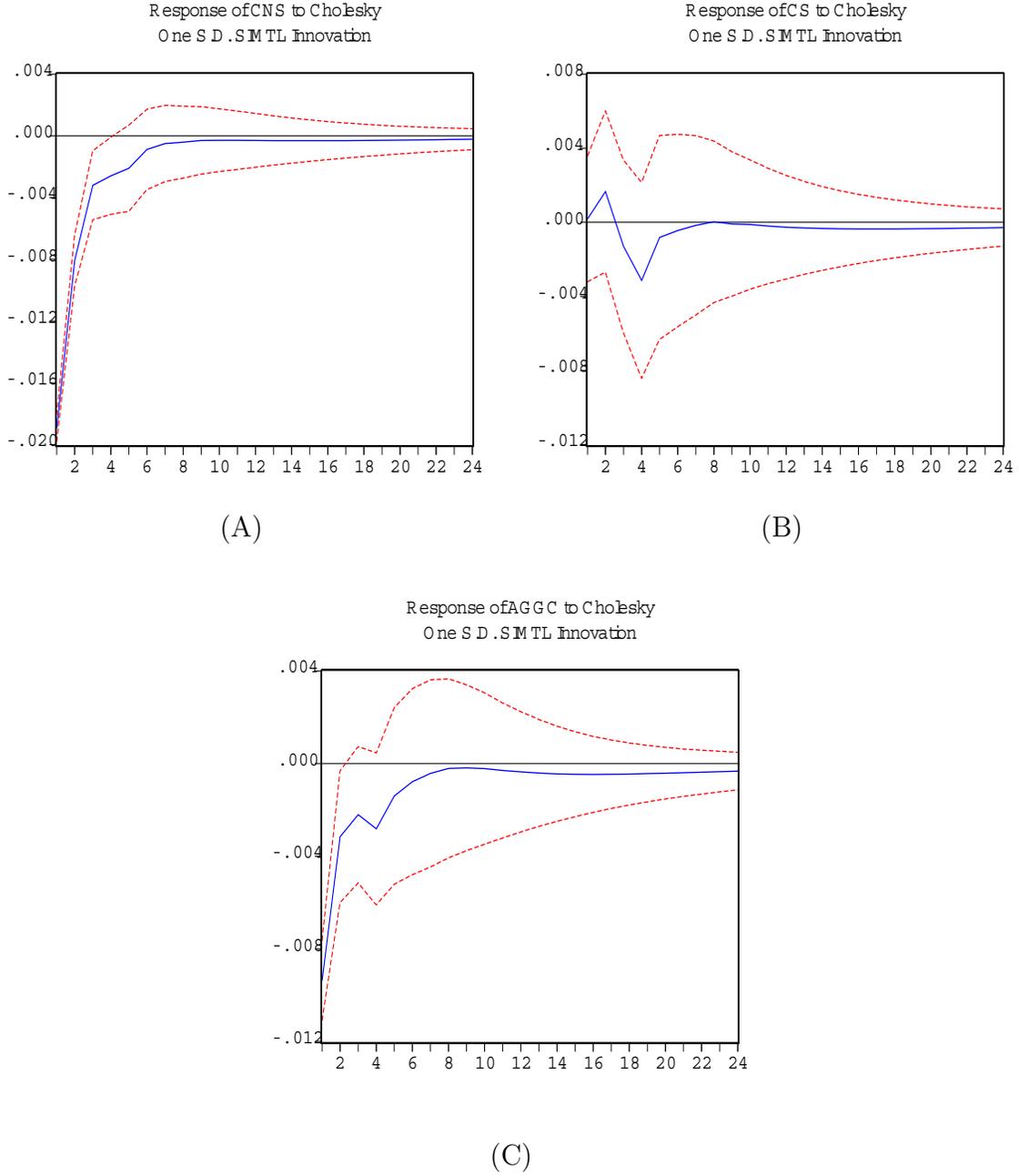


Figure 2: Consumption response to transitory income tax changes. Figures show impulse responses of aggregate (C), stockholders (B) and non-stockholders (A) consumption to transitory shocks of labor income tax rate.